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Correlation of Intuitionistic Fuzzy Sets

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Abstract

When we deal with crisp data, it is common to find the correlation between variables. In this paper, we propose a method to calculate the correlation coefficient for intuitionistic fuzzy data, by adopting the concepts from the conventional statistics. The value of the correlation coefficient computed from our formula not only provides us the strength of the relationship of intuitionistic fuzzy sets, but also shows that the intuitionistic fuzzy sets are positively or negatively related.

Key words : Intuitionistic fuzzy sets, Correlation coefficient.

1. Introduction

Intuitionistic fuzzy set was first suggested by Atanassov [1], which is a generalization of a Zadeh fuzzy set. Some relevant basic notions can be found in [1, 2, 3, 4, 5].

Let X be a crisp set. An intuitionistic fuzzy set A in X is an object having the form

$$A = \{(x, \mu_A(x), \nu_A(x)) : x \in X\}$$

where $\mu_A : X \to [0,1]$ and $\nu_A : X \to [0,1]$ denote membership function and nonmembership function, respectively, of A and satisfy $0 \le \mu_A(x) + \nu_A(x) \le 1$ for every $x \in X$. Note that a Zadeh fuzzy set, written down as an intuitionistic one, is of the form $A = \{(x, \mu_A(x), 1-\mu_A(x)) : x \in X\}$.

The concept of correlation is often used in statistics. Correlation analysis can be employed to study the nature of the relations between the variables. It is interesting to see how the notion of correlation can be extended to fuzzy sets. In 1985, Murthy et al. [17] proposed a measure of correlation between two membership functions satisfying some assumptions, which lies in the interval $[-1,1]$. Sahnoun et al. [18] used the Bhattacharyya's coefficient [6] to compute a consistency measure between two fuzzy statements. The value of the Bhattacharyya's coefficient also lies in $[0,1]$. Yu [9] described the correlation of A and B in the collection $\mathcal{F}([a,b])$ of all fuzzy numbers whose supports are included in a closed interval $[a,b]$. Chauduri and Bhattacharyya [18] used the Spearman's rank correlation coefficient [11] to define the correlation coefficient between two fuzzy sets which lies in the interval $[-1,1]$.

In 1991, Gerstenkorn and Mańko [12] defined the correlation of intuitionistic fuzzy sets A and B in finite crisp set $X = \{x_1, x_2, \ldots, x_n\}$ as follows: $C_{\mathrm{GM}}(A,B) = \sum_{i=1}^{n}(\mu_A(x_i)\mu_B(x_i) + \nu_A(x_i)\nu_B(x_i))$, and the correlation coefficient of A and B was given by

$$\rho_{\mathrm{GM}}(A,B) = \frac{C_{\mathrm{GM}}(A,B)}{\sqrt{C_{\mathrm{GM}}(A,A)\cdot C_{\mathrm{GM}}(B,B)}}.$$

In 1995, Hong and Hwang [13] defined the correlation of intuitionistic fuzzy sets A and B in a probability space $(X, \mathcal{B}, P)$ as follows:

$$C_{\mathrm{HH}}(A,B) = \int_X (\mu_A\mu_B + \nu_A\nu_B)dP,$$

and the correlation coefficient of A and B was given by

$$\rho_{\mathrm{HH}}(A,B) = \frac{C_{\mathrm{HH}}(A,B)}{\sqrt{C_{\mathrm{HH}}(A,A)\cdot C_{\mathrm{HH}}(B,B)}}.$$

Note that if $X = \{x_1, x_2, \ldots, x_n\}$ and a probability P is given by $P(A) = |A|/n$, where $|A|$ is the cardinality of A, then ρ_{HH} is exactly ρ_{GM}. As they stated, the values of ρ_{GM} and ρ_{HH} lie in the interval $[0,1]$. Recently, Hung and Wu [15] used the concept of centroid to define the correlation of intuitionistic fuzzy sets A and B in a crisp set X as follows:

$$C_{\mathrm{HW}}(A,B) = m(\mu_A)m(\mu_A) + m(\nu_A)m(\nu_B),$$

and the correlation coefficient of A and B was given by

$$\rho_{\mathrm{HW}}(A,B) = \frac{C_{\mathrm{HW}}(A,B)}{\sqrt{C_{\mathrm{HW}}(A,A)\cdot C_{\mathrm{HW}}(B,B)}},$$

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where $m(\mu_A) = \frac{\int_X x\mu_A(x)dx}{\int_X \mu_A(x)dx}$, $m(\mu_A) = \frac{\int_X x\mu_A(x)dx}{\int_X \mu_A(x)dx}$, $m(\mu_A) = \frac{\int_X x\mu_A(x)dx}{\int_X \mu_A(x)dx}$ and $m(\mu_A) = \frac{\int_X x\mu_A(x)dx}{\int_X \mu_A(x)dx}$ are the centroids of μ_A, ν_A, μ_B and ν_B, respectively. The values of ρ_{HW} lies in the interval $[-1, 1]$. Thus, unlike the correlation coefficients ρ_{GM} and ρ_{HH}, $\rho_{HW}(A, B)$ tells us not only the degree of the relationship between A and B but also whether A and B are positively or negatively related.

