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Fuzzy (r, s)-semi-preopen sets and fuzzy (r, s)-semi-precontinuous maps

Seok Jong Lee and Jin Tae Kim

Department of Mathematics, Chungbuk National University, Cheongju 361-763, Korea

Abstract

In this paper, we introduce the concepts of fuzzy (r, s)-semi-preopen sets and fuzzy (r, s)-semi-precontinuous mappings on intuitionistic fuzzy topological spaces in Šostak's sense. The relations among fuzzy (r, s)-semicontinuous, fuzzy (r, s)-precontinuous, and fuzzy (r, s)-semi-precontinuous mappings are discussed. The concepts of fuzzy (r, s)-semi-preinterior, fuzzy (r, s)-semi-preclosure, fuzzy (r, s)-semi-preneighborhood, and fuzzy (r, s)-quasi-semi-preneighborhood are given. Using these concepts, the characterization for the fuzzy (r, s)-semi-precontinuous mapping is obtained. Also, we introduce the notions of fuzzy (r, s)-semi-preopen and fuzzy (r, s)-semi-preclosed mappings on intuitionistic fuzzy topological spaces in Šostak's sense, and then we investigate some of their characteristic properties.

Key words : fuzzy (r, s)-semi-preopen set, fuzzy (r, s)-semi-precontinuous mapping, fuzzy (r, s)-semi-preopen mapping, fuzzy (r, s)-semi-preclosed mapping

1. Introduction

The concept of fuzzy topological spaces was introduced by Chang [2]. These spaces and its generalizations are later studied by several authors, one of which, developed by Šostak [14], used the idea of degree of openness. This type of generalization of fuzzy topological spaces was later rephrased by Chattopadhyay and his colleagues [3], and by Ramadan [13].

As a generalization of fuzzy sets, the concept of intuitionistic fuzzy sets was introduced by Atanassov [1]. Recently, Çoker and his colleagues [4, 7] introduced intuitionistic fuzzy topological spaces using intuitionistic fuzzy sets. Using the idea of degree of openness and degree of nonopenness, Çoker and Demirci [6] defined intuitionistic fuzzy topological spaces in Šostak's sense as a generalization of smooth topological spaces and intuitionistic fuzzy topological spaces. Thakur and Singh [15] introduced the concepts of fuzzy semi-preopen sets and fuzzy semi-precontinuous mappings on Chang's fuzzy topological spaces.

In this paper, we introduce the concepts of fuzzy (r, s)-semi-preopen sets and fuzzy (r, s)-semi-precontinuous mappings on intuitionistic fuzzy topological spaces in Šostak's sense. The relations among fuzzy (r, s)-semicontinuous, fuzzy (r, s)-precontinuous, and fuzzy (r, s)-semi-precontinuous mappings are discussed. The concepts of fuzzy (r, s)-semi-preinterior, fuzzy (r, s)-semi-preclosure, fuzzy (r, s)-semi-preneighborhood, and fuzzy (r, s)-quasi-semi-preneighborhood are given. Using these concepts, the characterization for the fuzzy (r, s)-semi-precontinuous mapping is obtained. Also, we introduce the notions of fuzzy (r, s)-semi-preopen and fuzzy (r, s)-semi-preclosed mappings on intuitionistic fuzzy topological spaces in Šostak's sense, and then we investigate some of their characteristic properties.

2. Preliminaries

For the nonstandard definitions and notations we refer to [9, 10].

Definition 2.1. ([6]) Let X be a nonempty set. An *intuitionistic fuzzy topology in Šostak's sense*(SoIFT for short) $\mathcal{T} = (\mathcal{T}_1, \mathcal{T}_2)$ on X is a mapping $\mathcal{T} : I(X) \to I \otimes I$ which satisfies the following properties :

(1) $\mathcal{T}_1(\underline{0}) = \mathcal{T}_1(\underline{1}) = 1$ and $\mathcal{T}_2(\underline{0}) = \mathcal{T}_2(\underline{1}) = 0$.

(2) $\mathcal{T}_1(A \cap B) \geq \mathcal{T}_1(A) \wedge \mathcal{T}_1(B)$ and $\mathcal{T}_2(A \cap B) \leq \mathcal{T}_2(A) \vee \mathcal{T}_2(B)$.

(3) $\mathcal{T}_1(\bigcup A_i) \geq \bigwedge \mathcal{T}_1(A_i)$ and $\mathcal{T}_2(\bigcup A_i) \leq \bigvee \mathcal{T}_2(A_i)$.

The $(X, \mathcal{T}) = (X, \mathcal{T}_1, \mathcal{T}_2)$ is said to be an *intuitionistic fuzzy topological space in Šostak's sense*(SoIFTS for short). Also, we call $\mathcal{T}_1(A)$ a *gradation of openness* of A and $\mathcal{T}_2(A)$ a *gradation of nonopenness* of A.

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Definition 2.2. ([5, 8]) Let $(X, \mathcal{T}_1, \mathcal{T}_2)$ be a SoIFTS and $(r, s) \in I \otimes I$. Then

(1) an intuitionistic fuzzy point $x_{(\alpha,\beta)}$ in X is said to be *quasi-coincident* with the intuitionistic fuzzy set A in X, denoted by $x_{(\alpha,\beta)}\mathrm{q}A$, if and only if $\mu_A(x) > \beta$ or $\gamma_A(x) < \alpha$.

(2) two intuitionistic fuzzy sets A and B in X are said to be *quasi-coincident*, denoted by $A\mathrm{q}B$, if and only if there exists an element $x \in X$ such that $\mu_A(x) > \gamma_B(x)$ or $\gamma_A(x) < \mu_B(x)$.

The word 'not quasi-coincident' will be abbreviated as $\tilde{\mathrm{q}}$.

Definition 2.3. ([12]) Let A be an intuitionistic fuzzy set in a SoIFTS $(X, \mathcal{T}_1, \mathcal{T}_2)$ and $(r, s) \in I \otimes I$. Then A is said to be

(1) *fuzzy (r, s)-preopen* if $A \subseteq \mathrm{int}(\mathrm{cl}(A, r, s), r, s)$,

(2) *fuzzy (r, s)-preclosed* if $\mathrm{cl}(\mathrm{int}(A, r, s), r, s) \subseteq A$.

