Bull. Korean Math. Soc. **44** (2007), No. 3, pp. 407–414

SOME EXAMPLES OF QUASI-ARMENDARIZ RINGS

Ebrahim Hashemi

Abstract. In [12], McCoy proved that if R is a commutative ring, then whenever $g(x)$ is a zero-divisor in $R[x]$, there exists a nonzero $c \in R$ such that $cg(x) = 0$. In this paper, first we extend this result to monoid rings. Then for a monoid M, we give some examples of M-quasi-Armendariz rings which are a generalization of quasi-Armendariz rings. Every reduced ring is M-quasi-Armendariz for any unique product monoid M and any strictly totally ordered monoid $(M, \leq)$. Also $T_4(R)$ is M-quasi-Armendariz when R is reduced and M-Armendariz.

1. Introduction

Throughout this paper R denotes an associative ring with identity. Rege and Chhawchharia [15] introduced the notion of an Armendariz ring. A ring R is called *Armendariz* if whenever polynomials $f(x) = a_0+a_1x+\cdots+a_nx^n, g(x) = b_0 + b_1x + \cdots + b_mx^m \in R[x]$ satisfy $f(x)g(x) = 0$, then $a_ib_j = 0$ for each i, j. Some properties of Armendariz rings have been studied in Rege and Chhawchharia [15], Armendariz [1], Anderson and Camillo [2], and Kim and Lee [9]. According to Hirano [5], a ring R is called to be *quasi-Armendariz* if whenever polynomials $f(x) = a_0+a_1x+\cdots+a_nx^n, g(x) = b_0+b_1x+\cdots+b_mx^m \in R[x]$ satisfy $f(x)R[x]g(x) = 0$, then $a_iRb_j = 0$ for each i, j. In [5], Hirano studied some properties of quasi-Armendariz rings. In [17], Zhongkui studied a generalization of Armendariz rings, which is called M-Armendariz rings, where M is a monoid. A ring R is called *M-Armendariz* if whenever $\alpha = a_1g_1 + \cdots + a_ng_n, \beta = b_1h_1 + \cdots + b_mh_m \in R[M]$, with $g_i, h_j \in M$ satisfy $\alpha\beta = 0$, then $a_ib_j = 0$ for each i, j. Recall that a monoid M is called a *u.p.-monoid* (unique product monoid) if for any two nonempty finite subset $A, B \subseteq M$ there exists an element $g \in M$ uniquely presented in the form ab where $a \in A$ and $b \in B$. The class of u.p.-monoid is quite large and important (see [3, 13, 14]). For example, this class includes the right or left ordered monoids, submonoids of a free group, and torsion-free nilpotent groups. Every u.p.-monoid M has no non-unity element of finite order. For $\alpha = a_1g_1 + \cdots + a_ng_n \in R[M]$ with $a_i \neq 0$ for each i, *length*(α) is defined to be $n - k + 1$.

Received December 14, 2005.
2000 *Mathematics Subject Classification.* 16S36, 16N60, 16P60.
Key words and phrases. Armendariz rings, quasi-Armendariz rings, monoid rings, unique product monoid rings.

In this paper, for a monoid M, we give some examples of M-quasi-Armendariz rings which are a generalization of quasi-Armendariz rings. Every reduced ring is M-quasi-Armendariz for any unique product monoid M and any strictly totally ordered monoid $(M, \leq)$. Also, $T_4(R)$ is M-quasi-Armendariz when R is reduced and M-Armendariz.

2. Some examples of quasi-Armendariz rings

McCoy [12] proved that if R is a commutative ring, then whenever $g(x)$ is a zero-divisor in $R[x]$ there exists a nonzero element $c \in R$ such that $cg(x) = 0$. Hirano [5] extend this result to a non commutative ring as follow. If $r_{R[x]}(f(x)R[x]) \neq 0$ for $f(x) \in R[x]$, then $r_{R[x]}(\alpha R[x]) \cap R \neq 0$. We shall generalize this result to monoid rings as follows:

Theorem 2.1. *Let M be a u.p.-monoid or $(M, \leq)$ be a totally ordered monoid. Let α be an element of $R[M]$. If $r_{R[M]}(\alpha R[M]) \neq 0$ then $r_{R[M]}(\alpha R[M]) \cap R \neq 0$.*

