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SOME VECTOR IMPLICIT COMPLEMENTARITY PROBLEMS WITH CORRESPONDING VARIATIONAL INEQUALITY PROBLEMS

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Abstract. Some existence theorems of solutions of a new class of generalized vector F-implicit complementarity problems with the corresponding generalized vector F-implicit variational inequality problems were established.

1. Introduction with preliminaries

After Lemke [22] introduced complementarity problems, there have been many discussions on the problems and the corresponding variational inequality problems [4-8, 10-20, 23-25]. Implicit complementarity problems were originally considered with the dynamic programming approach of stochastic impulse and continuous optimal control [2, 3, 18]. In 1991, Chang and Huang [5] considered the relation between multi-valued implicit complementarity problems and multi-valued implicit variational inequality problems.

In [24], the following scalar F-complementarity problems and the corresponding variational inequality problems were considered, where K is a closed convex cone of a Banach space X with a topological dual X^*, $T : K \to X^*$ a mapping and $F : K \to \mathbb{R}$ a function; Find $x \in K$ such that

$$\langle Tx, x\rangle + F(x) = 0, \ \langle Tx, y\rangle + F(y) \geq 0, \quad \forall y \in K,$$

and find $x \in K$ such that

$$\langle Tx, y - x\rangle + F(y) - F(x) \geq 0, \quad \forall y \in K.$$

In 2004, Huang and Li [12] studied a new class of scalar F-implicit complementarity problems and the corresponding F-implicit variational inequality problems in Banach spaces. Later, in 2006, Li and Huang [23] extended some results in [12] to the vector case and presented the equivalent relation between

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F-implicit complementarity problem and F-implicit variational inequality problem. They also obtained some new existence theorems for solutions of their problems by using F-KKM theorem under some suitable assumptions without monotonicity.

In [21], the authors considered the following vector F-implicit complementarity problem (GVF-ICP); Find $x \in K$ such that

$$\langle N(Ax, Bx), g(x)\rangle + F(g(x)) = 0$$

and

$$\langle N(Ax, Bx), g(y)\rangle + F(g(y)) \geq 0 \quad \text{for} \quad y \in K,$$

and the following corresponding vector F-implicit variational inequality problem (GVF-IVIP); Find $x \in K$ such that

$$\langle N(Ax, Bx), g(y) - g(x)\rangle + F(g(y)) - F(g(x)) \geq 0 \quad \text{for} \quad y \in K,$$

where K is a nonempty closed convex cone of a Banach space X, (Y, P) is an ordered Banach space induced by a pointed closed convex cone P, $L(X, Y)$ the space of all continuous linear mappings from X into Y and $A,\ B : K \to L(X, Y)$, $g : K \to K$, $F : K \to Y$ and $N : L(X, Y) \times L(X, Y) \to L(X, Y)$ are mappings.

This paper considers the following generalized multi-valued vector F-implicit complementarity problem (GMVF-ICP); finding $x \in K$ such that for all $y \in K$, there exists $T \in N(Ax, Bx)$ satisfying

$$\langle T,\ g(x)\rangle + v = 0 \quad \text{for some } v \in F(g(x))$$

and

$$\langle T,\ f(y)\rangle + w \in C(x), \quad \text{for all } w \in F(f(y)),$$

and the following generalized multi-valued vector F-implicit variational inequality problem (GMVF-IVIP); finding $x \in K$ such that for all $y \in K$, there exists $T \in N(Ax, Bx)$ satisfying

$$\begin{aligned}&\langle T,\ f(y) - g(x)\rangle + w - v \in C(x),\\ &\quad \text{for some } v \in F(g(x)) \text{ and for all } w \in F(f(y)),\end{aligned}$$

where, $f, g : K \to K$ and $A,\ B : K \to L(X, Y)$ are mappings, $F : K \to 2^Y$ and $N : L(X, Y) \times L(X, Y) \to 2^{L(X,Y)}$ are multi-valued mappings, and $C : K \to 2^Y$ a multi-valued mapping with nonempty convex pointed cone values.