Definition 2.4. ([12]) Let $(X, \mathcal{T}_1, \mathcal{T}_2)$ be a SoIFTS. For each $(r, s) \in I \otimes I$ and for each $A \in I(X)$, the *fuzzy (r, s)-preinterior* is defined by
$\mathrm{pint}(A, r, s)$

$$= \bigcup\{B \in I(X) \mid B \subseteq A,\ \ B \text{ is fuzzy } (r, s)\text{-preopen}\}$$

and the *fuzzy (r, s)-preclosure* is defined by
$\mathrm{pcl}(A, r, s)$

$$= \bigcap\{B \in I(X) \mid A \subseteq B,\ \ B \text{ is fuzzy } (r, s)\text{-preclosed}\}.$$

Definition 2.5. ([11, 12]) Let $f : (X, \mathcal{T}_1, \mathcal{T}_2) \to (Y, \mathcal{U}_1, \mathcal{U}_2)$ be a mapping from a SoIFTS X to a SoIFTS Y and $(r, s) \in I \otimes I$. Then f is called

(1) a *fuzzy (r, s)-semiclosed* mapping if $f(A)$ is a fuzzy (r, s)-semiclosed set in Y for each fuzzy (r, s)-closed set A in X,

(2) a *fuzzy (r, s)-precontinuous* mapping if $f^{-1}(B)$ is a fuzzy (r, s)-preopen set in X for each fuzzy (r, s)-open set B in Y,

(3) a *fuzzy (r, s)-preopen* mapping if $f(A)$ is a fuzzy (r, s)-preopen set in Y for each fuzzy (r, s)-open set A in X,

(4) a *fuzzy (r, s)-preclosed* mapping if $f(A)$ is a fuzzy (r, s)-preclosed set in Y for each fuzzy (r, s)-closed set A in X.

Definition 2.6. ([10]) Let $x_{(\alpha,\beta)}$ be an intuitionistic fuzzy point in a SoIFTS $(X, \mathcal{T}_1, \mathcal{T}_2)$ and $(r, s) \in I \otimes I$. Then an intuitionistic fuzzy set A in X is called

(1) a *fuzzy (r, s)-neighborhood* of $x_{(\alpha,\beta)}$ if there is a fuzzy (r, s)-open set B in X such that $x_{(\alpha,\beta)} \in B \subseteq A$,

(2) a *fuzzy (r, s)-semineighborhood* of $x_{(\alpha,\beta)}$ if there is a fuzzy (r, s)-semiopen set B in X such that $x_{(\alpha,\beta)} \in B \subseteq A$.

3. Fuzzy (r, s)-semi-preopen sets and fuzzy (r, s)-semi-precontinuous mappings

Now, we define the notions of fuzzy (r, s)-semi-preopen sets and fuzzy (r, s)-semi-precontinuous mappings on intuitionistic fuzzy topological spaces in Šostak's sense, and then we investigate some of their properties.

Theorem 3.1. Let A be an intuitionistic fuzzy set in a SoIFTS $(X, \mathcal{T}_1, \mathcal{T}_2)$ and $(r, s) \in I \otimes I$. Then A is a fuzzy (r, s)-preopen set in X if and only if there is a fuzzy (r, s)-open set B in X such that $A \subseteq B \subseteq \mathrm{cl}(A, r, s)$.

Proof. Let A be a fuzzy (r, s)-preopen set in X. Then $A \subseteq \mathrm{int}(\mathrm{cl}(A, r, s), r, s)$. Put $B = \mathrm{int}(\mathrm{cl}(A, r, s), r, s)$. Then B is a fuzzy (r, s)-open set in X and $A \subseteq B \subseteq \mathrm{cl}(A, r, s)$.
Conversely, let B be a fuzzy (r, s)-open set in X such that $A \subseteq B \subseteq \mathrm{cl}(A, r, s)$. Then $A \subseteq B = \mathrm{int}(B) \subseteq \mathrm{int}(\mathrm{cl}(A, r, s), r, s)$. Hence A is a fuzzy (r, s)-preopen set. □

Definition 3.2. Let A be an intuitionistic fuzzy set in a SoIFTS $(X, \mathcal{T}_1, \mathcal{T}_2)$ and $(r, s) \in I \otimes I$. Then A is called

(1) a *fuzzy (r, s)-semi-preopen* set if there is a fuzzy (r, s)-preopen set B in X such that $B \subseteq A \subseteq \mathrm{cl}(B, r, s)$.

(2) a *fuzzy (r, s)-semi-preclosed* set if there is a fuzzy (r, s)-preclosed set B in X such that $\mathrm{int}(B, r, s) \subseteq A \subseteq B$.

Theorem 3.3. Let A be an intuitionistic fuzzy set in a SoIFTS $(X, \mathcal{T}_1, \mathcal{T}_2)$ and $(r, s) \in I \otimes I$. Then the following statements are equivalent :

(1) A is a fuzzy (r, s)-semi-preopen set.

(2) A^c is a fuzzy (r, s)-semi-preclosed set.

Proof. Straightforward. □

Remark 3.4. It is clear that every fuzzy (r, s)-semiopen(resp. fuzzy (r, s)-semiclosed) set and every fuzzy (r, s)-preopen(resp. fuzzy (r, s)-preclosed) set is fuzzy (r, s)-semi-preopen(resp. fuzzy (r, s)-semi-preclosed) for each $(r, s) \in I \otimes I$. However, the following example shows that all of the converses need not be true.

