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COMMON FIXED POINT FOR MULTIVALUED MAPPINGS IN INTUITIONISTIC FUZZY METRIC SPACES

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ABSTRACT. The purpose of this paper is to obtain some common fixed point theorems for multivalued mappings in intuitionistic fuzzy metric space. We extend some earlier results.

Introduction

As a generalization of fuzzy sets introduced by Zadeh [12], Atanassov [2] introduced the concept of intuitionistic fuzzy sets. Recently, using the idea of intuitionistic fuzzy sets, Park [7] introduced the notion of intuitionistic fuzzy metric spaces with the help of continuous t-norms and continuous t-conorms as a generalization of fuzzy metric spaces due to George and Veeramani [3]. Jungck and Rhoades [4] gave more generalized concept weak compatibility then compatibility. Recently, many authors have studied fixed point theory in intuitionistic fuzzy metric spaces (see [1], [6], [7], [10], [11]).

In this paper, we prove common fixed point theorem in intuitionistic fuzzy metric space. We extend results of Kubiaczyk and Sharma [5] to intuitionistic fuzzy metric space.

We begin with some definitions.

1. Preliminaries

Definition 1 ([8]). A binary operation $* : [0,1] \times [0,1] \to [0,1]$ is a continuous t-norm if $*$ is satisfying the following conditions:

(a) $*$ is commutative and associative;
(b) $*$ is continuous;
(c) $a * 1 = a$ for all $a \in [0,1]$;
(d) $a * b \leq c * d$ whenever $a \leq c$ and $b \leq d$, and $a, b, c, d \in [0,1]$.

Definition 2 ([8]). A binary operation $\Diamond : [0,1] \times [0,1] \to [0,1]$ is a continuous t-conorm if $\Diamond$ is satisfying the following conditions:

(a) $\Diamond$ is commutative and associative;

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(b) $\Diamond$ is continuous;
(c) $a\Diamond 0 = a$ for all $a \in [0,1]$;
(d) $a\Diamond b \leq c\Diamond d$ whenever $a \leq c$ and $b \leq d$, and $a,b,c,d \in [0,1]$.

Definition 3 ([1]). A 5-tuple $(X, M, N, *, \Diamond)$ is said to be an intuitionistic fuzzy metric space (shortly IFM-space) if X is an arbitrary set, $*$ is a continuous t-norm, $\Diamond$ is a continuous t-conorm and M, N are fuzzy sets on $X^2 \times (0,\infty)$ satisfying the following conditions: for all x, y, $z \in X$, s, $t > 0$,
(IFM-1) $M(x,y,t) + N(x,y,t) \leq 1$;
(IFM-2) $M(x,y,0) = 0$;
(IFM-3) $M(x,y,t) = 1$ for all $t > 0$ if and only if $x = y$;
(IFM-4) $M(x,y,t) = M(y,x,t)$;
(IFM-5) $M(x,y,t)*M(y,z,s) \leq M(x,z,t+s)$ for all $x,y,z \in X$ and $s,t > 0$;
(IFM-6) $M(x,y,.) : [0,\infty) \to [0,1]$ is left continuous;
(IFM-7) $\lim_{t\to\infty} M(x,y,t) = 1$ for all $x,y \in X$;
(IFM-8) $N(x,y,0) = 1$;
(IFM-9) $N(x,y,t) = 0$ for all $t > 0$ if and only if $x = y$;
(IFM-10) $N(x,y,t) = N(y,x,t)$;
(IFM-11) $N(x,y,t)\Diamond N(y,z,s) \geq N(x,z,t+s)$ for all $x,y,z \in X$ and $s,t > 0$;
(IFM-12) $N(x,y,.) : [0,\infty) \to [0,1]$ is right continuous;
(IFM-13) $\lim_{t\to\infty} N(x,y,t) = 0$ for all $x,y \in X$;

Then (M,N) is called an intuitionistic fuzzy metric on X. The functions $M(x,y,t)$ and $N(x,y,t)$ denote the degree of nearness and the degree of non-nearness between x and y with respect to t, respectively.