(GMVF-ICP) and (GMVF-IVIP) generalize known problems as follows;

(1) For the case of single-valued mappings N and F, and the constant closed cone $C(x) = C$ for all $x \in K$, (GMVF-ICP) and (GMVF-IVIP) are, respectively, reduced to (GVF-ICP), and (GVF-IVIP) considered in [21].

(2) If we define $N(A, B) = \{A\}$, f the identity mapping and F a single-valued mapping in (GMVF-ICP), then we obtain the following vector

F-implicit complementarity problem (VF-ICP) of finding $x \in K$ such that

$$\langle A(x), g(x)\rangle + F(g(x)) = 0 \quad \text{and} \quad \langle A(x), y\rangle + F(y) \in P, \quad \forall y \in K,$$

which was considered and studied in [23] for a constant cone P. The following vector F-implicit variational inequality problem (VF-IVIP) of finding $x \in K$ such that

$$\langle A(x), y - g(x)\rangle + F(y) - F(g(x)) \in P, \quad \forall y \in K,$$

is a particular form of (GMVF-IVIP), considered and studied in [23].

(3) The following are all special cases of (GMVF-ICP) considered previously.

(3-1) Find $x \in K$ such that

$$\langle A(x), g(x)\rangle + F(g(x)) = 0 \quad \text{and} \quad \langle A(x), y\rangle + F(y) \in C(x), \quad \forall y \in K,$$

which were considered by Huang and Li [12].

(3-2) Find $x \in K$ such that

$$\langle A(x), x\rangle + F(x) = 0 \quad \text{and} \quad \langle A(x), y\rangle + F(y) \in C(x), \quad \forall y \in K,$$

which was studied by Yin et al. [24].

(3-3) Find $x \in K$ such that

$$\langle A(x), g(x)\rangle = 0 \quad \text{and} \quad \langle A(x), y\rangle \in C(x), \quad \forall y \in K,$$

which was studied by Isac [14, 15].

(3-4) Find $x \in K$ such that

$$\langle A(x), x\rangle = 0 \quad \text{and} \quad \langle A(x), y\rangle \in C(x), \quad \forall y \in K,$$

which was studied by Chen and Yang [6].

2. Main results

The following KKM mapping is a very useful mapping in nonlinear analysis.

Definition 2.1. Let K be a nonempty convex subset of a topological vector space X. A multi-valued mapping $T : K \to 2^X$ is said to be a KKM mapping if

$$conv A \subseteq \bigcup_{x \in A} T(x), \quad \forall A \in \mathcal{F}(K),$$

where *conv* denotes the convex hull and $\mathcal{F}(K)$ a family of finite subsets of K.

Definition 2.2 ([1])**.** Let X and Y be normed spaces. A multi-valued mapping $T : X \to 2^Y$ is said to be a process if its graph is a cone.

Note that a multi-valued mapping $T : X \to 2^Y$ is a process if and only if

$$\forall x \in X, \ \forall \lambda > 0, \ \lambda F(x) = F(\lambda x) \text{ and } 0 \in F(0).$$

If a process F is a single-valued mapping then it is said to be positively homogeneous.

The following F-KKM theorem is a useful key in our result.

Lemma 2.1 ([9]). *Let K be a nonempty subset of a Hausdorff topological vector space X and $T : K \to 2^X$ be a KKM mapping. Suppose that $T(x)$ is closed for each $x \in K$, and $T(y)$ is compact for some $y \in X$, then $\bigcap_{x \in K} T(x)$ is nonempty.*

Throughout this section, K is a nonempty closed convex cone of a Banach space X with a topological dual X^*, Y is also a Banach space and $C : K \to 2^Y$ is a multi-valued mapping with nonempty convex pointed cone values. The following theorem shows that (GMVF-ICP) and (GMVF-IVIP) are equivalent.

Theorem 2.1. (i) *If x solves* (GMVF-ICP), *then it also solves* (GMVF-IVIP).

