Two New Types of Candidate Symbol Sorting Schemes for Complexity Reduction of a Sphere Decoder

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ABSTRACT

The computational complexity of a sphere decoder (SD) is conventionally reduced by decoding order scheme which sorts candidate symbols in the ascending order of the Euclidean distance from the output of a zero-forcing (ZF) receiver. However, since the ZF output may not be a reliable sorting reference, we propose two types of sorting schemes to allow faster decoding. The first is to use the newly found lattice points in the previous search round instead of the ZF output (Type I). Since these lattice points are closer to the received signal than the ZF output, they can serve as a more reliable sorting reference for finding the maximum likelihood (ML) solution. The second sorting scheme is to sort candidate symbols in descending order according to the number of candidate symbols in the following layer, which are called child symbols (Type II). These two proposed sorting schemes can be combined with layer sorting for more complexity reduction. Through simulation, the Type I and Type II sorting schemes were found to provide 12% and 20% complexity reduction respectively over conventional sorting schemes. When they are combined with layer sorting, Type I and Type II provide an additional 10-15% complexity reduction while maintaining detection performance.

Key Words : Sphere Decoder, Candidate Symbol, Zero-Forcing(ZF)

Ⅰ. Introduction

Sphere decoders have been gaining attention due to their moderate computational complexity while achieving ML performance [1]-[3]. Unlike ML, an SD accesses only the symbols within the predefined sphere, in which it searches for the ML solution based on a depth-first search strategy. An SD represents the symbols over tree representation through QR decomposition of the channel matrix and performs a recursively bounded layer search. We refer to the symbols within the upper and lower bounds as candidate symbols. Once one of the candidates is accessed, the search goes through the following layer. If there is no candidate symbol in the following layer, the SD accesses another candidate symbol back in the previous layer and then continues the search in the following layer again.

The earlier access of a reliable candidate symbol is therefore critical to speed up decoding time, leading to the reduction of computation complexity. To reduce the complexity of an SD, a commonly used scheme is to order the candidate symbols [4]-[7], which alleviates the resultant complexity by giving priority of access to the candidate symbol which has the shortest Euclidean distance from the output of ZF. However, the sorting criteria based on the ZF output may not be reliable enough to search for the ML solution. In this paper, we propose two types of sorting schemes which allow the SD to expedite finding of the ML solution. The first is to use the lattice vector found in the previous search round as the

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논문번호 : KICS2007-05-241, 접수일자 : 2007년 5월 31일, 최종논문접수일자 : 2007년 8월 30일

reference vector for next search round (Type I). This updated lattice vector is closer to the received signal vector than the ZF output, leading to a more reliable reference symbol vector for finding the ML solution in the following round. The second is to sort the candidate symbols in descending order of the multiplicity of candidate symbols that belong to the following layer, which are called child symbols (Type II).

Since an SD searches over the tree representation of the signal space based on a depth-first search strategy, reliable accessing in the upper layer is critical to speed up processing. If the upper layer symbol is incorrect, the following search in the corresponding sub-tree will be in vain, resulting in a longer search time to find the final detection. The sorted QR decomposition (SQRD) [8] can be applied in layer sorting of an SD while doing the proposed symbol sorting schemes. SQRD introduces negligible extra complexity and has been already shown to be effective in a layer structure detection receiver such as V-BLAST. Through simulation, it is observed that Type I and Type II sorting schemes provide 12% and 20% complexity reduction respectively over conventional sorting schemes. When they are combined with layer sorting, they provide 35% and 40% complexity reduction over conventional sorting.

The remainder of this paper is organized as follows. In Section II, the MIMO system model and sphere decoding algorithm are described. In Section III, the proposed candidate symbol sorting schemes and layer sorting are presented. The simulation results are shown in Section IV, and the conclusion in Section V.