Remark 1. Every fuzzy metric space $(X, M, *)$ is an intuitionistic fuzzy metric space if X of the form $(X, M, 1-M, *, \Diamond)$ such that t-norm $*$ and t-conorm $\Diamond$ are associated, i.e. $x\Diamond y = 1 - ((1-x)*(1-y))$ for any $x,y \in [0,1]$. But the converse is not true.

Example 1 ([7]). Let (X,d) be a metric space. Denote $a*b = ab$ and $a\Diamond b = \min\{1, a+b\}$ for all $a,b \in [0,1]$ and let M_d and N_d be fuzzy sets on $X^2 \times (0,\infty)$ defined as follows:

$$M_d(x,y,t) = \frac{t}{t+d(x,y)}, \quad N_d(x,y,t) = \frac{d(x,y)}{t+d(x,y)}.$$

Then (M_d, N_d) is an intuitionistic fuzzy metric on X. We call this intuitionistic fuzzy metric induced by a metric d the standard intuitionistic fuzzy metric.

Remark 2. Note the above example holds even with the t-norm $a*b = \min\{a,b\}$ and the t-conorm $a\Diamond b = \max\{a,b\}$ and hence (M_d, N_d) is an intuitionistic fuzzy metric with respect to any continuous t-norm and continuous t-conorm.

Example 2 ([7]). Let $X = \mathbb{N}$. Define $a*b = \max\{0, a+b-1\}$ and $a\Diamond b = a+b-ab$ for all $a,b \in [0,1]$ and let M and N be fuzzy sets on $X^2 \times (0,\infty)$ as follows:

$$M(x,y,t)=\begin{cases}\frac{x}{y} \text{ if } x\le y,\\ \frac{y}{x} \text{ if } y\le x,\end{cases},\quad N(x,y,t)=\begin{cases}\frac{y-x}{y} \text{ if } x\le y,\\ \frac{x-y}{x} \text{ if } y\le x,\end{cases}$$

for all $x, y \in X$ and $t > 0$. Then $(X, M, N, *, \Diamond)$ is an IFM-space.

Remark 3. Note that, in the above example, t-norm $*$ and t-conorm $\Diamond$ are not associated. And there exists no metric d on X satisfying

$$M(x,y,t)=\frac{t}{t+d(x,y)},\ N(x,y,t)=\frac{d(x,y)}{t+d(x,y)},$$

where $M(x,y,t)$ and $N(x,y,t)$ are as defined in above example. Also note the above functions (M, N) is not an intuitionistic fuzzy metric with the t-norm and t-conorm defined as $a * b = \min\{a, b\}$ and $a\Diamond b = \max\{a, b\}$.

Lemma 1 ([1]). *In intuitionistic fuzzy metric space X, $M(x, y, .)$ is non-decreasing and $N(x, y, .)$ is non-increasing for all $x, y \in X$.*

Lemma 2. *Let $(X, M, N, *, \Diamond)$ be an intuitionistic fuzzy metric space. If there exists $k \in (0, 1)$ such that $M(x, y, kt) \ge M(x, y, t)$ and $N(x, y, kt) \le N(x, y, t)$ for $x, y \in X$. Then $x = y$.*

Proof. Since $M(x, y, kt) \ge M(x, y, t)$ and $N(x, y, kt) \le N(x, y, t)$, then using results of Sharma [9] we have $M(x, y, t) \ge M(x, y, \frac{t}{k})$ and $N(x, y, t) \le N(x, y, \frac{t}{k})$. By repeated application of above inequalities, we have

$$M(x,y,t)\ge M(x,y,\frac{t}{k})\ge M(x,y,\frac{t}{k^2})\ge\cdots\ge M(x,y,\frac{t}{k^n})\ge\cdots$$

and

$$N(x,y,t)\le N(x,y,\frac{t}{k})\le N(x,y,\frac{t}{k^2})\le\cdots\le N(x,y,\frac{t}{k^n})\le\cdots$$

for $n \in N$, which tend to 1 and 0 as $n \to \infty$, respectively. Thus $M(x, y, t) = 1$ and $N(x, y, t) = 0$ for all $t > 0$ and we get $x = y$. □

Lemma 3. *Let $(X, M, N, *, \Diamond)$ be an intuitionistic fuzzy metric space. If there exists a number $k \in (0, 1)$ such that*

(1.1) $M(y_{n+2}, y_{n+1}, qt) \ge M(y_{n+1}, y_n, t)$, $N(y_{n+2}, y_{n+1}, qt) \le N(y_{n+1}, y_n, t)$

for all $t > 0$ and $n = 1, 2, \ldots$, then $\{y_n\}$ is a Cauchy sequence in X.