(ii) *Let $F : K \to 2^Y$ satisfy $2F(x) \subset F(2x)$ for all $x \in K$ and $0 \in F(0)$. If x is a solution of* (GMVF-IVIP) *with $2g(x) \in f(K)$ and $0 \in f(K)$, then it also solves* (GMVF-ICP).

Proof. By the definitions of (GMVF-ICP) and (GMVF-IVIP), (i) easily holds. Now let $x \in K$ solve (GMVF-IVIP), then for all $y \in K$, there exists $T \in N(Ax, Bx)$ such that

$$\begin{gathered} \langle T,\ f(y) - g(x)\rangle + w - v \in C(x), \\ \text{for some } v \in F(g(x)) \text{ and for all } w \in F(f(y)). \end{gathered} \tag{2.1}$$

Since $0 \in f(K)$, there exists $y_1 \in K$ such that $f(y_1) = 0$. By substituting w with 0 in (2.1), we obtain that

$$\langle T, -g(x)\rangle + 0 - v \in C(x).$$

In (2.1), since $2v \in 2F(g(x)) \subset F(2g(x)) \subset F(f(K))$, taking $y_2 \in K$ such that $f(y_2) = 2g(x)$ and $w = 2v$, we obtain that

$$\langle T, g(x)\rangle + 2v - v \in C(x).$$

Then $\langle T, g(x)\rangle + v \in C(x) \cap -C(x)$. Since $C(x)$ is pointed, we have

$$\langle T, g(x)\rangle + v = 0. \tag{2.2}$$

By adding (2.1) and (2.2), we have

$$\langle T,\ f(y)\rangle + w \in C(x).$$

Therefore x is a solution of (GMVF-ICP). □

Example 2.1. Let $X = Y = \mathbb{R}$, and $K = C(x) = [0, \infty)$, for all $x \in X$. Define a multi-valued mapping $N : L(X,Y) \times L(X,Y) \to 2^{L(X,Y)}$ by $N(s,t) = \{s,t\}$ for each $s,\ t \in L(X,Y)$. Let $f,\ g : K \to K$ be mappings defined by $g(x) = x$ and $f(x) = x^2 + 2$ and $A,\ B : K \to L(X,Y)$ be defined by $\langle A(x), z\rangle = (x+1)z$ and $\langle B(x), z\rangle = 2z$ for each $z \in X$, for each $x \in K$. Assume that $F : K \to 2^Y$ is defined by

$$F(x) = [-x,\ x^2] \quad \text{for each } x \in X.$$

Then $2F(x) \subset F(2x)$ for $x \in X$. Also, for $y \in K$,

$$\begin{aligned}&\langle A(x),\ f(y)-g(x)\rangle + w - v\\ &= (x+1)(y^2+2-x)+w-v\\ &\geq (x+1)(y^2+2-x)-y^2-2-v\\ &= x(y^2-x+1)-v \quad \text{for all} \quad w \in F(f(y)).\end{aligned}$$

The solution set S_{AV} of (GMVF-IVIP) for A is $[0,2]$, but the solution set S_{AC} of (GMVF-ICP) for A is $\{0\}$. To show the existence of $v \in F(g(x))$ such that $\langle A(x), g(x)\rangle + v = 0$, it must be satisfied that $(x+1)x - x \leq 0$. For example, if $x = 1$,

$$\langle A(1), g(1)\rangle + v = 2 + v > 0 \quad \text{for each } v \in F(g(1)) = [-1,1].$$

On the other hand,

$$\begin{aligned}\langle B(x), f(y)-g(x)\rangle + w - v &= 2(y^2+2-x)+w-v\\ &\geq y^2-2x+2-v \quad \text{for all } w \in F(f(y)).\end{aligned}$$

To have a solution of (GMVF-IVIP) for B, it must hold that $y^2-2x+2-(-x) \geq 0$ for each $y \in K$; hence

$$\langle A(x), f(y)\rangle + w \geq 2(y^2+2)-y^2-2 = y^2+2 \geq 0 \quad \text{for all} \quad w \in F(f(y))$$

and

$$\langle A(x), g(x)\rangle + v = 2x + v.$$

Therefore the solution set S_{BV} of (GMVF-IVIP) for B is $[0,2]$, while the solution set S_{BC} of (GMVF-ICP) for B is $\{0\}$. Thus the solution set of (GMVF-IVIP) is $S_{AV} \cup S_{BV} = [0,2]$ and that of (GMVF-ICP) is $S_{AC} \cup S_{BC} = \{0\}$.