Example 3.5. Let $X = \{x, y\}$ and let A_1, A_2, A_3, and A_4 be intuitionistic fuzzy sets in X defined as

$$A_1(x) = (0.2, 0.8),\ \ A_1(y) = (0.3, 0.5);$$

$$A_2(x) = (0.8, 0.1),\ \ A_2(y) = (0.8, 0.1);$$

$$A_3(x) = (0.5, 0.2),\ \ A_3(y) = (0.2, 0.5);$$

and

$$A_4(x) = (0.1, 0.9),\ \ A_4(y) = (0.2, 0.6).$$

Define $\mathcal{T} : I(X) \to I \otimes I$ by

$$\mathcal{T}(A) = (\mathcal{T}_1(A), \mathcal{T}_2(A)) = \begin{cases} (1,0) & \text{if } A = \underline{0}, \underline{1}, \\ (\frac{1}{2}, \frac{1}{3}) & \text{if } A = A_1, \\ (0,1) & \text{otherwise.} \end{cases}$$

Then clearly $\mathcal{T}$ is a SoIFT on X. Since $A_2 \subseteq \underline{1} = \text{int}(\text{cl}(A_2, \frac{1}{2}, \frac{1}{3}), \frac{1}{2}, \frac{1}{3})$, A_2 is a fuzzy $(\frac{1}{2}, \frac{1}{3})$-preopen set and hence A_2 is fuzzy $(\frac{1}{2}, \frac{1}{3})$-semi-preopen. But A_2 is not a fuzzy $(\frac{1}{2}, \frac{1}{3})$-semiopen set, because $A_2 \nsubseteq \text{cl}(\text{int}(A_2, \frac{1}{2}, \frac{1}{3}), \frac{1}{2}, \frac{1}{3}) = A_1^c$. Since $A_4 \subseteq \text{int}(\text{cl}(A_4, \frac{1}{2}, \frac{1}{3}), \frac{1}{2}, \frac{1}{3}) = A_1$, A_4 is a fuzzy $(\frac{1}{2}, \frac{1}{3})$-preopen set. Also, A_3 is a fuzzy $(\frac{1}{2}, \frac{1}{3})$-semi-preopen set, because $A_4 \subseteq A_3 \subseteq \text{cl}(A_4, \frac{1}{2}, \frac{1}{3}) = A_1^c$. But A_3 is not a fuzzy $(\frac{1}{2}, \frac{1}{3})$-preopen set, because $A_3 \nsubseteq \text{int}(\text{cl}(A_3, \frac{1}{2}, \frac{1}{3}), \frac{1}{2}, \frac{1}{3}) = A_1$.

Theorem 3.6. Let A be an intuitionistic fuzzy set in a SoIFTS $(X, \mathcal{T}_1, \mathcal{T}_2)$ and $(r, s) \in I \otimes I$. Then the following statements are true :

(1) For each fuzzy (r, s)-semi-preopen set B in X, $B \subseteq A \subseteq \text{cl}(B, r, s)$ implies that A is fuzzy (r, s)-semi-preopen in X.

(2) For each fuzzy (r, s)-semi-preclosed set B in X, $\text{int}(B, r, s) \subseteq A \subseteq B$ implies that A is fuzzy (r, s)-semi-preclosed in X.

Proof. (1) Let C be a fuzzy (r, s)-preopen set in X such that $C \subseteq B \subseteq \text{cl}(C, r, s)$. Then clearly $C \subseteq A$ and $B \subseteq \text{cl}(C, r, s)$ implies that $\text{cl}(B, r, s) \subseteq \text{cl}(C, r, s)$. Thus $C \subseteq A \subseteq \text{cl}(B, r, s) \subseteq \text{cl}(C, r, s)$. Hence A is a fuzzy (r, s)-semi-preopen set in X.
(2) Similar to (1). □

Theorem 3.7. Let $(X, \mathcal{T}_1, \mathcal{T}_2)$ be a SoIFTS and $(r, s) \in I \otimes I$.

(1) If $\{A_i\}$ is a family of fuzzy (r, s)-semi-preopen sets in X, then $\bigcup A_i$ is fuzzy (r, s)-semi-preopen.

(2) If $\{A_i\}$ is a family of fuzzy (r, s)-semi-preclosed sets in X, then $\bigcap A_i$ is fuzzy (r, s)-semi-preclosed.

Proof. (1) Let $\{A_i\}$ be a collection of fuzzy (r, s)-semi-preopen sets in X. Then for each i, there is a fuzzy (r, s)-preopen set B_i in X such that $B_i \subseteq A_i \subseteq \text{cl}(B_i, r, s)$. So

$$\bigcup B_i \subseteq \bigcup A_i \subseteq \bigcup \text{cl}(B_i, r, s) \subseteq \text{cl}(\bigcup B_i, r, s)$$

and ,$\bigcup B_i$ is fuzzy (r, s)-preopen. Hence $\bigcup A_i$ is a fuzzy (r, s)-semi-preopen set.
(2) It follows from (1) using Theorem 3.3. □

The following example shows that the intersection(resp. union) of two fuzzy (r, s)-semi-preopen(resp. fuzzy (r, s)-semi-preclosed) sets need not be a fuzzy (r, s)-semi-preopen(resp. fuzzy (r, s)-semi-preclosed) set for each $(r, s) \in I \otimes I$.

Example 3.8. Let $X = \{x, y\}$ and let A_1 and A_2 be intuitionistic fuzzy sets in X defined as

$$A_1(x) = (0.1, 0.7),\ \ A_1(y) = (0.4, 0.3);$$

and

$$A_2(x) = (0.8, 0.1),\ \ A_2(y) = (0.2, 0.4).$$

Define $\mathcal{T} : I(X) \to I \otimes I$ by

$$\mathcal{T}(A) = (\mathcal{T}_1(A), \mathcal{T}_2(A)) = \begin{cases} (1,0) & \text{if } A = \underline{0}, \underline{1}, \\ (\frac{1}{2}, \frac{1}{3}) & \text{if } A = A_1, \\ (0,1) & \text{otherwise.} \end{cases}$$

Then clearly $\mathcal{T}$ is a SoIFT on X. Since A_1 is fuzzy $(\frac{1}{2}, \frac{1}{3})$-open, A_1 is a fuzzy $(\frac{1}{2}, \frac{1}{3})$-semi-preopen set. Since $A_2 \subseteq \underline{1} = \text{int}(\text{cl}(A_2, \frac{1}{2}, \frac{1}{3}), \frac{1}{2}, \frac{1}{3})$, A_2 is fuzzy $(\frac{1}{2}, \frac{1}{3})$-preopen and hence A_2 is a fuzzy $(\frac{1}{2}, \frac{1}{3})$-semi-preopen set. But $A_1 \cap A_2$ is not a fuzzy $(\frac{1}{2}, \frac{1}{3})$-semi-preopen set in X, because there is no fuzzy $(\frac{1}{2}, \frac{1}{3})$-preopen set B in X such that $B \subseteq A_1 \cap A_2 \subseteq \text{cl}(B, \frac{1}{2}, \frac{1}{3})$.