Proof. For $t > 0$ and $k \in (0, 1)$, we have

$$M(y_2,y_3,kt)\ge M(y_1,y_2,t)\ge M(y_0,y_1,t/k) \text{ or}$$
$$M(y_2,y_3,t)\ge M(y_0,y_1,t/k^2)$$

and

$$N(y_2,y_3,kt)\le N(y_1,y_2,t)\le N(y_0,y_1,t/k) \text{ or}$$
$$N(y_2,y_3,t)\le N(y_0,y_1,t/k^2).$$

By simple induction with the condition (1.1), we have for all $t > 0$ and $n = 0, 1, 2, \ldots$

(1.2)

$$M(y_{n+1}, y_{n+2}, t) \geq M(y_1, y_2, t/k^n), \quad N(y_{n+1}, y_{n+2}, t) \leq N(y_1, y_2, t/k^n).$$

Thus by (1.2) and (IFM-5) and (IFM-11), for any positive integer p and real number $t > 0$, we have

$$\begin{aligned} M(y_n, y_{n+p}, t) &\geq M(y_n, y_{n+1}, t/p) \overset{p-times}{* \cdots *} M(y_{n+p-1}, y_{n+p}, t/p) \\ &\geq M(y_1, y_2, t/pk^{n-1}) \overset{p-times}{* \cdots *} M(y_1, y_2, t/pk^{n+p-2}) \end{aligned}$$

and

$$\begin{aligned} N(y_n, y_{n+p}, t) &\leq N(y_n, y_{n+1}, t/p) \overset{p-times}{* \cdots *} N(y_{n+p-1}, y_{n+p}, t/p) \\ &\leq N(y_1, y_2, t/pk^{n-1}) \overset{p-times}{* \cdots *} N(y_1, y_2, t/pk^{n+p-2}) \end{aligned}$$

which $\to 1$ and $\to 0$ as n $\to \infty$, respectively. Thus $\lim_{n\to\infty} M(y_n, y_{n+p}, t) = 1$ and $\lim_{n\to\infty} N(y_n, y_{n+p}, t) = 0$. Which implies that $\{y_n\}$ is a Cauchy sequence in X. This completes the proof. □

Definition 4 ([1]). Let $(X, M, N, *, \diamondsuit)$ be an intuitionistic fuzzy metric space.

(a) A sequence $\{x_n\}$ in X is said to be Cauchy sequence if for each $t > 0$ and $p > 0$, $\lim_{n\to\infty} M(x_{n+p}, x_n, t) = 1$ and $\lim_{n\to\infty} N(x_{n+p}, x_n, t) = 0$.

Then a sequence $\{x_n\}$ in X converging to x in X if for each $t > 0$,

(b) $\lim_{n\to\infty} M(x_n, x, t) = 1$ and $\lim_{n\to\infty} N(x_n, x, t) = 0$.

(Since $*$ and $\diamondsuit$ are continuous, the limit is uniquely determined from (IFM-5) and (IFM-11)).

(c) An intuitionistic fuzzy metric space is said to be complete if and only if every Cauchy sequence is convergent. It is called compact if every sequence contains a convergent subsequence.

Definition 5. Let $(X, M, N, *, \diamondsuit)$ be a intuitionistic fuzzy metric space. Consider $I : X \to X$ and $T : X \to CB(X)$. A point $z \in X$ is called a coincidence point of I and T if and only if $Iz \in Tz$.

