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Bayesian Estimation of the Reliability Function of the Burr Type XII Model under Asymmetric Loss Function*

Chansoo Kim[1)]

Abstract

In this paper, Bayes estimates for the parameters k, c and reliability function of the Burr type XII model based on a type II censored samples under asymmetric loss functions viz., LINEX and SQUAREX loss functions are obtained. An approximation based on the Laplace approximation method (Tierney and Kadane, 1986) is used for obtaining the Bayes estimators of the parameters and reliability function. In order to compare the Bayes estimators under squared error loss, LINEX and SQUAREX loss functions respectively and the maximum likelihood estimator of the parameters and reliability function, Monte Carlo simulations are used.

Keywords: Burr type XII distribution; Laplace approximation; LINEX loss function; SQUAREX loss function.

1. Introduction

The two-parameter Burr Type XII distribution was first introduced in the literature Burr (1942). Its probability density function, cumulative distribution function are given by respectively,

$$\begin{aligned} f(x) &= ckx^{c-1}(1+x^c)^{-(k+1)}, x, c, k > 0, \\ F(x) &= 1-(1+x^c)^{-k} \end{aligned} \tag{1.1}$$

and the reliability function, $R(t) = (1+t^c)^{-k}$.

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1) Assistant Professor, Department of Applied Statistics, Kongju National University, 182 Shinwan-dong, Kongju, Chungnam, Korea.
E-mail: chanskim@kongju.ac.kr

Its capacity to assume various shapes often permits a good fit when used to describe biological, clinical or other experimental data. The usefulness and properties of the Burr distribution as a failure model are discussed by Dubey (1972, 1973). Some early works can be found in Papadopoulos (1978), Al-Hussaini and Jaheen (1992, 1994), Moore and Papadopoulos (2000) and Ali Mousa and Jaheen (2002).

In the most of approaches, the squared error loss function was used for obtaining the Bayes estimates. But, the symmetric nature of squared error loss (SEL) gives equal weight to overestimation as well as underestimation, while in the estimation of parameter of lifetime model overestimation may be more serious than underestimation or vice-versa. Inappropriateness of SEL has been noticed by Zellner (1986). A number of asymmetric loss functions were proposed for their use. Among these, one of the most popular asymmetric loss function is the LINEX loss function which is introduced by Varian (1975) and further properties of the LINEX loss function have been investigated by Zellner (1986). Thompson and Basu (1996) introduced a further generalization of the LINEX loss function, the SQUAREX loss function, in the context of system reliability estimation. Chaturvedi *et al.* (2000) showed that a Bayes estimator under SQUAREX loss function is a weighted average of a Bayes estimator under LINEX loss function and a Bayes estimator under squared error loss function.

The SQUAREX loss function has the following form;

$$L(\Delta) = b(e^{a\Delta} + d\Delta^2 - a\Delta - 1), \tag{1.2}$$

where $|a| \neq 0$, $b, d \geq 0$ and $\Delta = \hat{\theta} - \theta$ denotes the scalar estimation error in using $\hat{\theta}$ to estimate θ. If $d = 0$, SQUAREX and LINEX loss functions are identical. Hence, SQUAREX loss function represents a generalization of LINEX loss function; it is a richer family of asymmetric loss functions that are appropriate when estimating system reliability.

In this paper, Bayes estimators for the parameters k, c and reliability function $R(t)$ of the Burr type XII model are obtained based on a type II censored samples in section 2. In Section 3, the Laplace approximation(Tierney and Kadane, 1986) is used for obtaining the Bayes estimators of the parameters and reliability function. In Section 4, in order to compare MLE and Bayes estimators, these estimators are computed via Monte Carlo simulation study.

2. Bayes Estimation

Let $X_1 < X_2 < \cdots < X_r$ be a censored sample of size r obtained from Burr type XII distribution with probability density function (1.1). Then the likelihood function of c and k on $\mathbf{x} = (x_1, \ldots, x_r)$ is given by

$$\begin{aligned} L(k,c|\mathbf{x}) &= \frac{n!}{(n-r)!} c^r k^r \prod_{i=1}^{r} \left(\frac{x_i^{c-1}}{1+x_i^c} \right) \\ &\quad \times \exp\left(-k \left(\sum_{i=1}^{r} \log(1+x_i^c) + (n-r)\log(1+x_r^c) \right) \right). \qquad (2.1) \end{aligned}$$