Definition 3.9. Let $(X, \mathcal{T}_1, \mathcal{T}_2)$ be a SoIFTS. For each $(r, s) \in I \otimes I$ and for each $A \in I(X)$, the *fuzzy* (r, s)*-semi-preinterior* is defined by

$$\text{spint}(A, r, s) = \bigcup\{B \in I(X) \mid B \subseteq A,\ B \text{ is fuzzy } (r, s)\text{-semi-preopen}\}$$

and the *fuzzy* (r, s)*-semi-preclosure* is defined by

$$\text{spcl}(A, r, s) = \bigcap\{B \in I(X) \mid A \subseteq B,\ B \text{ is fuzzy } (r, s)\text{-semi-preclosed}\}.$$

Obviously $\text{spcl}(A, r, s)$ is the smallest fuzzy (r, s)-semi-preclosed set which contains A, and $\text{spint}(A, r, s)$ is the greatest fuzzy (r, s)-semi-preopen set which is contained in A. Also, $\text{spcl}(A, r, s) = A$ for any fuzzy (r, s)-semi-preclosed set A, and $\text{spint}(A, r, s) = A$ for any fuzzy (r, s)-semi-preopen set A. Moreover, we have

$$\begin{aligned} \text{int}(A, r, s) &\subseteq \text{pint}(A, r, s) \subseteq \text{spint}(A, r, s) \subseteq A \\ &\subseteq \text{spcl}(A, r, s) \subseteq \text{pcl}(A, r, s) \subseteq \text{cl}(A, r, s). \end{aligned}$$

Also, we have the following results :
(1) $\text{spcl}(\underline{0}, r, s) = \underline{0}$, $\text{spcl}(\underline{1}, r, s) = \underline{1}$.
(2) $\text{spcl}(A, r, s) \supseteq A$.
(3) $\text{spcl}(A \cup B, r, s) \supseteq \text{spcl}(A, r, s) \cup \text{spcl}(B, r, s)$.
(4) $\text{spcl}(\text{spcl}(A, r, s), r, s) = \text{spcl}(A, r, s)$.
(5) $\text{spint}(\underline{0}, r, s) = \underline{0}$, $\text{spint}(\underline{1}, r, s) = \underline{1}$.
(6) $\text{spint}(A, r, s) \subseteq A$.
(7) $\text{spint}(A \cap B, r, s) \subseteq \text{spint}(A, r, s) \cap \text{spint}(B, r, s)$.
(8) $\text{spint}(\text{spint}(A, r, s), r, s) = \text{spint}(A, r, s)$.

Definition 3.10. Let A be an intuitionistic fuzzy set and $x_{(\alpha,\beta)}$ an intuitionistic fuzzy point in a SoIFTS $(X, \mathcal{T}_1, \mathcal{T}_2)$ and $(r, s) \in I \otimes I$. Then A is called

(1) a *fuzzy (r, s)-semi-preneighborhood* of $x_{(\alpha,\beta)}$ if there is a fuzzy (r, s)-semi-preopen set B in X such that $x_{(\alpha,\beta)} \in B \subseteq A$.

(2) a *fuzzy (r, s)-quasi-semi-preneighborhood* of $x_{(\alpha,\beta)}$ if there is a fuzzy (r, s)-semi-preopen set B in X such that $x_{(\alpha,\beta)} q B \subseteq A$.

Theorem 3.11. Let A be an intuitionistic fuzzy set in a SoIFTS $(X, \mathcal{T}_1, \mathcal{T}_2)$ and $(r, s) \in I \otimes I$. Then A is fuzzy (r, s)-semi-preopen if and only if A is a fuzzy (r, s)-semi-preneighborhood of $x_{(\alpha,\beta)}$ for each intuitionistic fuzzy point $x_{(\alpha,\beta)} \in A$.

Proof. Straightforward. □

Theorem 3.12. Let A be an intuitionistic fuzzy set in a SoIFTS $(X, \mathcal{T}_1, \mathcal{T}_2)$ and $(r, s) \in I \otimes I$. Then an intuitionistic fuzzy point $x_{(\alpha,\beta)}$ is contained in $\text{spcl}(A, r, s)$ if and only if every fuzzy (r, s)-quasi-semi-preneighborhood of $x_{(\alpha,\beta)}$ is quasi-coincident with A.

Proof. Suppose $x_{(\alpha,\beta)} \in \text{spcl}(A, r, s)$ and there exists a fuzzy (r, s)-quasi-semi-preneighborhood B of $x_{(\alpha,\beta)}$ such that $A \tilde{q} B$. Then there is a fuzzy (r, s)-semi-preopen set C in X such that $x_{(\alpha,\beta)} q C \subseteq B$, which shows that $A \tilde{q} C$ and hence $A \subseteq C^c$. Since C^c is fuzzy (r, s)-semi-preclosed in X, $\text{spcl}(A, r, s) \subseteq C^c$. Thus $x_{(\alpha,\beta)} \in C^c$. But $x_{(\alpha,\beta)} \notin C^c$, because $x_{(\alpha,\beta)} q C$. This is a contradiction.

Conversely, suppose every fuzzy (r, s)-quasi-semi-preneighborhood of $x_{(\alpha,\beta)}$ is quasi-coincident with A. If $x_{(\alpha,\beta)} \notin \text{spcl}(A, r, s)$, then there is a fuzzy (r, s)-semi-preclosed set B in X such that $A \subseteq B$ and $x_{(\alpha,\beta)} \notin B$. So B^c is a fuzzy (r, s)-semi-preopen set in X such that $x_{(\alpha,\beta)} q B^c$ and $B^c \tilde{q} A$. This is a contradiction. □

Definition 3.13. Let $f : (X, \mathcal{T}_1, \mathcal{T}_2) \to (Y, \mathcal{U}_1, \mathcal{U}_2)$ be a mapping from a SoIFTS X to a SoIFTS Y and $(r, s) \in I \otimes I$. Then f is called a *fuzzy (r, s)-semi-precontinuous* mapping if $f^{-1}(B)$ is a fuzzy (r, s)-semi-preopen set in X for each fuzzy (r, s)-open set B in Y.

Remark 3.14. It is clear that every fuzzy (r, s)-semicontinuous and every fuzzy (r, s)-precontinuous mapping is fuzzy (r, s)-semi-precontinuous for each $(r, s) \in I \otimes I$. However, the following examples show that all of the converses need not be true.