Proof. We prove it for a u.p.-monoid. The other case is similar. Let $\alpha = a_1g_1 + \cdots + a_ng_n$. If $n = 1$, then assertion is clear. Let $n \geq 2$. Assume that $\beta = b_1h_1 + \cdots + b_mh_m \in R[M]$ be a nonzero element of minimal length in $r_{R[M]}(\alpha R[M])$. Since $(\alpha R[M]\beta) = 0$, $\alpha R\beta = 0$. Since M is a u.p.-monoid, there exists i, j with $1 \leq i \leq n$, $1 \leq j \leq m$ such that a_ih_j is uniquely presented by considering two subsets $A = \{g_1, \ldots, g_n\}$, $B = \{h_1, \ldots, h_m\}$ of M. Thus $a_icb_jg_ih_j = 0$ for each $c \in R$ and hence $a_iRb_j = 0$. Thus $0 = \alpha(R[M]a_iR[M])(b_1h_1 + \cdots + b_mh_m) = \alpha R[M](a_iR[M](b_1h_1 + \cdots + b_{j-1}g_{j-1} + b_{j+1}h_{j+1} + \cdots + b_mh_m))$. By hypothesis, $a_iR(b_1h_1 + \cdots + b_{j-1}g_{j-1} + b_{j+1}h_{j+1} + \cdots + b_mh_m) = 0$. Therefore $a_iRb_t = 0$ for each $1 \leq t \leq m$. Hence $(a_1g_1 + \cdots + a_{i-1}g_{i-1} + a_{i+1}g_{i+1} + \cdots + a_ng_n)(R[M]\beta) = 0$. Since M is u.p.-monoid, there exist r, s with $r \in \{1, \ldots, i-1, i+1, \ldots, n\}$ and $s \in \{1, \ldots, m\}$ such that g_rh_s is uniquely presented by considering two subsets $A = \{g_1, \ldots, g_{i-1}, g_{i+1}, \ldots, g_n\}$, $B = \{h_1, \ldots, h_m\}$ of M. Thus $a_rcb_sg_rh_s = 0$ for each $c \in R$ and hence $a_rRb_s = 0$. Thus $0 = \alpha(R[M]a_rR[M])(b_1h_1 + \cdots + b_mh_m) = \alpha R[M](a_rR[M](b_1h_1 + \cdots + b_{s-1}g_{s-1} + b_{s+1}h_{s+1} + \cdots + b_mh_m)$. By hypothesis, $a_rR(b_1h_1 + \cdots + b_{j-1}g_{j-1} + b_{j+1}h_{j+1} + \cdots + b_mh_m) = 0$. Therefore $a_rRb_t = 0$ for each $1 \leq t \leq m$. Repeating this process, we obtain $a_iRb_m = 0$ for each $1 \leq i \leq n$. Hence $b_m \in r_{R[M]}(\alpha R[M])$. Therefore $r_{R[M]}(\alpha R[M]) \cap R \neq 0$. □

Corollary 2.2 ([5], Theorem 2.2). *Let $f(x)$ be an element of $R[x]$. If $r_{R[x]}(f(x)R[x]) \neq 0$, then $r_{R[x]}(f(x)R[x]) \cap R \neq 0$.*

We investigate a generalization of quasi-Armendariz rings which we call an M-quasi-Armendariz ring.

Definition 2.3. Let M be a monoid. We say that R is *M-quasi-Armendariz*, if $\alpha = a_1g_1 + \cdots + a_ng_n, \beta = b_1h_1 + \cdots + b_mh_m \in R[M]$ satisfy $\alpha R[M]\beta = 0$, then $a_iRb_j = 0$ for each i, j.

If $M = (\mathbb{N} \cup \{0\})$, then R is M-quasi-Armendariz if and only if R is quasi-Armendariz. If R is reduced and M-Armendariz, then R is M-quasi-Armendariz.