The log-likelihood function is proportional to

$$\begin{aligned} l(k,c) &\propto r\log c + r\log k \\ &+ (c-1)\sum_{i=1}^{r} \log x_i - (k+1)\log(1+x_i^c) - (n-r)k\log(1+x_r^c). \qquad (2.2) \end{aligned}$$

Assuming that the parameters c and k are both unknown, the maximum likelihood estimate (MLE) $\hat{k}_{MLE}$ and $\hat{c}_{MLE}$ of k and c can be obtained iteratively by solving two equations which differentiate the equation in (2.2) with respect to k and c and equate each to zero. For a given t, the MLE $\hat{R}(t)_{MLE}$ can be obtained by replacing k by $\hat{k}_{MLE}$ and c by $\hat{c}_{MLE}$ in (1.1).

Since the parameter k and c are assumed to be unknown, Al-Hussaini and Jaheen (1992) suggested a bivariate prior density as the following forms;

$$\pi(k,c) = \pi_1(k|c)\pi_2(c), \qquad (2.3)$$

where

$$\pi_1(k|c) = \frac{c^{\alpha+1}}{\Gamma(\alpha+1)} \gamma^{\alpha+1} k^{\alpha} \exp\left(-\frac{kc}{\gamma}\right), \alpha > -1, \gamma > 0 \qquad (2.4)$$

is a gamma prior density function when c is known and

$$\pi_2(c) = \frac{1}{\Gamma(\delta)\beta^{\delta}} c^{\delta-1} \exp\left(-\frac{c}{\beta}\right), \beta > 0, \delta > 0 \qquad (2.5)$$

is a gamma prior density function. Therefore, the bivariate prior density of k and c can be written as

$$\pi(k,c) = \frac{c^{\delta+\alpha} k^{\alpha}}{\Gamma(\delta)\Gamma(\alpha+1)\gamma^{\delta}\beta^{\delta}} \exp\left\{ -c\left(\frac{k}{\gamma} + \frac{1}{\beta} \right) \right\}. \qquad (2.6)$$

It follows from (2.1) and (2.6) that the joint posterior density function of k and c given $\mathbf{X}$ is proportion to

$$\pi(k, c|x) \propto c^{r+\delta+\alpha} k^{\alpha+r} \prod_{i=1}^{r} \left(\frac{x_i^{c-1}}{1+x_i^c} \right)$$

$$\times \exp\left\{ -k \left(\sum_{i=1}^{r} \ln(1+x_i^c) + (n-r)\ln(1+x_r^c) + \frac{c}{\gamma} \right) - \frac{c}{\beta} \right\}. \quad (2.7)$$

Let $\phi(c, k)$ be a function of the parameters k and c. Then the Bayes estimator $\hat{\phi}_{SEL}$ of a function $\phi(k, c)$ relative to SEL takes the form

$$\hat{\phi}_{SEL} = E(\phi(k, c)|X), \quad (2.8)$$

where $E(\cdot|X)$ denotes a posterior expectation.

Under LINEX loss function given in (1.2) with $d = 0$, the Bayes estimator $\hat{\phi}_{LIX}$ of a function $\phi(c, k)$ is obtained by

$$\begin{aligned} \hat{\phi}_{LIX} &= -\frac{1}{a} \ln \left(E(e^{-a\phi(k,c)}|X) \right) \\ &= -\frac{1}{a} \ln \left(\int\int e^{-a\phi(k,c)} \pi(k, c|x) dk dc \right). \end{aligned} \quad (2.9)$$

The above Bayes estimators $\hat{\phi}_{SEL}$ and $\hat{\phi}_{LIX}$ in (2.8) and (2.9) can not be derived in a closed form. Therefore, in such situations, we can use numerical integration technique, which can be computationally intensive, especially in a high dimensional parameter space. One can also use approximate methods such as the approximate form due to Lindley (1980) or that of Tierney and Kadane (1986). We adopt here the Tierney and Kadane (1986) approximation.

Also, under SQUAREX loss function given in (1.2), the Bayes estimator $\hat{\phi}_{SQX}$ of a function $\phi(c, k)$ is

$$\hat{\phi}_{SQX} = \hat{\phi}_{LIX} + \frac{1}{a} \ln \left(1 + \frac{2d}{a} (\hat{\phi}_{SEL} - \hat{\phi}_{SQX}) \right) \quad (2.10)$$

where $\hat{\phi}_{SEL}$ and $\hat{\phi}_{SQX}$ are the Bayes estimators relative to SEL and LINEX loss function. The above Bayes estimator (2.10) can be solved numerically to get $\hat{\phi}_{SQX}$, but explicit expressions for this estimator can not be obtained.