II. System Model and Sphere Decoder Algorithm

2.1 MIMO System Model

The MIMO system with N_t transmit antennas and N_r receive antennas is considered, where $N_r \geq N_t$. The received signal vector is given by

$$\tilde{\mathbf{r}} = \widetilde{\mathbf{M}}\tilde{\mathbf{u}} + \tilde{\mathbf{w}}, \tag{1}$$

where $\widetilde{\mathbf{M}}$ is an $N_r \times N_t$ flat fading channel matrix and $\tilde{\mathbf{w}}$ is a complex Gaussian noise vector of which elements have zero means and variance of $2\sigma^2$. $\tilde{\mathbf{u}} = [\widetilde{u_1}, \widetilde{u_2}, \ldots, \widetilde{u_{N_t}}]$ is the transmit signal vector. It is assumed each element of $\tilde{\mathbf{u}}$ is independently drawn from a complex constellation such as QAM. This complex system can be transformed to equivalent real one as

$$\boldsymbol{r} = \boldsymbol{Mu} + \boldsymbol{w}, \tag{2}$$

where

$$\mathbf{r} = [Re(\tilde{\mathbf{r}})^T \quad Im(\tilde{\mathbf{r}})^T]^T, \tag{3}$$

$$\mathbf{M} = \begin{bmatrix} Re(\widetilde{\mathbf{M}})^T & -Im(\widetilde{\mathbf{M}})^T \\ Im(\widetilde{\mathbf{M}})^T & Re(\widetilde{\mathbf{M}})^T \end{bmatrix}, \tag{4}$$

$$\mathbf{u} = [Re(\tilde{\mathbf{u}})^T \quad Im(\tilde{\mathbf{u}})^T]^T. \tag{5}$$

$$\mathbf{w} = [Re(\tilde{\mathbf{w}})^T \quad Im(\tilde{\mathbf{w}})^T]^T. \tag{6}$$

SD searches those of only lattice points which are inside the hyper-sphere with initial radius C given by

$$\| \mathbf{r} - \mathbf{M}\mathbf{u} \|^2 < C^2. \tag{7}$$

2.2 Sphere decoder algorithm

The original sphere decoder algorithm for MIMO systems[2] is described as follows. Assuming $2N_r = 2N_t = n$, (6) can be written as

$$\begin{aligned} &\| \mathbf{r} - \mathbf{M}\mathbf{u} \|^2 \\ &= \boldsymbol{\xi}\mathbf{M}\mathbf{M}^T\boldsymbol{\xi}^T = \boldsymbol{\xi}\mathbf{R}^T\mathbf{R}\boldsymbol{\xi}^T = \| \mathbf{R}\boldsymbol{\xi}^T \|^2 \\ &= \sum_{i=1}^{n} q_{ii}\left(\xi_i + \sum_{j=i+1}^{n} q_{ij}\xi_j\right)^2 \leq C^2 \end{aligned} \tag{8}$$

where

$$\boldsymbol{\xi} = \boldsymbol{\rho} - \mathbf{u}, \tag{9}$$

$$\boldsymbol{\rho} = \mathbf{M}^{-1}\mathbf{r}, \tag{10}$$

$$q_{ii} = r_{ii}^2 \quad \text{for } i = 1, ..., n \ , \tag{11}$$

$$q_{ij} = r_{ij} / r_{ii} \quad \text{for } j = i+1, ..., n, \tag{12}$$

and $\mathbf{R}$ is an upper triangular matrix obtained form Cholesky's decomposition of $\mathbf{M}^T\mathbf{M}$. In (8), the SD finds the lattice vector inside the sphere with its radius C by performing the bounded layer search as

$$\left\lceil -\sqrt{\frac{T_i}{q_{ii}}} + S_i \right\rceil \le u_i \le \left\lfloor \sqrt{\frac{T_i}{q_{ii}}} + S_i \right\rfloor, \tag{13}$$

where

$$S_i = \rho_i + \sum_{l=i+1}^{n} q_{il}\xi_l, \tag{14}$$

$$T_i = R^2 - \sum_{l=i+1}^{n} q_{ll}\left(\xi_l + \sum_{j=l+1}^{n} q_{lj}\xi_j\right)^2. \tag{15}$$