We denote by $CB(X)$ the set of all non-empty bounded and closed subsets of X. We have

$$\begin{aligned} M^{\nabla}(B, y, t) &= \max\{M(b, y, t); b \in B\}, \\ N^{\triangle}(B, y, t) &= \min\{N(b, y, t);\ b \in B\} \end{aligned}$$

and

$$\begin{aligned} M_{\nabla}(A, B, t) &\geq \min\left\{\min_{a\in A} M^{\nabla}(a, B, t), \min_{b\in B} M^{\nabla}(A, b, t)\right\}, \\ N_{\triangle}(A, B, t) &\leq \max\left\{\max_{a\in A} N^{\triangle}(a, B, t), \max_{b\in B} N^{\triangle}(A, b, t)\right\} \end{aligned}$$

for all A, B in X and $t > 0$.

2. Main results

Theorem 1. *Let $(X, M, N, *, \diamondsuit)$ be a complete intuitionistic fuzzy metric space with continuous t-norm $*$ and continuous t-conorm $\diamondsuit$ defined by $t * t \geq t$ and $(1-t)\diamondsuit(1-t) \leq (1-t)$ for all $t \in [0,1]$. Let $P : X \to C(X)$ such that*

$$M_\nabla(Px, Py, kt) \geq M(x, y, t),\ N_\triangle(Px, Py, kt) \leq N(x, y, t) \tag{2.1}$$

for all $x, y \in X$ and $0 < k < 1$. Then P has a fixed point. These means that there exists a point u in X such that $u \in Pu$.

Proof. For any $x_0 \in X$, we define a sequence $\{x_n\}$ in X, as follows : $x_1 \in Px_0$ such that $M(x_0, x_1, t) = M^\nabla(x_0, Px_0, t)$ and $N(x_0, x_1, t) = N^\triangle(x_0, Px_0, t)$, $x_2 \in Px_1$ such that

$$M(x_1, x_2, t) = M^\nabla(x_1, Px_1, t) \text{ and } N(x_1, x_2, t) = N^\triangle(x_1, Px_1, t).$$

Inductively, we write $x_{n+1} \in Px_n$ such that $M(x_n, x_{n+1}, t) = M^\nabla(x_n, Px_n, t)$ and $N(x_n, x_{n+1}, t) = N^\triangle(x_n, Px_n, t)$.

We shall prove that $\{x_n\}$ is a Cauchy sequence. By (2.1), we write

$$\begin{aligned} M(x_n, x_{n+1}, kt) &= M^\nabla(x_n, Px_n, kt) \geq M_\nabla(Px_{n-1}, Px_n, kt) \\ &\geq M(x_{n-1}, x_n, t) \end{aligned}$$

and

$$\begin{aligned} N(x_n, x_{n+1}, kt) &= N^\triangle(x_n, Px_n, kt) \leq N_\triangle(Px_{n-1}, Px_n, kt) \\ &\leq N(x_{n-1}, x_n, t). \end{aligned}$$

So by Definition 4 and Lemma 3, $\{x_n\}$ is a Cauchy sequence. As X is complete, there exists $\lim_{n\to\infty} x_n = x$. We show that x is a fixed point of Px.

$$\begin{aligned} M^\nabla(x, Px, t) &\geq M(x, x_n, t/2) * M^\nabla(x_n, Px, t/2) \\ &\geq M(x, x_n, t/2) * M_\nabla(Px_{n-1}, Px, t/2) \\ &\geq M(x, x_n, t/2) * M(x_{n-1}, x, t/2) \end{aligned}$$

and

$$\begin{aligned} N^\triangle(x, Px, t) &\leq N(x, x_n, t/2)\diamondsuit N^\triangle(x_n, Px, t/2) \\ &\leq N(x, x_n, t/2)\diamondsuit N_\triangle(Px_{n-1}, Px, t/2) \\ &\leq N(x, x_n, t/2)\diamondsuit N(x_{n-1}, x, t/2). \end{aligned}$$

Letting $n \to \infty$, we write

$$M^\nabla(x, Px, t) \geq 1 * 1 \geq 1 \text{ and } N^\triangle(x, Px, t) \leq 0\diamondsuit 0 \leq 0.$$

This implies that x is a fixed point of Px. □

Now we extend Theorem 1 for the sequences of mappings. We prove the following :