Remark 2.1 For small values of a, the SQUAREX loss function is almost symmetric and not far from a squared error loss function. On expanding $e^{a\Delta} = 1 + a\Delta + (a^2\Delta^2/2)$, $L(\Delta) = b\left((a^2/2) + d\right)\Delta^2$, a squared error loss function. In this case, the Bayes estimates of the parameters k and c and reliability function under SQUAREX and LINEX loss functions are not far different from those obtained with a squared error loss function.

Remark 2.2 2. If d in equation (2.10) is small in comparison to a so that the terms of order $O\left((d/a)\right)$ is negligible, then (2.10) can be approximately rewritten as

$$\hat{\phi}_{APM} = \frac{a^2\hat{\phi}_{LIX} + 2d\hat{\phi}_{SEL}}{a^2 + 2d}. \tag{2.11}$$

The above Bayes estimator $\hat{\phi}_{APM}$ under the SQUAREX loss function is a weighted average of the Bayes estimator under LINEX loss function and the Bayes estimator under the squared error loss function.

3. Laplace Approximation

Let $\theta = (c, k)$ and $l(\theta; x)$ be the likelihood function of θ based on the n observations and $\pi(\theta|x)$ denote the posterior distribution of θ. Then the posterior mean of a function $\phi(\theta)$ can be written as

$$E(\phi(\theta)|X) = \int \phi(\theta)\pi(\theta|x)d\theta = \frac{\int e^{nL^*}}{\int e^{nL}}, \tag{3.1}$$

where

$$\begin{aligned} L(\theta) &= \frac{1}{n}\ln \pi(\theta|x), \\ L^*(\theta) &= L(\theta) + \frac{1}{n}\ln \phi(\theta). \end{aligned}$$

Following Tierney and Kadane (1986), equation (3.1) can be approximated as following forms;

$$\begin{aligned} E(\phi(\theta)|X) &= \left(\frac{|\Sigma^*|}{|\Sigma|}\right)^{\frac{1}{2}} \exp\left(n(L^*(\hat{\theta}^*) - L(\hat{\theta}))\right) \\ &= \left(\frac{|\Sigma^*|}{|\Sigma|}\right)^{\frac{1}{2}} \frac{\phi(\hat{\theta}^*)\pi(\hat{\theta}^*|x)}{\pi(\hat{\theta}|x)}, \end{aligned} \tag{3.2}$$

where $\hat{\theta}^*$ and $\hat{\theta}$ maximize $L^*(\theta)$ and $L(\theta)$, respectively and Σ^* and Σ are minus the inverse Hassians of $L^*(\theta)$ and $L(\theta)$ at $\hat{\theta}^*$ and $\hat{\theta}$ respectively.

We apply this approximation to obtain the Bayes estimators of the parameters k, c and the reliability function $R(t)$ given by (1.1). In this case, the functions L^* and L are given by respectively

$$L(c,k) = \frac{1}{n}\Bigg((r+\delta+\alpha)\ln c + (\alpha+r)\ln k + (c-1)\sum_{i=1}^{r}\ln x_i - \frac{c}{\beta}$$

$$-(k+1)\sum_{i=1}^{r}\ln(1+x_i^c) - (n-r)k\ln(1+x_r^c) - \frac{kc}{\gamma}\Bigg) \tag{3.3}$$

and

$$L^*(c,k) = L(c,k) + \frac{1}{n}\ln\phi(c,k). \tag{3.4}$$

Let $L_1 = \partial L(c,k)/\partial c$, $L_2 = \partial L(c,k)/\partial k$, $L_{11} = \partial^2 L(c,k)/\partial c^2$, $L_{12} = \partial^2 L(c,k)/\partial c\partial k$ and $L_{22} = \partial^2 L(c,k)/\partial k^2$ be the first and second derivatives of $L(c,k)$. Then, the posterior mode $(\hat{c},\hat{k})$ is obtained by equating L_1 and L_2 to zero and solving the resulting nonlinear equations in c and k. From L_{11}, L_{12} and L_{22}, one can see that

$$|\Sigma| = \frac{1}{L_{11}L_{22} - L_{12}^2} \tag{3.5}$$

evaluated at the posterior mode $(\hat{c},\hat{k})$.