The bounds in (13) can be updated recursively by using the following equations:

$$T_{i-1} = T_i - q_{ii}(S_i - u_i)^2. \tag{16}$$

When a vector inside the sphere is found, its squared distance from the center (the received points) is given by

$$\hat{d}^2 = C^2 - T_1 + q_{11}(S_1 - u_1)^2. \tag{17}$$

This value is compared with the minimum squared distance (initially set equal to C) found so far in the search. If it is smaller C then we have a new candidate lattice vector. The search continues until all the vectors inside the sphere are tested. For more detail, the reader is referred to [2].

Ⅲ. Two New Two Types of Candidate Symbols Sorting Schemes and Layer Sorting

3.1 Conventional candidate symbol sorting scheme

The conventional method of candidate symbol sorting [6] arranges candidate symbols in the i th layer in ascending order of the Euclidean distance to ZF output in (10). The SD first visits the candidate symbol which is closer to the ρ_i in Euclidean distance. For example, Figure 1 shows that when the black dot is ρ_i and the circles are candidate symbols in the i th layer, the SD visits them in the order of the number in the circle. The ZF output, however, may not be a reliable reference for finding the ML solution, since the ZF solution is not close to the ML solution. In the next subsection, a modified sorting scheme is proposed that provides faster accessing of the ML solution.

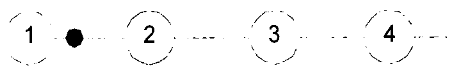

Fig. 1 Example of a conventional candidate symbol sorting scheme

Throughout this paper, the ZF output ρ_i is set for the initial reference point. However, the initial reference ρ_i can be set from other linear or non-linear receivers such as MMSE or SIC for more reliability [7].

3.2 Type 1: Reference update sorting

We propose an updated lattice vector sorting scheme. If the SD finds a lattice vector inside of sphere within the initial radius, the search radius is reduced to (17) The SD starts the search process all over again with this new smaller radius to find any better lattice vector. The proposed sorting scheme uses the lattice vector found in the previous search round as a reference vector for the next search round. This update of the reference using the lattice vector found in the previous search round is expected to help expedite the finding of the ML solution. An example is presented in Fig.2.

Fig.2 shows an example of an update of the reference, where the radius of the SD is reduced through four search rounds and the reference vector is updated as follows:

$$\rho \rightarrow \hat{\mathbf{u}}_1 \rightarrow \hat{\mathbf{u}}_2 \rightarrow \hat{\mathbf{u}}_3. \tag{18}$$

where $\hat{\mathbf{u}}_i$ is the lattice vector founded in the i th search round. In (13), the candidate symbols in the i th layer are sorted in ascending order based on their Euclidean distances from the i th element of the updated reference lattice vector as follows:

$$\left| u_i - \hat{u}_{i,k} \right|, \tag{19}$$

where k is the number of the search round.

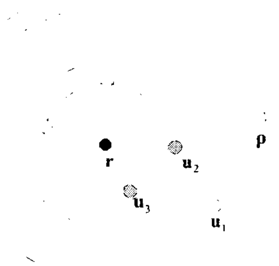

Fig. 2 Example of the type I sorting scheme

3.3 Type II- Child multiplicity sorting

The sphere decoder performs a recursive bounded search over the tree representation of the signal space based on a depth-first search strategy. In (13) and (16) the search bound in the $i-1$th layer is dependent on the candidate symbol selected in the i th layer. However, the conventional sorting scheme does not consider this recursive characteristic in bound. It is just ordering the candidate symbols in the i th layer according to their Euclidean distance from the corresponding output of the ZF receiver. To introduce the recursive characteristic in the search bound, the proposed sorting scheme sorts the candidate symbols in descending order based on the number of candidate symbols that belong to the following layers, which we call child symbols.