Example 3.15. Let $X = \{x, y\}$ and let A_1, A_2 and B be intuitionistic fuzzy sets in X defined as

$$A_1(x) = (0.2, 0.7), \quad A_1(y) = (0.3, 0.5);$$

$$A_2(x) = (0.7, 0.2), \quad A_2(y) = (0.7, 0.2);$$

and

$$B(x) = (0.7, 0.2), \quad B(y) = (0.6, 0.3).$$

Define $\mathcal{T} : I(X) \to I \otimes I$ and $\mathcal{U} : I(X) \to I \otimes I$ by

$$\mathcal{T}(A) = (\mathcal{T}_1(A), \mathcal{T}_2(A)) = \begin{cases} (1, 0) & \text{if } A = \underline{0}, \underline{1}, \\ (\frac{1}{2}, \frac{1}{3}) & \text{if } A = A_1, \\ (0, 1) & \text{otherwise}; \end{cases}$$

and

$$\mathcal{U}(A) = (\mathcal{U}_1(A), \mathcal{U}_2(A)) = \begin{cases} (1, 0) & \text{if } A = \underline{0}, \underline{1}, \\ (\frac{1}{2}, \frac{1}{3}) & \text{if } A = A_2, \\ (0, 1) & \text{otherwise}. \end{cases}$$

Then clearly $\mathcal{T}$ and $\mathcal{U}$ are SoIFTs on X. Consider a mapping $f : (X, \mathcal{T}) \to (X, \mathcal{U})$ defined by $f(x) = x$ and $f(y) = y$. Then it is easy to see that B is a fuzzy $(\frac{1}{2}, \frac{1}{3})$-preopen set in $(X, \mathcal{T})$ and $B \subseteq f^{-1}(A_2) = A_2 \subseteq \text{cl}(B, \frac{1}{2}, \frac{1}{3}) = \underline{1}$. So $f^{-1}(A_2) = A_2$ is a fuzzy $(\frac{1}{2}, \frac{1}{3})$-semi-preopen set in $(X, \mathcal{T})$ and hence f is a fuzzy $(\frac{1}{2}, \frac{1}{3})$-semi-precontinuous mapping. But $f^{-1}(A_2) = A_2$ is not a fuzzy $(\frac{1}{2}, \frac{1}{3})$-semiopen set in $(X, \mathcal{T})$, because $A_2 \nsubseteq \text{cl}(\text{int}(A_2, \frac{1}{2}, \frac{1}{3}), \frac{1}{2}, \frac{1}{3}) = A_1^c$ in $(X, \mathcal{T})$. Hence f is not fuzzy $(\frac{1}{2}, \frac{1}{3})$-semicontinuous.

Example 3.16. Let $X = \{x, y\}$ and let A_1, A_2 and B be intuitionistic fuzzy sets in X defined as

$$A_1(x) = (0.2, 0.5), \quad A_1(y) = (0.3, 0.3);$$

$$A_2(x) = (0.5, 0.3), \quad A_2(y) = (0.3, 0.4);$$

and

$$B(x) = (0.2, 0.6), \quad B(y) = (0.2, 0.4).$$

Define $\mathcal{T} : I(X) \to I \otimes I$ and $\mathcal{U} : I(X) \to I \otimes I$ by

$$\mathcal{T}(A) = (\mathcal{T}_1(A), \mathcal{T}_2(A)) = \begin{cases} (1, 0) & \text{if } A = \underline{0}, \underline{1}, \\ (\frac{1}{2}, \frac{1}{3}) & \text{if } A = A_1, \\ (0, 1) & \text{otherwise}; \end{cases}$$

and

$$\mathcal{U}(A) = (\mathcal{U}_1(A), \mathcal{U}_2(A)) = \begin{cases} (1, 0) & \text{if } A = \underline{0}, \underline{1}, \\ (\frac{1}{2}, \frac{1}{3}) & \text{if } A = A_2, \\ (0, 1) & \text{otherwise}. \end{cases}$$

Then clearly $\mathcal{T}$ and $\mathcal{U}$ are SoIFTs on X. Consider a mapping $f : (X, \mathcal{T}) \to (X, \mathcal{U})$ defined by $f(x) = x, f(y) = y$. Then it is easy to see that B is a fuzzy $(\frac{1}{2}, \frac{1}{3})$-preopen set in $(X, \mathcal{T})$ and $B \subseteq f^{-1}(A_2) = A_2 \subseteq \mathrm{cl}(B, \frac{1}{2}, \frac{1}{3}) = A_1^c$. So $f^{-1}(A_2) = A_2$ is a fuzzy $(\frac{1}{2}, \frac{1}{3})$-semi-preopen set in $(X, \mathcal{T})$ and hence f is a fuzzy $(\frac{1}{2}, \frac{1}{3})$-semi-precontinuous mapping. But $f^{-1}(A_2) = A_2$ is not a fuzzy $(\frac{1}{2}, \frac{1}{3})$-preopen set in $(X, \mathcal{T})$, because $A_2 \nsubseteq \mathrm{int}(\mathrm{cl}(A_2, \frac{1}{2}, \frac{1}{3}), \frac{1}{2}, \frac{1}{3}) = A_1$ in $(X, \mathcal{T})$. Hence f is not fuzzy $(\frac{1}{2}, \frac{1}{3})$-precontinuous.

Theorem 3.17. Let $f : (X, \mathcal{T}_1, \mathcal{T}_2) \to (Y, \mathcal{U}_1, \mathcal{U}_2)$ be a mapping from a SoIFTS X to a SoIFTS Y and $(r, s) \in I \otimes I$. Then the following statements are equivalent :

(1) f is fuzzy (r, s)-semi-precontinuous.

(2) For each fuzzy (r, s)-closed set B in Y, $f^{-1}(B)$ is a fuzzy (r, s)-semi-preclosed set in X.

(3) For each intuitionistic fuzzy point $x_{(\alpha,\beta)}$ in X and each fuzzy (r, s)-open set B in Y such that $f(x_{(\alpha,\beta)}) \in B$, there is a fuzzy (r, s)-semi-preopen set A in X such that $x_{(\alpha,\beta)} \in A$ and $f(A) \subseteq B$.