Theorem 2. *Let $(X, M, N, *, \diamondsuit)$ be a complete intuitionistic fuzzy metric space with continuous t-norm $*$ and continuous t-conorm $\diamondsuit$ defined by $t * t \geq t$ and $(1-t)\diamondsuit(1-t) \leq (1-t)$ for all $t \in [0,1]$. Let $T_n : X \to CB(X)$ $(n \in N)$ and continuous mapping $I : X \to X$ be such that $T_n(X) \subset I(X)$ where I commute with T_n for every $n \in N$ and there exists $q \in (0,1)$ such that*

$$\begin{aligned} &M_\nabla(T_i x, T_j y, qt) \\ (2.2) \qquad \geq\ &\min\{M(Ix, Iy, t), M^\nabla(Ix, T_i x, t), M^\nabla(Iy, T_j y, t), \\ &M^\nabla(Ix, T_j y, (2-\alpha)t), M^\nabla(Iy, T_i x, t)\} \end{aligned}$$

and

$$\begin{aligned} &N_\triangle(T_i x, T_j y, qt) \\ (2.3) \qquad \leq\ &\max\{N(Ix, Iy, t), N^\triangle(Ix, T_i x, t), N^\triangle(Iy, T_j y, t), \\ &N^\triangle(Ix, T_j y, (2-\alpha)t), N^\triangle(Iy, T_i x, t)\} \end{aligned}$$

for all $x, y \in X, \alpha \in (0,2)$ and $t > 0$ for every $i, j \in N$ $(i \neq j)$. Then there exists a common coincidence point of T_n and I, i.e. there exists a point z in X such that $Iz \in \cap\, T_n z$, $n \in N$.

Proof. Let $x_0 \in X$ and $x_1 \in X$ such that $Ix_1 \in T_1 x_0$, $y_1 = Ix_1$ and the inequalities hold

$$M(x_0, y_1, qt) = M(x_0, Ix_1, qt) \geq M^\nabla(x_0, T_1 x_0, qt) - \varepsilon/2,$$

$$N(x_0, y_1, qt) = N(x_0, Ix_1, qt) \leq N^\triangle(x_0, T_1 x_0, qt) + \varepsilon/2$$

$x_2 \in X$ such that $Ix_2 \in T_2 x_1, y_2 = Ix_2$ and

$$M(y_1, y_2, qt) = M(Ix_1, Ix_2, qt) \geq M^\nabla(y_1, T_2 x_1, qt) - \varepsilon/2^2,$$

$$N(y_1, y_2, qt) = N(Ix_1, Ix_2, qt) \leq N^\triangle(y_1, T_2 x_1, qt) + \varepsilon/2^2.$$

Inductively, we construct a sequence $\{y_n\}$ in X such that

$$M(y_n, y_{n+1}, qt) = M(Ix_n, Ix_{n+1}, qt) \geq M^\nabla(y_n, T_{n+1} x_n, qt) - \varepsilon/2^n,$$

$$N(y_n, y_{n+1}, qt) = N(Ix_n, Ix_{n+1}, qt) \leq N^\triangle(y_n, T_{n+1} x_n, qt) + \varepsilon/2^n.$$

Now we show that $\{y_n\}$ is a Cauchy sequence. By (2.2), for all $t > 0$ and $\alpha = 1 - k$ with $k \in (0,1)$, we write

$$\begin{aligned} &M(y_n, y_{n+1}, qt) \\ \geq\ &M^\nabla(y_n, T_{n+1} x_n, qt) - \varepsilon/2^n \geq M_\nabla(T_n x_{n-1}, T_{n+1} x_n, qt) - \varepsilon/2^n \\ \geq\ &\min\{M(Ix_{n-1}, Ix_n, t), M^\nabla(Ix_{n-1}, T_n x_{n-1}, t), \\ &M^\nabla(Ix_n, T_{n+1} x_n, t), M^\nabla(Ix_{n-1}, T_{n+1} x_n, (2-\alpha)t), \\ &M^\nabla(Ix_n, T_n x_{n-1}, t)\} - \varepsilon/2^n \\ \geq\ &\min\{M(Ix_{n-1}, Ix_n, t), M(Ix_{n-1}, Ix_n, t), M(Ix_n, Ix_{n+1}, t), \\ &M(Ix_{n-1}, Ix_{n+1}, (1+k)t), M(Ix_n, Ix_n, t)\} - \varepsilon/2^n \end{aligned}$$