In Example 2.1, if we put $f(x) = x^2$ then f is onto and thus Theorem 2.1 holds; in this case, the common solution set for (GMVF-IVIP) is $\{0\}$.

Since every process F satisfies $F(2x) = 2F(x)$ for each $x \in X$ and $0 \in F(0)$, Theorem 2.1 has the following corollary.

Corollary 2.1. *Let $F : K \to 2^Y$ be a process. If $0 \in f(K)$ and $2g(K) \subset f(K)$, then* (GMVF-IVIP) *and* (GMVF-ICP) *are equivalent.*

The mapping $F : K \to 2^Y$ in Example 2.1 is a multi-valued mapping which is not a process but satisfies condition of (ii) in Theorem 2.1.

Corollary 2.2. *Let N and F be single-valued mappings and $f = g$.*

(i) *If x solves* (GVF-ICP), *then it also solves* (GVF-IVIP).

(ii) *Let K be a closed convex cone, $F : K \to Y$ positively homogeneous and f onto. If x solves* (GVF-IVIP), *then it also solves* (GVF-ICP).

Corollary 2.3 ([23]). (i) *If x solves* (VF-ICP), *then it also solves* (VF-IVIP).

(ii) *Let K be a closed convex cone and $F : K \to Y$ positively homogeneous. If x solves* (VF-IVIP), *then it also solves* (VF-ICP).

The following theorem improves and extends Theorem 3.2 in [23].

Theorem 2.2. *Assume that*

(a) *the set* $H = \{x \in K$: *there exists* $T \in N(Ax, Bx)$ *such that* $\langle T,\ f(y) - g(x)\rangle + w - v \in C(x)$ *for all* $w \in F(f(y))$ *and for some* $v \in F(g(x))\}$ *is closed in* K *for all* $y \in K$;

(b) *there exists a mapping* $h : K \times K \to Y$ *such that*

(i) $h(x,x) \in C(x)$ *for all* $x \in K$;

(ii) $\langle T,\ f(y)-g(x)\rangle+w-v-h(x,y) \in C(x)$ *for* $x,\ y \in K$, $T \in N(Ax, Bx)$, $w \in F(f(y))$ *and for some* $v \in F(g(x))$;

(iii) *the set* $\{y \in K : h(x,y) \notin C(x)\}$ *is convex for all* $x \in K$;

(c) *there exists a nonempty convex compact subset* D *of* K *such that for each* $x \in K \backslash D$, *there exists* $y \in D$ *such that*

$$\langle T, f(y) - g(x)\rangle + w - v \notin C(x)$$

for all $T \in N(Ax, Bx)$, *for some* $w \in F(f(y))$, *for all* $v \in F(g(x))$. *Then, the solution set of (GVF-IVIP) is a nonempty compact subset of* K.

Proof. Define

$$\begin{aligned} G(y) = \{x \in D :\ & \text{there exists } T \in N(Ax, Bx) \text{ such that} \\ & \langle T, f(y) - g(x)\rangle + w - v \in C(x) \text{ for all } w \in F(f(y)) \\ & \text{and for some } v \in F(g(x))\} \end{aligned}$$

for each $y \in K$. Then $G : K \to 2^K$ is a multi-valued mapping. By assumption (a), $G(y) = H \cap D$ is closed in D. We have to show that $\bigcap_{y \in K} G(y) \neq \emptyset$, because every element of $\bigcap_{y \in K} G(y)$ is a solution of (GMF-IVIP). First we claim that $\{G(y) : y \in K\}$ has the finite intersection property.