In conventional statistics, from the random sample, we can consider some statistics that allow us to analyze the set of data and to obtain some information about the population distribution. When we deal with a set of data, our interest often is on the information about the relationships among the variables. We often employ the correlation coefficient rather than covariance of two random variables, because the correlation coefficient is not only a measure to describe the type and the strength of the linear relationship of two random variables, but also is bounded between -1 and 1.

In this paper, we extend the correlation between crisp sets to intuitionistic fuzzy sets, we adopt the method from conventional statistics to develop the formula for the correlation coefficient between the intuitionistic fuzzy sets, because we obtain grades of membership function and grades of nonmembership function for each individual observation of the random sample, even though the membership and nonmembership functions of the intuitionistic fuzzy sets can be either discrete or continuous. The correlation coefficient computed in this paper, which lie in $[-1, 1]$, gives us more information than the correlation coefficients ρ_{GM} and ρ_{HH}, and not only provides the strength of the relationship between the intuitionistic fuzzy sets, but also shows that the intuitionistic fuzzy sets are positively or negatively related, which is better than the correlation coefficients ρ_{GM} and ρ_{HH}, because they provide only the strength of the relation.

2. Correlation coefficient of intuitionistic fuzzy sets

When we have a random sample from a crisp set, with the corresponding membership grades and nonmembership grades of some specific intuitionistic fuzzy set defined on each observation, we are interested in finding the sample estimates of the mean of the differences of membership grades and nonmembership grades and the variations of the differences among the sample of that intuitionistic fuzzy set. Now we first define the sample mean and sample variance of the differences of membership grades and nonmembership grades of an intuitionistic fuzzy set as follows:

Definition 2.1. Let $\{x_1, x_2, \ldots, x_n\}$ be a random sample of size n from X, having the membership grades $\mu_A(x_i)$ and the nonmembership grades $\nu_A(x_i)$ of an intuitionistic fuzzy set A defined on a crisp set X. The sample mean and sample variance of the differences $\rho_A(x_i) = \mu_A(x_i) - \nu_A(x_i)$ of membership grades and nonmembership grades of A defined as follows:

$$\bar{\rho}_A = \frac{\sum_{i=1}^n \rho_A(x_i)}{n}, \quad S_A^2 = \frac{\sum_{i=1}^n \left(\frac{\rho_A(x_i) - \bar{\rho}_A}{2}\right)^2}{n-1}, \quad (1)$$

where S_A^2 represents the degree of variations of $\rho_A(x_i)$ of A over $\{x_1, x_2, \ldots, x_n\}$, and then the sample standard deviation will be $S_A = \sqrt{S_A^2}$.

Now, let us consider the situation when there are two intuitionistic fuzzy sets A and B defined on a crisp set X, then A and B can be expressed as

$$A = \{(x, \mu_A(x), \nu_A(x)) : x \in X\},$$
$$B = \{(x, \mu_B(x), \nu_B(x)) : x \in X\},$$

where $\mu_A, \mu_B : X \to [0, 1]$ and $\nu_A, \nu_B : X \to [0, 1]$.

If we have a random sample from a crisp set, with corresponding pairs of membership grades and pairs of nonmembership grades of two intuitionistic fuzzy sets we have interested in, likely we will compare the grades of membership functions and the grades of nonmembership functions of the two intuitionistic fuzzy sets to see if there is any linear relationship between the two intuitionistic fuzzy sets, we need a formula for the sample correlation coefficient of two intuitionistic fuzzy sets to show the relationship between them.

Definition 2.2. Assume that there is a random sample $\{x_1, x_2, \ldots, x_n\}$ of X,having the membership grades $\mu_A(x_i)$ and $\mu_B(x_i)$ and the nonmembership grades $\nu_A(x_i)$ and $\nu_B(x_i)$ of intuitionistic fuzzy sets A and B defined on a crisp set X, respectively, let us define the correlation coefficient $r_{A,B}$ between the intuitionistic fuzzy sets A and B as follows:

$$r_{A,B} = \frac{\sum_{i=1}^n \frac{(\rho_A(x_i) - \bar{\rho}_A)}{2} \cdot \frac{(\rho_B(x_i) - \bar{\rho}_B)}{2} / (n-1)}{S_A \cdot S_B}, \quad (2)$$

where $\bar{\rho}_A$ and $\bar{\rho}_B$ denote the averages of $\rho_A(x_i)$ and $\rho_B(x_i)$ of intuitionistic fuzzy sets A and B over the random sample, S_A and S_B are the sample standard deviations of the differences $\rho_A(x_i)$ and $\rho_B(x_i)$ of intuitionistic fuzzy sets A and B, respectively.