In (16), T_{i-1} is determined by the previously visited candidate symbol u_i, and T_{i-1} determines the bound for the candidate symbol in the $i-1$ th layer as (13). The proposed sorting scheme gives accessing priority to the candidate symbol u_i that increases the value of T_{i-1} so that the bound in the $i-1$th layer has more candidate symbols. Therefore, the candidate symbols u_i are sorted in the ascending order of the metric,

$$(S_i - u_i)^2 \tag{20}$$

Fig.3 shows an example where the numbers in the circle are in accessing order.

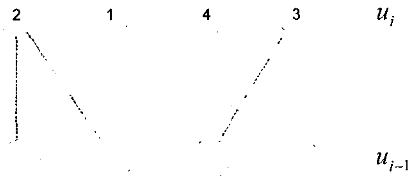

Fig. 3 Example of Type II scheme

Fig.4 is the flowchart of the sphere decoder employing the proposed sorting schemes. For further explanation of the flowchart, the reader is referred to [4]. In the flowchart, the function "*sort_Type_I*($\mathbf{y}_i, \hat{u}_i, \rho_i$)" and *sort_Type_II*($\mathbf{y}_i, S_i$) sort the candidate symbols in ascending order according to (19) and (20) respectively.

3.4 Layer sorting scheme

In (16) the previously visited candidate symbol determines the bound for the candidate symbol in the next layer. The reliable accessing of candidate symbols in the upper layer is therefore a critical problem, since incorrect accessing in the upper layer results in an unnecessary search in the sub-layer. In the V-BLAST detection scheme, the sorted QR decomposition (SQRD) is used to alleviate this problem [8]. It is known that SQRD also can be applied to sphere decoding.

The SQRD finds the permutation of $\mathbf{M}$ that maximizes diagonal elements r_{ii} running from n to 1. As shown in (11), this leads to a bigger value of q_{ii} in the upper layer. Consequently, the upper bound has reduced the number of candidate symbols. SQRD introduces negligible extra com-