Let $\{y_1, \ldots, y_n\}$ be a finite subset of K and set $E = \overline{\text{conv}}(D \cup \{y_1, \ldots, y_n\})$. Then E is a compact and convex subset of K. Define multi-valued mappings $F_1,\ F_2 : E \to 2^E$ as follows: for each $y \in E$,

$$\begin{aligned} F_1(y) = \{x \in E :\ & \text{there exists } T \in N(Ax, Bx) \text{ such that} \\ & \langle T, f(y) - g(x)\rangle + w - v \in C(x) \ \text{ for all } w \in F(f(y)) \\ & \text{and for some } v \in F(g(x))\} \end{aligned}$$

and

$$F_2(y) = \{x \in E : h(x,y) \in C(x)\}.$$

To show that F_2 is a KKM-mapping, suppose that there exists a finite subset $\{u_1, \ldots, u_m\}$ of E and $\lambda_i \geq 0$ $(i = 1, 2, \ldots, m)$ with $\sum_{i=1}^{m} \lambda_i = 1$ such that

$$u = \sum_{i=1}^{m} \lambda_i u_i \notin \bigcup_{i=1}^{m} F_2(u_i).$$

Since $h(u, u_i) \notin C(u)$ for each $i = 1, \ldots, m$ and $\{y \in K : h(x, y) \notin C(x)\}$ is convex, it follows that

$$h\left(u, \sum_{i=1}^{m} \lambda_i u_i\right) = h(u, u) \notin C(u),$$

which is a contradiction to the assumption (i) of (b). Therefore F_2 is a KKM-mapping. From the assumption (ii) of (b) and the fact that $C(x)$ is a cone, we have $F_2(y) \subset F_1(y)$ for each $y \in E$. Hence F_1 is also a KKM-mapping. Since $F_1(y) = H \cap E$ and H is closed in K, $F_1(y)$ is a closed subset of a compact set E and thus $F_1(y)$ is compact. By Lemma 2.1,

$$\bigcap_{y \in E} F_1(y) \neq \emptyset.$$

By assumption (c), each element of $\bigcap_{y \in E} F_1(y)$ can not belong to $K \backslash D$ but to D. Therefore $\bigcap_{y \in E} F_1(y) \subset G(y_j)$ for $j = 1, 2, \ldots, n$, that is, $\bigcap_{j=1}^{n} G(y_j) \neq \emptyset$. Hence $\{G(y) : y \in K\}$ is a family of closed subsets of the compact set D, having the finite intersection property. Therefore $\bigcap_{y \in K} G(y) \neq \emptyset$ and it's a compact subset of K. That is, there exists $x \in K$ such that for all $y \in K$ there exists $T \in N(Ax, Bx)$ such that

$$\langle T, f(y) - g(x) \rangle + w - v \in C(x) \text{ for all } w \in F(f(y)) \text{ and for some } v \in F(g(x)).$$

□

Note that a multi-valued mapping $F : K \to 2^Y$ is upper semicontinuous if for any closed set $C \subset Y$, the set $F^-(C) = \{x \in K : F(x) \cap C \neq \emptyset\}$ is closed in K. If $C : K \to 2^Y$ is a multi-valued mapping with closed set values N, F are upper semicontinuous and A, B, g, f are continuous, then condition (a) of Theorem 2.2 holds. Therefore we obtain the following existence result of solution for the (GMVF-IVIP).

Theorem 2.3. *Let $N : L(X,Y) \times L(X,Y) \to 2^{L(X,Y)}$ and $F : K \to 2^Y$ be upper semicontinuous multi-valued mappings and $C(x)$ be closed for each $x \in K$. Assume that*

(a) *A, $B : K \to L(X,Y)$ and f, $g : K \to K$ are continuous;*

(b) *there exists a mapping $h : K \times K \to Y$ such that*

(i) *$h(x,x) \in C(x)$ for all $x \in K$;*

(ii) *$\langle T, f(y) - g(x)\rangle + w - v - h(x,y) \in C(x)$, for x, $y \in K$, $T \in N(Ax, Bx)$, for all $w \in F(f(y))$ for some $v \in F(g(x))$;*