Proposition 2.4. *Let M be a u.p.-monoid and R be a reduced ring. Then R is M-quasi-Armendariz.*

Proof. Let $\alpha = a_1g_1+\cdots+a_ng_n$ and $\beta = b_1h_1+\cdots+b_mh_m \in R[M]$ be such that $\alpha R[M]\beta = 0$. We show that $a_iRb_j = 0$ for each i, j. We proceed by induction on m. It is clear for $m = 1$. Since M is a u.p.-monoid, there exists i, j with $1 \leq i \leq n$ and $1 \leq j \leq m$ such that g_ih_j is uniquely present by considering two subsets $A = \{g_1, \dots, g_n\}$ and $B = \{h_1, \dots, h_m\}$ of M. Thus $a_iRb_jg_ih_j = 0$ and that $a_iRb_j = 0$. Thus $0 = (a_1g_1+\cdots+a_ng_n)R[M]a_i(b_1h_1+\cdots+b_mh_m) = (a_1g_1+\cdots+a_ng_n)R[M](a_ib_1h_1+\cdots+a_ib_{j-1}h_{j-1}+a_ib_{j+1}h_{j+1}+\cdots+a_ib_mh_m)$. By induction, it follows that $a_iRa_ib_q = 0$ for $q = 1, \dots, m$. Then $a_iRb_q = 0$, for each $q = 1, \dots, m$, since R is reduced. Thus $(a_1g_1+\cdots+a_{i-1}g_{i-1}+a_{i+1}g_{i+1}+ a_ng_n)R[M](b_1h_1+\cdots+b_mh_m) = 0$. Continuing this procedure yield $a_iRb_j = 0$ for each $1 \leq i \leq n$, $1 \leq j \leq m$. Therefore R is M-quasi-Armendariz. □

Let $(M, \leq)$ be an ordered monoid. If for any $g_1, g_2, h \in M$, $g_1 < g_2$ implies that $g_1h < g_2h$ and $hg_1 < hg_2$, then $(M, \leq)$ is called a strictly ordered monoid.

Proposition 2.5. *Let M be a strictly totally ordered monoid and R a reduced ring. Then R is M-quasi-Armendariz.*

Proof. Let $\alpha = a_1g_1+\cdots+a_ng_n$ and $\beta = b_1h_1+\cdots+b_mh_m \in R[M]$ be such that $\alpha R[M]\beta = 0$ and $g_1 < \cdots < g_n$, $h_1 < \cdots < h_m$. We use transfinite induction on the strictly totaly ordered set $(M, \leq)$ to show that $a_iRb_j = 0$ for each i, j. If there exist $1 \leq i \leq n$ and $1 \leq j \leq m$ such that $g_ih_j = g_1h_1$, then $g_1 \leq g_i$ and $h_1 \leq h_j$. If $g_1 < g_i$ then $g_1h_1 < g_ih_1 \leq g_ih_j = g_1h_1$ a contradiction. Thus $g_1 = g_i$. Similarly, $h_1 = h_j$. Hence $a_1Rb_1 = 0$. Now suppose that $\omega \in M$ is such that for any g_i and h_j with $g_ih_j < \omega$, $a_iRb_j = 0$. We will show that $a_iRb_j = 0$ for any g_i and h_j with $g_ih_j = \omega$. Set $X = \{(g_i, h_j) | g_ih_j = \omega\}$. Then X is a finite set. We write X as $\{(g_{i_t}, h_{j_t}) | t = 1, \dots, k\}$ such that $g_{i_1} < \cdots < g_{i_n}$. Since M is cancellative, $g_{i_1} = g_{i_2}$ and $g_{i_1}h_{j_1} = g_{i_2}h_{j_2} = \omega$ imply $h_{j_1} = h_{j_2}$. Since $\leq$ is a strict order, $g_{i_1} < g_{i_2}$ and $g_{i_1}h_{j_1} = g_{i_2}h_{j_2} = \omega$ imply $h_{j_2} < h_{j_1}$. Thus we have $h_{j_k} < \cdots < h_{j_2} < h_{j_1}$. Now

$$\sum_{(g_i,h_j)\in X} a_ib_j = \sum_{t=1}^{k} a_{i_t}b_{j_t} = 0. \tag{1}$$

For any $t \geq 2$, $g_{i_1}h_{j_t} < g_{i_t}h_{j_t} = \omega$, and thus, by induction hypothesis, we have $A_{i_1}Rb_{j_t} = 0$ for each $t = 2, \dots, k$. By multiplying a_{i_1} to Eq.(1), from the left hand-side, we have $a_{i_1}a_{i_1}b_{j_1} = 0$. Since R is reduced, we have $a_{i_1}b_{j_1} = 0$. Now