(4) For each intuitionistic fuzzy point $x_{(\alpha,\beta)}$ in X and each fuzzy (r, s)-neighborhood B of $f(x_{(\alpha,\beta)})$, $f^{-1}(B)$ is a fuzzy (r, s)-semi-preneighborhood of $x_{(\alpha,\beta)}$.

(5) For each intuitionistic fuzzy point $x_{(\alpha,\beta)}$ in X and each fuzzy (r, s)-neighborhood B of $f(x_{(\alpha,\beta)})$, there is a fuzzy (r, s)-semi-preneighborhood A of $x_{(\alpha,\beta)}$ such that $f(A) \subseteq B$.

(6) For each intuitionistic fuzzy set B in Y, $\mathrm{spcl}(f^{-1}(B), r, s) \subseteq f^{-1}(\mathrm{cl}(B, r, s))$.

(7) For each intuitionistic fuzzy set A in X, $f(\mathrm{spcl}(A, r, s)) \subseteq \mathrm{cl}(f(A), r, s)$.

(8) For each intuitionistic fuzzy set B in Y, $f^{-1}(\mathrm{int}(B, r, s)) \subseteq \mathrm{spint}(f^{-1}(B), r, s)$.

Proof. (1) $\Leftrightarrow$ (2) It is obvious.

(1) $\Rightarrow$ (3) Let $x_{(\alpha,\beta)}$ be an intuitionistic fuzzy point in X and B a fuzzy (r, s)-open set in Y such that $f(x_{(\alpha,\beta)}) \in B$. Then $x_{(\alpha,\beta)} \in f^{-1}(B)$. Put $A = f^{-1}(B)$. Then by (1), A is a fuzzy (r, s)-semi-preopen set in X such that $x_{(\alpha,\beta)} \in A$ and $f(A) = f(f^{-1}(B)) \subseteq B$.

(3) $\Rightarrow$ (1) Let B be a fuzzy (r, s)-open set in Y and $x_{(\alpha,\beta)} \in f^{-1}(B)$. Then $f(x_{(\alpha,\beta)}) \in B$. By (3), there is a fuzzy (r, s)-semi-preopen set $A_{x_{(\alpha,\beta)}}$ in X such that $x_{(\alpha,\beta)} \in A_{x_{(\alpha,\beta)}}$ and $f(A_{x_{(\alpha,\beta)}}) \subseteq B$. Thus $x_{(\alpha,\beta)} \in A_{x_{(\alpha,\beta)}} \subseteq f^{-1}(f(A_{x_{(\alpha,\beta)}})) \subseteq f^{-1}(B)$. So we have

$$\begin{aligned} f^{-1}(B) &= \bigcup\{x_{(\alpha,\beta)} \mid x_{(\alpha,\beta)} \in f^{-1}(B)\} \\ &\subseteq \bigcup\{A_{x_{(\alpha,\beta)}} \mid x_{(\alpha,\beta)} \in f^{-1}(B)\} \subseteq f^{-1}(B). \end{aligned}$$

Thus $f^{-1}(B) = \bigcup\{A_{x_{(\alpha,\beta)}} \mid x_{(\alpha,\beta)} \in f^{-1}(B)\}$ and hence $f^{-1}(B)$ is fuzzy (r, s)-semi-preopen in X. Therefore f is a fuzzy (r, s)-semi-precontinuous mapping.

(1) $\Rightarrow$ (4) Let $x_{(\alpha,\beta)}$ be an intuitionistic fuzzy point in X and B a fuzzy (r, s)-neighborhood of $f(x_{(\alpha,\beta)})$. Then there is a fuzzy (r, s)-open set C in Y such that $f(x_{(\alpha,\beta)}) \in C \subseteq B$ and hence $x_{(\alpha,\beta)} \in f^{-1}(C) \subseteq f^{-1}(B)$. Since f is fuzzy (r, s)-semi-precontinuous, $f^{-1}(C)$ is a fuzzy (r, s)-semi-preopen set in X. Thus $f^{-1}(B)$ is a fuzzy (r, s)-semi-preneighborhood of $x_{(\alpha,\beta)}$.

(4) $\Rightarrow$ (5) Let $x_{(\alpha,\beta)}$ be an intuitionistic fuzzy point in X and B a fuzzy (r, s)-neighborhood of $f(x_{(\alpha,\beta)})$. By (4), $A = f^{-1}(B)$ is a fuzzy (r, s)-semi-preneighborhood of $x_{(\alpha,\beta)}$ and $f(A) = f(f^{-1}(B)) \subseteq B$.

(5) $\Rightarrow$ (3) Let $x_{(\alpha,\beta)}$ be an intuitionistic fuzzy point in X and B a fuzzy (r, s)-open set in Y such that $f(x_{(\alpha,\beta)}) \in B$. Then B is a fuzzy (r, s)-neighborhood of $f(x_{(\alpha,\beta)})$. By (5), there is a fuzzy (r, s)-semi-preneighborhood A of $x_{(\alpha,\beta)}$ in X such that $x_{(\alpha,\beta)} \in A$ and $f(A) \subseteq B$. Thus there is a fuzzy (r, s)-semi-preopen set C in X such that $x_{(\alpha,\beta)} \in C \subseteq A$ and hence $f(C) \subseteq f(A) \subseteq B$.

(2) $\Rightarrow$ (6) Let B be an intuitionistic fuzzy set in Y. Then $\mathrm{cl}(B, r, s)$ is a fuzzy (r, s)-closed set in Y and $f^{-1}(B) \subseteq f^{-1}(\mathrm{cl}(B, r, s))$. By (2), $f^{-1}(\mathrm{cl}(B, r, s))$ is a fuzzy (r, s)-semi-preclosed set in X. Hence

$$\mathrm{spcl}(f^{-1}(B), r, s) \subseteq f^{-1}(\mathrm{cl}(B, r, s)).$$

(6) $\Rightarrow$ (2) Let B be a fuzzy (r, s)-closed set in Y. Then by (6),

$$\begin{aligned} f^{-1}(B) \subseteq \mathrm{spcl}(f^{-1}(B), r, s) &\subseteq f^{-1}(\mathrm{cl}(B, r, s)) \\ &= f^{-1}(B). \end{aligned}$$

Hence $f^{-1}(B) = \mathrm{spcl}(f^{-1}(B), r, s)$. Thus $f^{-1}(B)$ is a fuzzy (r, s)-semi-preclosed set in X.