Similar derivatives are needed to determine the mode $(\hat{c}^*,\hat{k}^*)$ of $L^*(c,k)$ and $|\Sigma^*|$. Let $L_1^* = \partial L^*(c,k)/\partial c$, $L_2^* = \partial L^*(c,k)/\partial k$, $L_{11}^* = \partial^2 L^*(c,k)/\partial c^2$, $L_{12}^* = \partial^2 L^*(c,k)/\partial c\partial k$ and $L_{22}^* = \partial^2 L^*(c,k)/\partial k^2$ be the first and second derivatives of $L^*(c,k)$. Differentiate (3.4) with respect to c and k and equate each result L_1^* and L_2^* to zero. The mode $(\hat{c}^*,\hat{k}^*)$ of c and k can be obtained iteratively by solving the two resulting equations. Using L_{11}^*, L_{12}^* and L_{22}^*, we get

$$|\Sigma^*| = \frac{1}{L_{11}^*L_{22}^* - L_{12}^{*2}} \tag{3.6}$$

evaluated at the posterior mode $(\hat{c}^*,\hat{k}^*)$.

Remark 3.1 All of these values of $L^*(c,k)$ and the first and second derivatives of $L^*(c,k)$ can be found if $\phi(c,k)$ has an explicit functional form in the parameters c and k.

Substituting from (3.5) and (3.6) in (3.2), the Bayes estimator $\hat{\phi}_{LIX}$ of a function $\phi(c, k)$ under LINEX loss function takes of the form

$$\hat{\phi}_{LIX} = -\frac{1}{a} \ln \left\{ \left(\frac{L_{11}L_{22} - L_{12}^2}{L_{11}^*L_{22}^* - L_{12}^{*2}} \right)^{\frac{1}{2}} \frac{\phi(\hat{c}^*, \hat{k}^*)\pi(\hat{c}^*, \hat{k}^*|x)}{\pi(\hat{c}, \hat{k}|x)} \right\}, \tag{3.7}$$

where $\pi(c, k|x)$ is the posterior density function given by (2.7) evaluated at the modes $(\hat{c}^*, \hat{k}^*)$ and $(\hat{c}, \hat{k})$ of the functions $L^*(c, k)$ and $L(c, k)$, respectively.

Under SQUAREX loss function, the Bayes estimator $\hat{\phi}_{SQX}$ of a function $\phi(c, k)$ given by (2.10) can be solved by Newton-Raphson iteration scheme using the nonlinear equations as following forms;

$$\exp(a(\phi - \hat{\phi}_{LIX})) + \frac{2d}{a}(\phi - \hat{\phi}_{SEL}) - 1 = 0. \tag{3.8}$$

The solution of equation given by (3.8) yields the Bayes estimator $\hat{\phi}_{SQX}$ under SQUAREX loss function.

Remark 3.2 From (3.7), the Bayes estimators $\hat{c}_{LIX}$, $\hat{k}_{LIX}$ and $\hat{R}(t)_{LIX}$ of the parameters c, k and the reliability function of $R(t)$ for a given $\phi(c, k)$ takes the following forms respectively.

1 If $\phi(c, k) = e^{-ac}$ then $L^*(c, k) = L(c, k) - ac/n$.

$$\hat{c}_{LIX} = -\frac{1}{a} \ln \left\{ \left(\frac{L_{11}L_{22} - L_{12}^2}{L_{11}^*L_{22}^* - L_{12}^{*2}} \right)^{\frac{1}{2}} \frac{e^{-a\hat{c}^*}\pi(\hat{c}^*, \hat{k}^*|x)}{\pi(\hat{c}, \hat{k}|x)} \right\},$$

where $L(c, k)$ is given by (3.3).