Eq.(1), becomes

$$(2) \qquad \sum_{t=2}^{k} a_{i_t} b_{j_t} = 0.$$

By multiplying a_{i_2} to Eq.(2), from the left hand-side, we obtain $a_{i_2}b_{j_2} = 0$ by the same way as above. Continuing this process, we can prove $a_ib_j = 0$ for any i, j with $g_ih_j = \omega$. Therefore, by transfinite induction, $a_ib_j = 0$ for any i, j. Thus $a_ib_j = 0$ for any i, j, since R is reduced. Therefore R is M-quasi-Armendariz. □

Corollary 2.6. *Let R be a reduced ring. Then R is Z-quasi-Armendariz, that is for any $\alpha = a_{-m}x^{-m} + \cdots + b_qx^q, \beta = b_{-n}x^{-n} + \cdots + b_qx^q \in R[x, x^{-1}]$, if $\alpha R[x, x^{-1}]\beta = 0$, then $a_iRb_j = 0$ for each i, j.*

Proposition 2.7. *Let M be a u.p.-monoid or $(M, \leq)$ be a strictly totally ordered monoid and I an ideal of R. If I is a reduced and R/I is M-quasi-Armendariz, then R is M-quasi-Armendariz.*

Proof. We prove it for u.p.-monoid. The other case is similar. Let $\alpha = a_1g_1 + \cdots + a_ng_n$ and $\beta = b_1h_1 + \cdots + b_mh_m \in R[M]$ be such that $\alpha R[M]\beta = 0$. Since M is a u.p.-monoid, there exists i, j with $1 \leq i \leq n$ and $1 \leq j \leq m$ such that g_ih_j is uniquely present by considering two subsets $A = \{g_1, \ldots, g_n\}$ and $B = \{h_1, \ldots, h_m\}$ of M. Thus $a_iRb_jg_ih_j = 0$ and that $a_iRb_j = 0$. Thus

$$\begin{aligned}0 &= (a_1g_1 + \cdots + a_ng_n)R[M]a_i(b_1h_1 + \cdots + b_mh_m)\\ &= (a_1g_1 + \cdots + a_ng_n)R[M](a_ib_1h_1 + \cdots\\ &\qquad + a_ib_{j-1}h_{j-1} + a_ib_{j+1}h_{j+1} + \cdots + a_ib_mh_m).\end{aligned}$$

Thus, by induction hypothesis, we have $a_iRa_ib_j = 0$ for each $j = 1, \ldots, m$. Note that in $(R/I)[M]$, $(\overline{a_1}g_1 + \cdots + \overline{a_n}g_n)R/I(\overline{b_1}h_1 + \cdots + \overline{b_m}h_m) = 0$. Thus we have $a_iRb_j \subseteq I$ for each i, j, since R/I is M-quasi-Armendariz. Hence $(a_ib_j)^2 = 0$ and that $a_ib_j = 0$ for $j = 1, \ldots, m$, since I is reduced and $a_ib_j \in I$. Thus $0 = (a_1g_1 + \cdots + a_{i-1}g_{i-1} + a_{i+1}g_{i+1} + \cdots + a_ng_n)R[M](b_1h_1 + \cdots + b_mh_m) = 0$. Therefore, by induction on $m+n$, we have $a_iRb_j = 0$ for each i, j. Consequently R is M-quasi-Armendariz. □

Recall that a monoid M is called torsion-free if the following property holds: if $g, h \in M$ and $k \geq 1$ are such that $g^k = h^k$, then $g = h$.

Corollary 2.8. *Let M be a commutative, cancellative and torsion-free monoid. If one of the following conditions holds, then R is M-quasi-Armendariz:*

(1) *R is reduced.*
(2) *R/I is M-quasi-Armendariz for some ideal I of R and I is reduced.*

Proof. If M is commutative, cancellative and torsion-free, then by [16] there exists a compatible strict total ordered $\leq$ on M. Now the results follows from Proposition 2.5 and 2.7. □

Proposition 2.9. *Let M be a cyclic group of order $n \geq 2$ and R a ring with $0 \neq 1$. Then R is not M-quasi-Armendariz.*

Proof. Suppose that $M = \{e, g, g^2, \dots, g^{n-1}\}$. Let $\alpha = 1e + 1g + 1g^2 + \cdots + 1g^{n-1}$ and $\beta = 1e + (-1)g$. Then $\alpha c \beta = 0$ for each $c \in R$ and that $\alpha R[M]\beta = 0$. Thus R is not M-quasi-Armendariz. $\square$