(iii) *the set $\{y \in K : h(x,y) \notin C(x)\}$ is convex for all $x \in K$;*

(c) *there exists a nonempty convex compact subset D of K such that for each $x \in K \backslash D$, there exists $y \in D$ such that*

$$\langle T, f(y) - g(x) \rangle + w - v \notin C(x)$$

for all $T \in N(Ax, Bx)$, for some $w \in F(f(y))$, for all $v \in F(g(x))$. Then, the solution set of (GVF-IVIP) *is a nonempty compact subset of K.*

If $N : L(X,Y) \times L(X,Y) \to 2^{L(X,Y)}$ and $F : K \to 2^Y$ are single-valued mappings, $f = g$ and $C : K \to 2^Y$ is a constant mapping, then we have the existence theorem for (GVF-IVIP) as a corollary.

Corollary 2.4 ([21]). *Assume that*

(a) *five mappings $N : L(X,Y) \times L(X,Y) \to L(X,Y)$, $g : K \to K$, A, $B : K \to L(X,Y)$ and $F : K \to Y$ are continuous;*

(b) *there exists a mapping $h : K \times K \to Y$ such that*

(i) *$h(x,x) \geq 0$ for all $x \in K$;*

(ii) *$\langle N(Ax, Bx),\ g(y) - g(x)\rangle + F(g(y)) - F(g(x)) - h(x,y) \geq 0$ for all x, $y \in K$;*

(iii) *the set $\{y \in K : h(x,y) \ngeq 0\}$ is convex for all $x \in K$;*

(c) *there exists a nonempty compact convex subset D of K such that for all $x \in K \backslash D$ there exists $y \in D$ such that*

$$\langle N(Ax, Bx),\ g(y) - g(x)\rangle + F(g(y)) - F(g(x)) \ngeq 0$$

Then (GVF-IVIP) *has a solution. Furthermore, the solution set of* (GVF-IVIP) *is closed.*

If $N(A, B) = \{A\}$, f is the identity mapping, F is a single-valued mapping and C is a constant mapping, then we obtain an existence theorem for (VF-IVIP).

Corollary 2.5 ([23]). *Let Y be an ordered Banach space induced by a pointed closed convex cone P. Assume that*

(a) *$A : K \to L(X,Y)$, $g : K \to K$ and $F : K \to Y$ are continuous;*

(b) *there exists a mapping $h : K \times K \to Y$ such that*

(i) *$h(x,x) \geq 0$ for all $x \in K$;*

(ii) *$\langle A(x),\ y - g(x)\rangle + F(y) - F(g(x)) - h(x,y) \geq 0$, for all x, $y \in K$;*

(iii) *the set $\{y \in K : h(x,y) \ngeq 0\}$ is convex, for all $x \in K$;*

(c) *there exists a nonempty compact, convex subset D of K, such that for all $x \in K \backslash D$, there exists $y \in D$ such that*

$$\langle f(x), y - g(x)\rangle + f(y) - F(g(x)) \ngeq 0.$$

Then (VF-IVIP) *has a solution. Furthermore, the solution set of* (VF-IVIP) *is closed.*

Theorem 2.4. *Let K be convex cone, $F : K \to Y$ satisfy $2F(x) \subset F(2x)$, for all $x \in K$ and $f : K \to K$ be a mapping such that $0 \in f(K)$ and $2g(K) \subset f(K)$. If assumptions of Theorem 2.2 are satisfied, then the solution set of* (GVF-ICP) *is nonempty and compact.*

Proof. The result follows from Theorem 2.1 and Theorem 2.2. □

Corollary 2.6. *We obtain the same results for* (VF-ICP) *and* (VF-IVIP) *considered in* [23].

References

[1] J.-P. Aubin and H. Frankowska, *Set-Valued Analysis*, Birkhäuser Boston (1990).