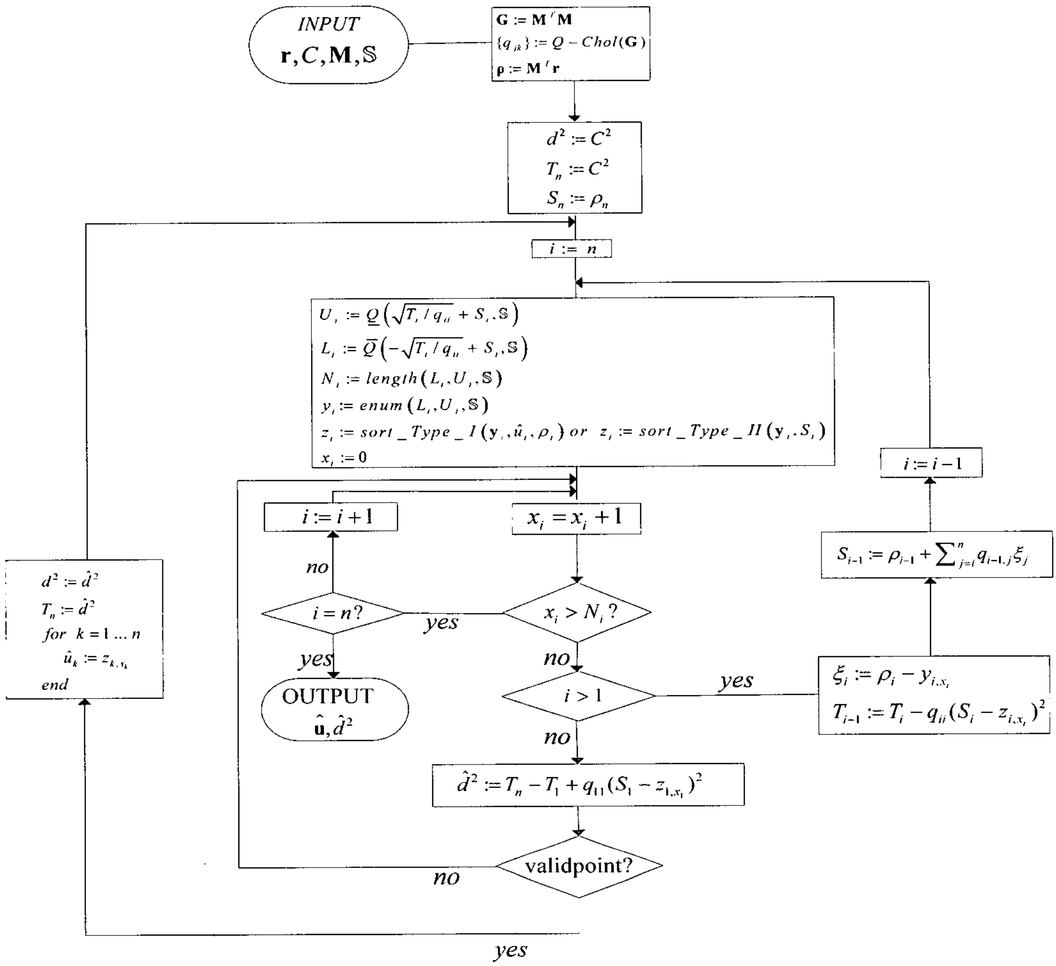

Fig. 4 A flowchart of the sphere decoder employing the proposed sorting schemes

plexity and can be combined with the proposed candidate symbol sorting scheme. The SQRD algorithm is given as follows:

(a) $\mathbf{R} = \mathbf{0}$, $\mathbf{Q} = \mathbf{M}$, $S = (1, \ldots, n)$
(b) for $i = 1, \ldots, n$
(c) $\quad k_i = \arg\min_{l = i, \ldots, n} \| \mathbf{q}_l \|^2$
(d) $\quad$ Exchange col. i and k_i in $\mathbf{Q}$, $\mathbf{R}$ and S
(e) $\quad r_{ii} = \|q_i\|$
(f) $\quad \mathbf{q}_i = \mathbf{q}_i / r_{i,i}$
(g) $\quad$ for $l = i+1, \ldots, n$
(h) $\qquad r_{i,l} = \mathbf{q}_i^H \cdot \mathbf{q}_l$
(i) $\qquad \mathbf{q}_l = \mathbf{q}_l - r_{i,l} \cdot \mathbf{q}_i$
(j) $\quad$ end
(k) end

Ⅳ. Simulation Results

A MIMO system with four transmit and receive antennas is considered, where uncoded 16QAM symbols are transmitted. The initial radius is $C^2 = \alpha m \sigma^2$, where $m = 2n$ and α satisfies following [3],

$$\int_0^{\alpha m} \frac{\lambda^{m/2-1}}{\Gamma(m/2)} e^{-\lambda} d\lambda = 0.99. \qquad (21)$$

Fig. 5 shows the computational complexity of the updated reference symbols sorting scheme (Type I). The measure of complexity is the average number of floating pointing operations generated by 10^4 transmit symbol vectors. It is observed that the proposed Type I candidate symbol sorting scheme reduces the complexity by about 12% over a conventional sorting scheme. Layer sorting provides an additional 10% complexity reduction. Fig. 6 compares candidate symbol sorting by child symbol (Type II) to conventional sorting. It shows that the Type II sorting scheme provides a reduction in complexity of around 20% in the operating signal to noise ratio (SNR) region. Fig. 7 shows a comparison of complexity between the two proposed candidate symbol sorting schemes. It shows that sorting by child symbol results in greater complexity reduction compared with the update reference sorting scheme. Fig. 8 compares the SER performance of an original sphere decoder with a sphere decoder using the proposed candidate symbol sorting schemes combined with layer sorting. The SER performance of the sphere decoder using the proposed candidate symbol sorting schemes is as good as the original sphere decoder. Therefore, the proposed sorting schemes reduce computational complexity while maintaining detection performance.