In spite of the values of the membership and nonmembership functions are constrained in $[0, 1]$, the value of the correlation coefficient obtained from (2) is in $[-1, 1]$. This shows not only the degree of the relationship between the intuitionistic fuzzy sets, but also the fact that these two sets are positively or negatively related.

Theorem 2.3. Let A and B be two intuitionistic fuzzy sets on a crisp set X having membership functions μ_A and μ_B and nonmembership functions ν_A and ν_B respectively, and let us take a random sample $\{x_1, x_2, \ldots, x_n\}$ from X having sequences of paired membership grades, $((\mu_A(x_1), \mu_B(x_1)), \ldots, (\mu_A(x_n), \mu_B(x_n)))$, and paired nonmembership grades, $((\nu_A(x_1), \nu_B(x_1)), (\nu_A(x_2), \nu_B(x_2)), \ldots, (\nu_A(x_n), \nu_B(x_n)))$, of intuitionistic fuzzy sets A and B, respectively. Then the sample correlation coefficient $r_{A,B}$ defined by (2) lies in $[-1, 1]$, that is, $|r_{A,B}| \le 1$.

Proof. For $\{x_1, x_2, \ldots, x_n\} \subset X$,

(a) If $\mu_B(x_i) = a\mu_A(x_i) + b$ and $\nu_B(x_i) = a\nu_A(x_i) + b$ for some $a > 0$ and b, then $r_{A,B} = 1$, if $\mu_B(x_i) = a\mu_A(x_i) + b$ and $\nu_B(x_i) = a\nu_A(x_i) + b$ for some $a < 0$ and b, then $r_{A,B} = -1$.

Let $\bar{\rho}_A$ and $\bar{\rho}_B$ be defined as in (1),

$$\bar{\rho}_A = \frac{\sum_{i=1}^n \rho_A(x_i)}{n} \text{ and } \bar{\rho}_B = \frac{\sum_{i=1}^n \rho_B(x_i)}{n}.$$

Since $\rho_B(x_i) = \mu_B(x_i) - \nu_B(x_i) = a\rho_A(x_i)$,

$$\bar{\rho}_B = \frac{\sum_{i=1}^n a\rho_A(x_i)}{n} = \frac{a\sum_{i=1}^n \rho_A(x_i)}{n} = a\bar{\rho}_A. \quad (3)$$

Let S_A^2 and S_B^2 be defined as in (1),

$$S_A^2 = \frac{\sum_{i=1}^n \left(\frac{\rho_A(x_i) - \bar{\rho}_A}{2}\right)^2}{n-1},$$

$$S_B^2 = \frac{\sum_{i=1}^n \left(\frac{\rho_B(x_i) - \bar{\rho}_B}{2}\right)^2}{n-1}.$$

Then by using (3),

$$\begin{aligned} S_B^2 &= \frac{\sum_{i=1}^n \left(\frac{a\rho_A(x_i) - a\bar{\rho}_B}{2}\right)^2}{n-1} \\ &= \frac{a^2\sum_{i=1}^n \left(\frac{\rho_A(x_i) - \bar{\rho}_B}{2}\right)^2}{n-1} \\ &= a^2 S_A^2, \end{aligned}$$

and then the sample standard deviation of B is $S_B = |a| \cdot S_A$.

The sample correlation coefficient $r_{A,B}$ of intuitionistic fuzzy sets A and B as defined in (2),

$$\begin{aligned} r_{A,B} &= \frac{\sum_{i=1}^n \frac{(\rho_A(x_i) - \bar{\rho}_A)}{2} \cdot \frac{(\rho_B(x_i) - \bar{\rho}_B)}{2} / (n-1)}{S_A \cdot S_B} \\ &= \frac{\sum_{i=1}^n \frac{(\rho_A(x_i) - \bar{\rho}_A)}{2} \cdot \frac{(a\rho_A(x_i) - a\bar{\rho}_A)}{2} / (n-1)}{S_A \cdot S_B} \\ &= \frac{a\sum_{i=1}^n (\frac{\rho_A(x_i) - \bar{\rho}_A}{2})^2 / (n-1)}{S_A \cdot S_B} \\ &= \frac{aS_A^2}{|a|S_A^2} = \frac{a}{|a|}. \end{aligned}$$

So that $r_{A,B} = 1$ if $a > 0$, and $r_{A,B} = -1$ if $a < 0$.