2 If $\phi(c, k) = e^{-ak}$ then $L^*(c, k) = L(c, k) - ak/n$.

$$\hat{k}_{LIX} = -\frac{1}{a} \ln \left\{ \left(\frac{L_{11}L_{22} - L_{12}^2}{L_{11}^*L_{22}^* - L_{12}^{*2}} \right)^{\frac{1}{2}} \frac{e^{-a\hat{k}^*}\pi(\hat{c}^*, \hat{k}^*|x)}{\pi(\hat{c}, \hat{k}|x)} \right\}.$$

3 If $\phi(c, k) = R(t) = \exp\left(-a(1 + t^c)^{-k}\right)$ then $L^*(c, k) = L(c, k) - (a/n)(1 + t^c)^{-k}$.

$$\hat{R}(t)_{LIX} = -\frac{1}{a} \ln \left\{ \left(\frac{L_{11}L_{22} - L_{12}^2}{L_{11}^*L_{22}^* - L_{12}^{*2}} \right)^{\frac{1}{2}} \frac{\exp\left(-a(1 + t^{\hat{c}^*})^{-\hat{k}^*}\right)\pi(\hat{c}^*, \hat{k}^*|x)}{\pi(\hat{c}, \hat{k}|x)} \right\}.$$

4 For $\phi(c,k)$ as given by 1, 2 and 3, the Bayes estimators relative to SQUAREX loss function, $\hat{c}_{SQX}$, $\hat{k}_{SQX}$ and $\hat{R}(t)_{SQX}$, are obtained by using (3.8).

4. Illustrative Examples

First, we use an numerical example from Wingo (1983) based on the failure times of certain electronic components and using type II censoring: 30 components were involved in the life test which was censored after 20 failures. The failure times (in month) are 0.1, 0.1, 0.2, 0.3, 0.4, 0.5, 0.5, 0.6, 0.7, 0.8, 0.9, 0.9, 1.2, 1.6, 1.8, 2.3, 2.5, 2.6, 2.9, 3.1.

For $n = 30$ and $r = 20$ and $t = 0.5$, the MLE and Bayes estimators under SEL, LINEX and SQAREX loss functions of the parameters c, k and $R(t)$ are computed and presented in Table 4.1 and Table 4.2.

In Table 4.1 and Table 4.2, the Bayes estimates of the parameters c and k and reliability function relative to LINEX and SQUAREX loss functions are sensitive to the values of the a and d. In Table 4.1, if d is small in comparison with a (for example, $a = 5$ and $d = 0.01$), it is seen that the Bayes estimates $\hat{c}_{SQX}$ and $\hat{k}_{SQX}$ and the approximation estimates $\hat{c}_{APM}$ and $\hat{k}_{APM}$ are almost same values and also the Bayes estimates relative to LINEX and SQUAREX loss functions

Table 4.1: MLE and Bayes estimates of the parameters c and k for various a and d

a	d	$\hat{c}_{MLE}$ $\hat{k}_{MLE}$	$\hat{c}_{SEL}$ $\hat{k}_{SEL}$	$\hat{c}_{LIX}$ $\hat{k}_{LIX}$	$\hat{c}_{SQX}$ $\hat{k}_{SQX}$	$\hat{c}_{APM}$ $\hat{k}_{APM}$
5	5	1.2912	1.3103	1.1681	1.2060	1.2087
		0.6378	0.6728	0.6171	0.6326	0.6330
2	2			1.2478	1.2786	1.2791
				0.6489	0.6608	0.6609
5	0.01			1.1681	1.1682	1.1682
				0.6178	0.6171	0.6171
0.01	5			1.3099	1.3103	1.3103
				0.6722	0.6728	0.6728
-5	5			1.5216	1.4670	1.4612
				0.7469	0.7265	0.7258

Table 4.2: MLE and Bayes estimates of the reliability of $R(t)$ for various a and d

a	d	$\hat{R}(t)_{MLE}$	$\hat{R}(t)_{SEL}$	$\hat{R}(t)_{LIX}$	$\hat{R}(t)_{SQX}$
5	5	0.8037	0.8020	0.7964	0.7980
1	5			0.8037	0.8022
.01	5			0.8055	0.8020
-5	5			0.8136	0.8103
5	.01			0.7964	0.7964

have almost the same. For small value a, the SQUAREX loss function is almost symmetric and not far from a SEL.

In Table 4.2, if a goes to a negative value, then it tends to give more weight to overestimation. Otherwise, it gives more weight to underestimation. For small value d, the estimators $\hat{R}(t)_{LIX}$ and $\hat{R}(t)_{SQX}$ are almost same.