Example 2.10. Let R be an M-Armendariz and reduced ring. Let

$$T_4(R) = \left\{ \begin{pmatrix} a & a_{12} & a_{13} & a_{14} \\ 0 & a & a_{23} & a_{24} \\ 0 & 0 & a & a_{34} \\ 0 & 0 & 0 & a \end{pmatrix} \mid a, a_{ij} \in R \right\}.$$

Then $T_4(R)$ is M-quasi-Armendariz. It is easy to see that there exists an isomorphism of rings $T_4(R)[M] \longrightarrow T_4(R[M])$ defined by:

$$\sum_{k=1}^{s} \begin{pmatrix} a^k & a_{12}^k & a_{13}^k & a_{1n}^k \\ 0 & a^k & a_{23}^k & a_{24}^k \\ 0 & 0 & a^k & a_{34}^k \\ 0 & 0 & 0 & a^k \end{pmatrix} g_k$$

$$\longrightarrow \begin{pmatrix} \sum_{k=1}^{s} a^k g_k & \sum_{k=1}^{s} a_{12}^k g_k & \sum_{k=1}^{s} a_{13}^k g_k & \sum_{k=1}^{s} a_{14}^k g_k \\ 0 & \sum_{k=1}^{s} a^k g_k & \sum_{k=1}^{s} a_{23}^k g_k & \sum_{k=1}^{s} a_{24}^k g_k \\ 0 & 0 & \sum_{k=1}^{s} a^k g_k & \sum_{k=1}^{s} a_{34}^k g_k \\ 0 & 0 & 0 & \sum_{k=1}^{s} a^k g_k \end{pmatrix}$$

$$= \begin{pmatrix} \alpha & \alpha_{12} & \alpha_{13} & \alpha_{14} \\ 0 & \alpha & \alpha_{23} & \alpha_{24} \\ 0 & 0 & \alpha & \alpha_{34} \\ 0 & 0 & 0 & \alpha \end{pmatrix}.$$

Let $\alpha = A_1 g_1 + \cdots + A_s g_s$ and $\beta = B_1 h_1 + \cdots + B_m h_m \in T_4(R)[M]$ such that $\alpha T_4(R)[M]\beta = 0$. We claim that $A_i T_4(R) B_j = 0$ for all $i = 1, \dots, s$, $j = 1, \dots, m$. Assume that

$$A_i = \begin{pmatrix} a_{11}^i & a_{12}^i & a_{13}^i & a_{14}^i \\ 0 & a_{22}^i & a_{23}^i & a_{24}^i \\ 0 & 0 & a_{33}^i & a_{34}^i \\ 0 & 0 & 0 & a_{44}^i \end{pmatrix}$$

and

$$B_j = \begin{pmatrix} b_{11}^i & b_{12}^i & b_{13}^i & b_{14}^i \\ 0 & b_{22}^i & b_{23}^i & b_{24}^i \\ 0 & 0 & b_{33}^i & b_{34}^i \\ 0 & 0 & 0 & b_{44}^i \end{pmatrix}$$

with $a^i_{tt} = a^i_{kk}$ and $b^j_{tt} = b^j_{kk}$ for each i, j, k, t. Let

$$X = \begin{pmatrix} \sum_{i=1}^s a^i_{11}g_i & \sum_{i=1}^s a^i_{12}g_i & \sum_{i=1}^s a^i_{13}g_i & \sum_{i=1}^s a^i_{14}g_i \\ 0 & \sum_{i=1}^s a^i_{22}g_i & \sum_{i=1}^s a^i_{23}g_i & \sum_{i=1}^s a^i_{24}g_i \\ 0 & 0 & \sum_{i=1}^s a^i_{33}g_i & \sum_{i=1}^s a^i_{34}g_i \\ 0 & 0 & 0 & \sum_{i=1}^s a^i_{44}g_i \end{pmatrix} = \begin{pmatrix} \alpha_{11} & \alpha_{12} & \alpha_{13} & \alpha_{14} \\ 0 & \alpha_{22} & \alpha_{23} & \alpha_{24} \\ 0 & 0 & \alpha_{33} & \alpha_{34} \\ 0 & 0 & 0 & \alpha_{44} \end{pmatrix}$$