(6) $\Rightarrow$ (7) Let A be an intuitionistic fuzzy set in X. Then $f(A)$ is an intuitionistic fuzzy set in Y. By (6),

$$\begin{aligned} \mathrm{spcl}(A, r, s) &\subseteq \mathrm{spcl}(f^{-1}(f(A)), r, s) \\ &\subseteq f^{-1}(\mathrm{cl}(f(A), r, s)). \end{aligned}$$

Thus $f(\mathrm{spcl}(A, r, s)) \subseteq \mathrm{cl}(f(A), r, s)$.

(7) $\Rightarrow$ (6) Let B be an intuitionistic fuzzy set in Y. Then $f^{-1}(B)$ is an intuitionistic fuzzy set in X. By (7),

$$\begin{aligned} f(\mathrm{spcl}(f^{-1}(B), r, s)) &\subseteq \mathrm{cl}(f(f^{-1}(B)), r, s) \\ &\subseteq \mathrm{cl}(B, r, s). \end{aligned}$$

4Hence

$$\begin{aligned}\text{spcl}(f^{-1}(B), r, s) &\subseteq f^{-1}(f(\text{spcl}(f^{-1}(B), r, s))) \\ &\subseteq f^{-1}(\text{cl}(B, r, s)).\end{aligned}$$

(1) $\Rightarrow$ (8) Let B be an intuitionistic fuzzy set in Y. Then $\text{int}(B, r, s)$ is a fuzzy (r, s)-open set in Y. Since f is fuzzy (r, s)-semi-precontinuous, $f^{-1}(\text{int}(B, r, s))$ is a fuzzy (r, s)-semi-preopen set in X. Thus

$$f^{-1}(\text{int}(B, r, s)) \subseteq \text{spint}(f^{-1}(B), r, s).$$

(8) $\Rightarrow$ (1) Let B be a fuzzy (r, s)-open set in Y. By (8),

$$\begin{aligned}f^{-1}(B) = f^{-1}(\text{int}(B, r, s)) &\subseteq \text{spint}(f^{-1}(B), r, s) \\ &\subseteq f^{-1}(B).\end{aligned}$$

Thus $f^{-1}(B) = \text{spint}(f^{-1}(B), r, s)$ and hence $f^{-1}(B)$ is a fuzzy (r, s)-semi-preopen set. Therefore f is fuzzy (r, s)-semi-precontinuous. □

4. Fuzzy (r, s)-semi-preopen and fuzzy (r, s)-semi-preclosed mappings

We define the notions of fuzzy (r, s)-semi-preopen and fuzzy (r, s)-semi-preclosed mappings on intuitionistic fuzzy topological spaces in Šostak's sense, and then we investigate some of their properties.

Definition 4.1. Let $f : (X, \mathcal{T}_1, \mathcal{T}_2) \to (Y, \mathcal{U}_1, \mathcal{U}_2)$ be a mapping from a SoIFTS X to a SoIFTS Y and $(r, s) \in I \otimes I$. Then f is called

(1) a *fuzzy (r, s)-semi-preopen* mapping if $f(A)$ is a fuzzy (r, s)-semi-preopen set in Y for each fuzzy (r, s)-open set A in X,

(2) a *fuzzy (r, s)-semi-preclosed* mapping if $f(A)$ is a fuzzy (r, s)-semi-preclosed set in Y for each fuzzy (r, s)-closed set A in X.

Remark 4.2. It is obvious that every fuzzy (r, s)-semiopen(resp. fuzzy (r, s)-semiclosed) and every fuzzy (r, s)-preopen(resp. fuzzy (r, s)-preclosed) mapping is fuzzy (r, s)-semi-preopen(resp. fuzzy (r, s)-semi-preclosed). However, the following examples show that all of the converses need not be true.

Example 4.3. Let $X = \{x, y\}$ and let A_1, A_2 and B be intuitionistic fuzzy sets in X defined as

$$A_1(x) = (0.9, 0.1), \quad A_1(y) = (0.5, 0.4);$$

$$A_2(x) = (0.4, 0.5), \quad A_2(y) = (0.4, 0.2);$$

and

$$B(x) = (0.5, 0.4), \quad B(y) = (0.3, 0.4).$$

Define $\mathcal{T} : I(X) \to I \otimes I$ and $\mathcal{U} : I(X) \to I \otimes I$ by

$$\mathcal{T}(A) = (\mathcal{T}_1(A), \mathcal{T}_2(A)) = \begin{cases} (1, 0) & \text{if } A = \underline{0}, \underline{1}, \\ (\frac{1}{2}, \frac{1}{3}) & \text{if } A = A_1, \\ (0, 1) & \text{otherwise;} \end{cases}$$

and

$$\mathcal{U}(A) = (\mathcal{U}_1(A), \mathcal{U}_2(A)) = \begin{cases} (1, 0) & \text{if } A = \underline{0}, \underline{1}, \\ (\frac{1}{2}, \frac{1}{3}) & \text{if } A = A_2, \\ (0, 1) & \text{otherwise.} \end{cases}$$

Then clearly $\mathcal{T}$ and $\mathcal{U}$ are SoIFTs on X. Consider a mapping $f : (X, \mathcal{T}) \to (X, \mathcal{U})$ defined by $f(x) = x$ and $f(y) = y$. Since $B \subseteq \text{int}(\text{cl}(B, \frac{1}{2}, \frac{1}{3}), \frac{1}{2}, \frac{1}{3}) = \underline{1}$ in $(X, \mathcal{U})$, B is a fuzzy $(\frac{1}{2}, \frac{1}{3})$-preopen set in $(X, \mathcal{U})$. Also, $f(A_1)$ is a fuzzy $(\frac{1}{2}, \frac{1}{3})$-semi-preopen set, because $B \subseteq f(A_1) = A_1 \subseteq \underline{1} = \text{cl}(B, \frac{1}{2}, \frac{1}{3})$ in $(X, \mathcal{U})$. Thus f is a fuzzy $(\frac{1}{2}, \frac{1}{3})$-semi-preopen mapping. But $f(A_1) = A_1$ is not a fuzzy $(\frac{1}{2}, \frac{1}{3})$-semiopen set in $(X, \mathcal{U})$, because $A_1 \not\subseteq \text{cl}(\text{int}(A_1, \frac{1}{2}, \frac{1}{3}), \frac{1}{2}, \frac{1}{3}) = \underline{0}$ in $(X, \mathcal{U})$. Thus f is not a fuzzy $(\frac{1}{2}, \frac{1}{3})$-semiopen mapping.