In order to compare MLE and Bayes estimators of the parameters k, c and reliability function $R(t)$ under LINEX and SQUAREX loss functions, Monte Carlo simulation is performed. The following steps summarize the simulation:

(1) For a given value $\alpha = 6$, $\delta = 3$, $\gamma = 5$ and $\beta = 7$, generate $c = 4.9195$ and $k = 7.6922$ (can be used as the true values) from the prior density functions in (2.4) and (2.5).

(2) Using the results for k and c from step (1), a sample size n is generated from the Burr type XII distribution using the inverse CDF.

(3) The MLE $\hat{R(t)}_{MLE}$ and Bayes estimators $\hat{R(t)}_{SEL}$, $\hat{R(t)}_{LIX}$ and $\hat{R(t)}_{SQX}$ under SEL, LINEX and SQUAREX loss functions of the reliability function $R(t)$ for some given value of $t = 0.7$ are computed. The true value of $R(t)$ with $c = 4.9195$ and $k = 7.6922$ is $R(0.7) = 0.2931$.

(4) Steps (1)–(3) are repeated 500 times.

(5) The average values and RMSE of the estimates are calculated over the 500 samples.

$$\text{RMSE} = \sqrt{\frac{1}{500}\sum_{i=1}^{500}(\hat{R}_i - R_0)^2}$$

where R_0 is true value of R and $\hat{R}$ is estimate of R.

Table 4.3 displays the MLE and the Bayes estimates relative to SEL, LINEX and SQUAREX loss functions of the reliability function $R(t)$ and its corresponding RMSE for different sample and censoring size. The true value of $R(t)$ with $c = 4.9195$ and $k = 7.6922$ is $R(0.7) = 0.2931$. It is seen that the Bayes estimates are better than the MLE and for LINEX loss function, the Bayes estimator performed better than the others in the sense of comparing the RMSE of the estimates. As sample increases, the RMSE decreases which is the case in our computer simulation.

Table 4.4 represents the MLE and Bayes estimates of the reliability $R(t)$ relative to symmetric and asymmetric loss functions (SEL, LINEX and SQAREX)

Table 4.3: MLE and Bayes estimates of the reliability $R(t)$ and RMSE for different sample, n and censoring sizes, $r(R(t)$=0.2931, t=0.7, $\alpha = 6$, $\delta = 3$, $\gamma = 5$, $\beta = 7$, a=1 and $d = 1$)

n	r	$\hat{R}(t)_{MLE}$	$\hat{R}(t)_{SEL}$	$\hat{R}(t)_{LIX}$	$\hat{R}(t)_{SQX}$
20	15	0.2786	0.3278	0.3270	0.3275
		(0.009157)	(0.006886)	(0.006258)	(0.006673)
	20	0.2930	0.3253	0.3244	0.3250
		(0.007254)	(0.006735)	(0.006215)	(0.006560)
50	40	0.2902	0.3078	0.3090	0.3082
		(0.003313)	(0.002655)	(0.002484)	(0.002596)
	50	0.2953	0.3075	0.3081	0.3077
		(0.002546)	(0.0025019)	0.002376)	(0.002459)
100	80	0.2907	0.2987	0.3001	0.2992
		(0.001650)	(0.001406)	(0.001337)	(0.001382)
	100	0.2933	0.2991	0.2998	0.2994
		(0.001386)	(0.001347)	(0.001300)	(0.001331)

Table 4.4: MLE and Bayes estimates of the reliability $R(t)$ and RMSE for different sample sizes $n(R(t)$=0.2931, t=0.7, $\alpha = 6$, $\delta = 3$, $\gamma = 5$, $\beta = 7$)

n	a	d	$\hat{R}(t)_{MLE}$	$\hat{R}(t)_{SEL}$	$\hat{R}(t)_{LIX}$	$\hat{R}(t)_{SQX}$
50	0.01	2	0.2953	0.3075	0.3092	0.3075
			(0.002546)	(0.002502)	(0.002427)	(0.002502)
	2	0.01			0.3069	0.3069
					(0.002329)	(0.002330)
100	2	0.01	0.2933	0.2991	0.2991	0.2992
			(0.001386)	(0.001346)	(0.001288)	(0.001288)

for different values of a and d. For small value of d, the Bayes estimates $\hat{R}(t)_{LIX}$ and $\hat{R}(t)_{SQX}$ and its RMSE have almost same value. While, for small value of a, the RMSE relative to asymmetric loss function are not far different from those obtained with SEL. One would expect that as the sample size increases, the RMSE decreases.

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