[2] A. Bensoussan, *Variational inequalities and optimal stopping time problems. D. L. Russel ed. : Calculus of Variations and Control Theory*, Academic Press (1976), 219–244.

[3] I. Capuzzo-Dolcetta and U. Mosco, *Implicit complementarity problems and quasi-variational inequalities, R. W. Cottle, R. Giannessi and J. L. Lions ed. : Variational Inequalities and Complementarity Problems*, Theory and Applications, John Wiley and Sons (1990), 75–87.

[4] A. Carbone, *A note on complementarity problem*, Internat. J. Math. Math. Sci. **21** (1998), no. 3, 621–623.

[5] S. S. Chang and N. J. Huang, *Generalized multivalued implicit complementarity problems in Hilbert spaces*, Math. Japonica **36**(1991), no. 6, 1093–1100.

[6] G. Y. Chen and X. Q. Yang, *The vector complementarity problems and its equivalences with the weak minimal element in ordered spaces*, J. Math. Anal. Appl. **153** (1990), 136–158.

[7] R. W. Cottle and G. B. Danzig, *Complementarity pivot theory of mathematical programming*, Linear Algebra Appl. **1** (1968), 103–125.

[8] R. W. Cottle and J. C. Yao, *Pseudo-monotone complementarity problems in Hilbert space*, J. Opti, Th. & Appl. **75** (1992), no. 2, 281–295.

[9] K. Fan, *A generalization of Tychonoff's fixed point theorem*, Math. Ann. **142** (1961), 305–310.

[10] Y. P. Fang and N. J. Huang, *The vector F-complementarity problem with demipseudomonotone mappings in Banach spaces*, Appl. Math. Lett. **16** (2003), 1019–1024.

[11] W. Guo, *Implicit complementarity problems in Banach spaces,* Northeast. Math. J. **13** (1997) 335–339.

[12] N. J. Huang and J. Li, *F-implicit complementarity problems in Banach spaces*, Z. Anal. Anwendungen **23** (2004), 293–302.

[13] G. Isac, *A special variational inequality and the implicit complementarity problems*, J. Fac. Sci. Univ. Tokyo **37** (1990), 109–127.

[14] ———, *Complementarity Problems*, Springer-Verlag, New York, 1992.

[15] ———, *A generalization of Karamardian's condition in complementarity theory*, Nonlinear Analysis Forum **4** (1999), 49–63.

[16] ———, *On the implicit complementarity problem in Hilbert spaces*, Bull. Australian Math. Soc. **32** (1985), 251–260.

[17] ———, *Condition $(S)^1_+$, Altman's condition and the scalar asymptotic derivative: applications to complementarity theory*, Nonlinear Analysis Forum **5** (2000), 1–13.

[18] ———, *Topological Methods in Complementarity Theory*, Kluwer Academic Publishers, Dordrecht, Boston, London, 2000.

[19] G. Isac and J. Li, *Complementarity problems, Karamardian's condition and a generalization of Harker-Pang condition*, Nonlinear Analysis Forum **6** (2001), no. 2, 383–390.

[20] S. Karamardian, *Generalized complementarity problem*, J. Optim. Theory Appl. **8** (1971), 161–168.

[21] B. S. Lee, M. Firdosh Khan, and Salahuddin, *Vector F-implicit complementarity problems with corresponding variational inequality problems,* Appl. Math. Lett. **20** (2007), 433–438.

[22] C. E. Lemke, *Bimatrix equilibrium points and mathematical programming*, Management Sci. **11** (1965), 681–689.

[23] J. Li and N. J. Huang, *Vector F-implicit complementarity problems in Banach spaces*, Appl. Math. Lett. **19** (2006), 467–471.

[24] H. Y. Yin, C. X. Xu, and Z. X. Zhang, *The complementarity problems and its equivalence with the least element problem*, Acta Math. Sinica **44** (2001), 679–686.

[25] S. Zhang and Y. Shu, *Complementarity problems with applications to mathematical programming,* Acta Math. Appl. Sinica **15** (1992) 380–388.

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