Example 4.4. Let $X = \{x, y\}$ and let A_1, A_2 and B be intuitionistic fuzzy sets in X defined as

$$A_1(x) = (0.5, 0.3), \quad A_1(y) = (0.2, 0.3);$$

$$A_2(x) = (0.2, 0.5), \quad A_2(y) = (0.2, 0.2);$$

and

$$B(x) = (0.2, 0.6), \quad B(y) = (0.1, 0.3).$$

Define $\mathcal{T} : I(X) \to I \otimes I$ and $\mathcal{U} : I(X) \to I \otimes I$ by

$$\mathcal{T}(A) = (\mathcal{T}_1(A), \mathcal{T}_2(A)) = \begin{cases} (1, 0) & \text{if } A = \underline{0}, \underline{1}, \\ (\frac{1}{2}, \frac{1}{3}) & \text{if } A = A_1, \\ (0, 1) & \text{otherwise;} \end{cases}$$

and

$$\mathcal{U}(A) = (\mathcal{U}_1(A), \mathcal{U}_2(A)) = \begin{cases} (1, 0) & \text{if } A = \underline{0}, \underline{1}, \\ (\frac{1}{2}, \frac{1}{3}) & \text{if } A = A_2, \\ (0, 1) & \text{otherwise.} \end{cases}$$

Then clearly $\mathcal{T}$ and $\mathcal{U}$ are SoIFTs on X. Consider a mapping $f : (X, \mathcal{T}) \to (X, \mathcal{U})$ defined by $f(x) = x$ and $f(y) = y$. Since $B \subseteq \text{int}(\text{cl}(B, \frac{1}{2}, \frac{1}{3}), \frac{1}{2}, \frac{1}{3}) = \text{int}(A_2^c, \frac{1}{2}, \frac{1}{3}) = A_2$ in $(X, \mathcal{U})$, B is fuzzy $(\frac{1}{2}, \frac{1}{3})$-preopen in $(X, \mathcal{U})$. Also, A_1 is a fuzzy $(\frac{1}{2}, \frac{1}{3})$-semi-preopen set in $(X, \mathcal{U})$, because $B \subseteq f(A_1) = A_1 \subseteq \text{cl}(B, \frac{1}{2}, \frac{1}{3}) = A_2^c$ in $(X, \mathcal{U})$. Hence f is a fuzzy $(\frac{1}{2}, \frac{1}{3})$-semi-preopen mapping. But $f(A_1) = A_1$ is not a fuzzy $(\frac{1}{2}, \frac{1}{3})$-preopen set in $(X, \mathcal{U})$, because $A_1 \not\subseteq \text{int}(\text{cl}(A_1, \frac{1}{2}, \frac{1}{3}), \frac{1}{2}, \frac{1}{3}) = \text{int}(A_2^c, \frac{1}{2}, \frac{1}{3}) = A_2$ in $(X, \mathcal{U})$. Thus f is not a fuzzy $(\frac{1}{2}, \frac{1}{3})$-preopen mapping.

Theorem 4.5. Let $f : (X, \mathcal{T}_1, \mathcal{T}_2) \to (Y, \mathcal{U}_1, \mathcal{U}_2)$ be a mapping from a SoIFTS X to a SoIFTS Y and $(r, s) \in I \otimes I$. Then f is a fuzzy (r, s)-semi-preopen mapping if and only if $f(\text{int}(A, r, s)) \subseteq \text{spint}(f(A), r, s)$ for each intuitionistic fuzzy set A in X.

Proof. Let f be a fuzzy (r, s)-semi-preopen mapping. Since $\text{int}(A, r, s)$ is fuzzy (r, s)-open in X, $f(\text{int}(A, r, s))$ is a fuzzy (r, s)-semi-preopen set in Y. Hence

$$\begin{aligned} f(\text{int}(A, r, s)) &= \text{spint}(f(\text{int}(A, r, s)), r, s) \\ &\subseteq \text{spint}(f(A), r, s). \end{aligned}$$

Conversely, let A be a fuzzy (r, s)-open set in X. By hypothesis, $f(A) = f(\text{int}(A, r, s)) \subseteq \text{spint}(f(A), r, s) \subseteq f(A)$. So $f(A) = \text{spint}(f(A), r, s)$. Thus $f(A)$ is a fuzzy (r, s)-semi-preopen set in Y. Hence f is a fuzzy (r, s)-semi-preopen mapping. □

Theorem 4.6. Let $f : (X, \mathcal{T}_1, \mathcal{T}_2) \to (Y, \mathcal{U}_1, \mathcal{U}_2)$ be a mapping from a SoIFTS X to a SoIFTS Y and $(r, s) \in I \otimes I$. Then f is a fuzzy (r, s)-semi-preclosed mapping if and only if $\text{spcl}(f(A), r, s) \subseteq f(\text{cl}(A, r, s))$ for each intuitionistic fuzzy set A in X.

Proof. Let f be a fuzzy (r, s)-semi-preclosed mapping. Since $\text{cl}(A, r, s)$ is a fuzzy (r, s)-closed set in X, $f(\text{cl}(A, r, s))$ is a fuzzy (r, s)-semi-preclosed set in Y. Since $f(A) \subseteq f(\text{cl}(A, r, s))$, we have $\text{spcl}(f(A), r, s) \subseteq f(\text{cl}(A, r, s))$. Conversely, let A be a fuzzy (r, s)-closed set in X. By hypothesis, $f(A) \subseteq \text{spcl}(f(A), r, s) \subseteq f(\text{cl}(A, r, s)) = f(A)$. So $f(A) = \text{spcl}(f(A), r, s)$. Thus $f(A)$ is a fuzzy (r, s)-semi-preclosed set in Y. Hence f is a fuzzy (r, s)-semi-preclosed mapping. □

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저 자 소 개

Seok Jong Lee

한국 퍼지 및 지능시스템학회 부회장
현재 충북대학교 수학과 교수
제 15권 1호(2005년 2월호) 참조
E-mail : sjl@cbnu.ac.kr

Jin Tae Kim

현재 충북대학교 수학과 박사과정
E-mail : kjtmath@hanmail.net