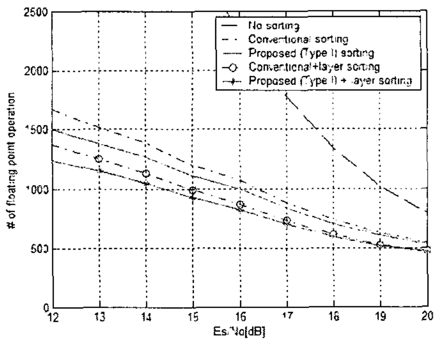

Fig. 5 Comparison of complexity between Proposed Type I sorting and conventional sorting

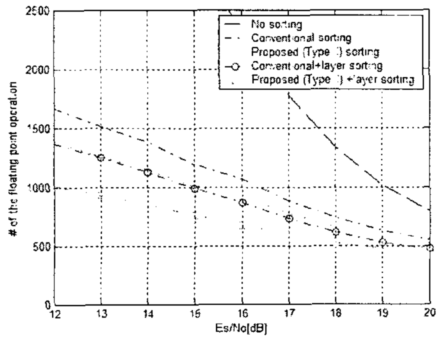

Fig. 6 Comparison of complexity between Proposed Type II sorting and conventional sorting

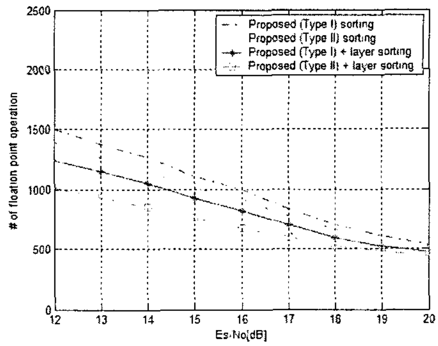

Fig. 7 Comparison of complexity between Proposed Type I sorting and Type II sorting

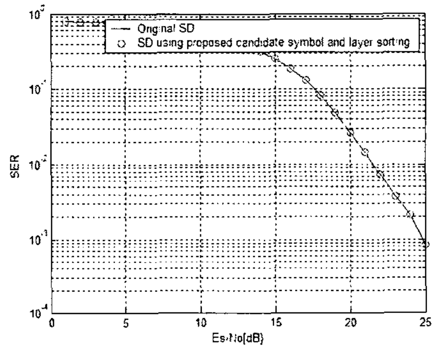

Fig. 8 Comparison of SER performance

V. Conclution

We proposed two types of candidate symbol sorting schemes combined with layer sorting which significantly reduce complexity compared with a conventional sorting scheme.

One of the proposed sorting schemes replaces ZF signal points with newly found lattice points in the previous search round for the reference of the next search round. The other scheme sorts candidate symbols of the current layer in descending order of the multiplicity of their child symbols, which are inside the radius of the following layer. We combine the two proposed candidate symbol sorting schemes with SQRD layer sorting in order to allow for more complexity reduction. Through a simulation, it was observed that sorting by updated reference symbol provides around 12% complexity reduction compared with conventional sorting schemes in the range of the practical operating SNR region. Sorting by child symbol results in around 20% complexity reduction over conventional sorting.

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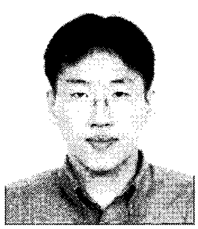

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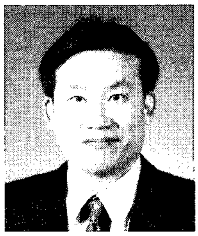

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