and

$$Y = \begin{pmatrix} \sum_{i=1}^s b^i_{11}g_i & \sum_{i=1}^s b^i_{12}g_i & \sum_{i=1}^s b^i_{13}g_i & \sum_{i=1}^s b^i_{14}g_i \\ 0 & \sum_{i=1}^s b^i_{22}g_i & \sum_{i=1}^s b^i_{23}g_i & \sum_{i=1}^s b^i_{24}g_i \\ 0 & 0 & \sum_{i=1}^s b^i_{33}g_i & \sum_{i=1}^s b^i_{34}g_i \\ 0 & 0 & 0 & \sum_{i=1}^s b^i_{44}g_i \end{pmatrix} = \begin{pmatrix} \beta_{11} & \beta_{12} & \beta_{13} & \beta_{14} \\ 0 & \beta_{22} & \beta_{23} & \beta_{24} \\ 0 & 0 & \beta_{33} & \beta_{34} \\ 0 & 0 & 0 & \beta_{44} \end{pmatrix}.$$

Then we have $XAY = 0$ for each $A \in T_4(R[M])$. We show that $\alpha_{ij}\beta_{jk} = 0$ for each $i = 1, 2, 3, 4$, $j = 1, 2, 3, 4$ and $k = 1, 2, 3, 4$. Since $XT_4(R[M])Y = 0$, we have

$$\begin{pmatrix} \alpha_{11} & \alpha_{12} & \alpha_{13} \\ 0 & \alpha_{22} & \alpha_{23} \\ 0 & 0 & \alpha_{33} \end{pmatrix} \begin{pmatrix} \beta_{11} & \beta_{12} & \beta_{13} \\ 0 & \beta_{22} & \beta_{23} \\ 0 & 0 & \beta_{33} \end{pmatrix} = 0$$

and

$$\begin{pmatrix} \alpha_{22} & \alpha_{23} & \alpha_{24} \\ 0 & \alpha_{33} & \alpha_{34} \\ 0 & 0 & \alpha_{44} \end{pmatrix} \begin{pmatrix} \beta_{22} & \beta_{23} & \beta_{24} \\ 0 & \beta_{33} & \beta_{34} \\ 0 & 0 & \beta_{44} \end{pmatrix} = 0.$$

By ([15], Proposition 1.7), $\alpha_{11}\beta_{11} = \alpha_{11}\beta_{12} = \alpha_{11}\beta_{13} = \alpha_{22}\beta_{22} = \alpha_{12}\beta_{23} = \alpha_{13}\beta_{33} = \alpha_{22}\beta_{23} = \alpha_{23}\beta_{33} = 0$ and $\alpha_{22}\beta_{22} = \alpha_{22}\beta_{23} = \alpha_{22}\beta_{24} = \alpha_{23}\beta_{34} = \alpha_{24}\beta_{44} = \alpha_{33}\beta_{34} = \alpha_{34}\beta_{44} = 0$. Since $XT_4(R[M])Y = 0$, we have $\alpha_{11}\beta_{14} + \alpha_{12}\beta_{24} + \alpha_{13}\beta_{34} + \alpha_{14}\beta_{44} = 0$. Since $R[M]$ is reduced and $\alpha_{11} = \alpha_{jj}$ and $\alpha_{ii}\beta_{in} = 0$ for each $i = 2, \ldots, n$, if we multiply this equation on the left side by α_{11}, then $\alpha_{11}\alpha_{11}\beta_{14} = 0$ and that $\alpha_{11}\beta_{14} = 0$. Hence $\alpha_{12}\beta_{24}+\alpha_{13}\beta_{34}+\alpha_{14}\beta_{44} = 0$. Also if we multiply this equation on the right side by β_{44}, then $\alpha_{14}\beta_{44}\beta_{44} = 0$ and that $\alpha_{14}\beta_{44} = 0$, since $\beta_{44} = \beta_{jj}$ for each j and $R[M]$ is reduced. Thus