(b) For the other cases that $\mu_B(x_i) = a\mu_A(x_i) + b$ and $\nu_B(x_i) = a\nu_A(x_i) + b$, $|r_{A,B}| \le 1$.

By the Cauchy-Schwarz inequality,

$$\begin{aligned} &(\sum_{i=1}^n (\rho_A(x_i) - \bar{\rho}_A)(\rho_B(x_i) - \bar{\rho}_B))^2 \\ &\le (\sum_{i=1}^n (\rho_A(x_i) - \bar{\rho}_A)^2) \cdot (\sum_{i=1}^n (\rho_B(x_i) - \bar{\rho}_B)^2), \end{aligned}$$

or

$$\frac{(\sum_{i=1}^n (\rho_A(x_i) - \bar{\rho}_A)(\rho_B(x_i) - \bar{\rho}_B))^2}{(\sum_{i=1}^n (\rho_A(x_i) - \bar{\rho}_A)^2) \cdot (\sum_{i=1}^n (\rho_B(x_i) - \bar{\rho}_B)^2)} \le 1,$$

and then

$$\begin{aligned} &\frac{(\sum_{i=1}^n \frac{(\rho_A(x_i) - \bar{\rho}_A)}{2} \cdot \frac{(\rho_B(x_i) - \bar{\rho}_B)}{2} / (n-1))^2}{\frac{\sum_{i=1}^n (\frac{\rho_A(x_i) - \bar{\rho}_A}{2})^2}{n-1} \cdot \frac{\sum_{i=1}^n (\frac{\rho_B(x_i) - \bar{\rho}_B}{2})^2}{n-1}} \\ &= \frac{(\sum_{i=1}^n \frac{(\rho_A(x_i) - \bar{\rho}_A)}{2} \cdot \frac{(\rho_B(x_i) - \bar{\rho}_B)}{2} / (n-1))^2}{S_A^2 \cdot S_B^2} \\ &= (r_{A,B})^2 \\ &\le 1. \end{aligned}$$

Thus we have $|r_{A,B}| \le 1$. □

Example 2.4. Let X = {All the girls in the university}, our interests are the estimates on the degree of charm and the degree of prettiness of the girls in this university, we cannot measure over all the girls in the campus, therefore, a sample of six girls was taken at random from the campus, $\{x_1, x_2, \ldots, x_6\}$ = {Kim, Park, Cho, Jung, Hwang, Yoon}.

Now, let us define two intuitionistic fuzzy sets over the crisp set X, A = {Charming girl} and B = {Pretty girl} and then we collect information and obtain the membership grades and nonmembership grades of these six girls concerning intuitionistic fuzzy sets A and B as follows:

Name	(μ_A, ν_A)	(μ_B, ν_B)
Kim	(0.79,0.18)	(0.72,0.26)
Park	(0.94,0.03)	(0.98,0.01)
Cho	(0.65,0.32)	(0.60,0.39)
Jung	(0.84,0.20)	(0.90,0.08)
Hwang	(1.00,0.00)	(0.83,0.16)
Yoon	(0.86,0.12)	(0.88,0.10)

By Definition 2.2, we can compute the sample estimates of the averages of ρ_A and ρ_B

$$\bar{\rho}_A = \frac{\sum_{i=1}^6 \rho_A(x_i)}{6} = 0.71, \ \bar{\rho}_B = \frac{\sum_{i=1}^6 \rho_B(x_i)}{6} = 0.65$$

and the sample variances of intuitionistic fuzzy sets A and B are computed over six samples,

$$S_A^2 = \frac{\sum_{i=1}^{6} \left(\frac{\rho_A(x_i) - \bar{\rho}_A}{2} \right)^2}{6-1} = 0.0142,$$

$$S_B^2 = \frac{\sum_{i=1}^{6} \left(\frac{\rho_B(x_i) - \bar{\rho}_B}{2} \right)^2}{6-1} = 0.0189.$$

Thus we can see that the girls in the university have more degrees of charm than the prettiness, according to this particular random sample girls.

Next, we will estimate the correlation coefficient between the intuitionistic fuzzy sets A and B, that is the correlation between the charm and the prettiness of the girls in this university. Using equation (3),

$$\begin{aligned} r_{A,B} &= \frac{\sum_{i=1}^{6} \frac{(\rho_A(x_i) - \bar{\rho}_A)}{2} \cdot \frac{(\rho_B(x_i) - \bar{\rho}_B)}{2} / (6-1)}{S_A \cdot S_B} \\ &= 0.7577. \end{aligned}$$

Thus this value gives us the information that the two intuitionistic fuzzy sets, {Charming girl} and {Pretty girl}, are positively and closely related with a strength of 0.7577.

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