$\alpha_{12}\beta_{24} + \alpha_{13}\beta_{34} = 0$. Hence

$$X \begin{pmatrix} \alpha_{12} & 0 & 0 & a \\ 0 & \alpha_{12} & \alpha_{13} & 0 \\ 0 & 0 & \alpha_{12} & 0 \\ 0 & 0 & 0 & \alpha_{12} \end{pmatrix} Y$$

$$= \begin{pmatrix} \alpha_{11}\alpha_{12}\beta_{11} & \cdots & \cdots & \alpha_{13}\alpha_{12}\beta_{34} \\ 0 & \alpha_{22}\alpha_{12}\beta_{22} & \cdots & \cdots \\ 0 & 0 & \alpha_{33}\alpha_{12}\beta_{33} & \cdots \\ 0 & 0 & 0 & \alpha_{nn}\alpha_{12}\beta_{nn} \end{pmatrix} = 0.$$

Thus $\alpha_{13}\alpha_{12}\beta_{34} = \alpha_{12}\alpha_{13}\beta_{34} = 0$, since $R[M]$ is reduced. Now multiplying $\alpha_{12}\beta_{24} + \alpha_{13}\beta_{34} = 0$ on the left by α_{12}, we obtain $\alpha_{12}\beta_{24} = \alpha_{13}\beta_{34} = 0$. Hence $a^i_{rs}b^j_{st} = 0$ for each $r, s, t, i, j \geq 1$, since $R[M]$ is M-Armendariz. Thus $a^i_{rs}cb^j_{st} = 0$, for each $c \in R$, since R is reduced. Consequently, $A_iCb_j = 0$ for each $C \in T_4(R)$. Therefore $T_4(R)$ is M-quasi-Armendariz.

Acknowledgement. This research is supported by the Shahrood University of Technology.

References

[1] E. P. Armendariz, *A note on extensions of Baer and p.p.-rings*, J. Austral. Math. Soc. **18** (1974), 470–473.

[2] D. D. Anderson and S. Camillo, *Armendariz rings and Gaussian rings*, Comm. Algebra **73** (1997), 14–17.

[3] G. F. Birkenmeier and J. K. Park, *Triangular matrix representations of ring extensions*, J. Algebra **265** (2003), no. 2, 103–122.

[4] G. F Birkenmeier, J. Y. Kim, and J. K. Park, *Polynomial extensions of Baer and quasi-Baer rings*, J. Pure Appl. Algebra **159** (2001), no. 1, 25–42.

[5] Y. Hirano, *On annihilator ideals of a polynomial ring over a noncommutative ring*, J. Pure Appl. Algebra **168** (2002), no. 1, 45–52.

[6] ———, *On ordered monoid rings over a quasi-Baer ring*, Comm. Algebra **29** (2001), no. 5, 2089–2095.

[7] C. Y. Hong, N. K. Kim, and T. K. Kwak, *On skew Armendariz rings*, Comm. Algebra **31** (2003), no. 1, 103–122.

[8] C. Huh, Y. Lee, and A. Smoktunowicz, *Armendariz rings and semicommutative rings*, Comm. Algebra **30** (2002), no. 2, 751–761.

[9] N. H Kim and Y. Lee, *Armendariz rings and reduced rings*. J. Algebra **223** (2000), no. 2, 477–488.

[10] J. Krempa, *Some examples of reduced rings*, Algebra Colloq. **3** (1996), no. 4, 289–300.

[11] T. K. Lee and T. L. Wong, *On Armendariz rings*, Houston J. Math. **29** (2003), no. 3, 583–593.

[12] N. H. McCoy, *Remarks on divisors of zero*, Amer. Math. Monthly **49** (1942), 286–296.

[13] J. Okninski, *Semigroup Algebra*, Monographs and Textbooks in Pure and Applied Mathematics, 138. Marcel Dekker, Inc., New York, 1991.

[14] D. S. Passman, *The Algebraic structure of group rings*, Pure and Applied Mathematics. Wiley-Interscience [John Wiley & Sons], New York-London-Sydney, 1977.

[15] M. B. Rege and S. Chhawchharia, *Armendariz rings*, Proc. Japan Acad. Ser. A Math. Sci. **73** (1997), no. 1, 14–17.

[16] P. Ribenboim, *Noetherian rings of generalized power series*, J. Pure Appl. Algebra **79** (1992), no. 3, 293–312.
[17] L. Zhongkui, *Armendariz rings relative to a monoid*, Comm. Algebra **33** (2005), no. 3, 649–661.

Department of Mathematics
Shahrood University of Technology
Shahrood 316-3619995161, Iran
E-mail address: eb_hashemi@yahoo.com or eb_hashemi@shahroodut.ac.ir