and by (2.3)

$$\begin{aligned}
& N(y_n, y_{n+1}, qt) \\
\leq\ & N^{\triangle}(y_n, T_{n+1}x_n, qt) + \varepsilon/2^n \leq N_{\triangle}(T_n x_{n-1}, T_{n+1}x_n, qt) + \varepsilon/2^n \\
\leq\ & \max\{N(Ix_{n-1}, Ix_n, t), N^{\triangle}(Ix_{n-1}, T_n x_{n-1}, t), \\
& N^{\triangle}(Ix_n, T_{n+1}x_n, t), N^{\triangle}(Ix_{n-1}, T_{n+1}x_n, (2-\alpha)t), \\
& N^{\triangle}(Ix_n, T_n x_{n-1}, t)\} + \varepsilon/2^n \\
\leq\ & \max\{N(Ix_{n-1}, Ix_n, t), N(Ix_{n-1}, Ix_n, t), N(Ix_n, Ix_{n+1}, t), \\
& N(Ix_{n-1}, Ix_{n+1}, (1+k)t), N(Ix_n, Ix_n, t)\} + \varepsilon/2^n
\end{aligned}$$

respectively. Now using (IFM-5) and (IFM-11), we write

(2.4)

$$\begin{aligned}
M(y_n, y_{n+1}, qt) \quad \geq \quad & \max\{M(y_{n-1}, y_n, t), M(y_{n-1}, y_n, t), M(y_n, y_{n+1}, t), \\
& M(y_{n-1}, y_n, t), M(y_n, y_{n+1}, \ kt), 1\} - \varepsilon/2^n,
\end{aligned}$$

(2.5)

$$\begin{aligned}
N(y_n, y_{n+1}, qt) \quad \leq \quad & \max\{N(y_{n-1}, y_n, t), N(y_{n-1}, y_n, t), N(y_n, y_{n+1}, t), \\
& N(y_{n-1}, y_n, t), N(y_n, y_{n+1}, \ kt), 1\} + \varepsilon/2^n.
\end{aligned}$$

Since t-norm $*$, t-conorm $\diamondsuit$, $M(x, y, \cdot)$ and $N(x, y, \cdot)$ are continuous, letting $k \to 1$ in (2.4) and (2.5), we have

$$M(y_n, y_{n+1}, qt) \geq \min\{M(y_{n-1}, y_n, t), M(y_n, y_{n+1}, t)\} - \varepsilon/2^n,$$

$$N(y_n, y_{n+1}, qt) \leq \max\{N(y_{n-1}, y_n, t), N(y_n, y_{n+1}, t)\} + \varepsilon/2^n$$

for $n = 1, 2, \ldots$ and so, for positive integers n, p and $\varepsilon \in (0, 1)$, we have

$$M(y_n, y_{n+1}, qt) \geq \min\{M(y_{n-1}, y_n, t), M(y_n, y_{n+1}, t/q^p)\} - \varepsilon/2^n,$$

$$N(y_n, y_{n+1}, qt) \leq \max\{N(y_{n-1}, y_n, t), N(y_n, y_{n+1}, t/q^p)\} + \varepsilon/2^n.$$

Since ε is arbitrary making $\varepsilon \to 0$, and $M(y_n, y_{n+1}, t/q^p) \to 1$ and $N(y_n, y_{n+1}, t/q^p) \to 0$ as $n \to \infty$, we obtain

$$M(y_n, y_{n+1}, qt) \geq M(y_{n-1}, y_n, t) \text{ and } N(y_n, y_{n+1}, qt) \leq N(y_{n-1}, y_n, t).$$

By Lemma 3, $\{y_n\}$ is a Cauchy sequence. Since X is complete, $\{y_n\}$ converges to a point z in X. Now, by (2.2) and (2.3) with $\alpha = 1$, we have

$$\begin{aligned}
& M^{\nabla}(IIx_n, T_m z, qt) \\
\geq\ & M_{\nabla}(T_n I x_{n-1}, T_m z, qt) \\
\geq\ & \min\{M(IIx_{n-1}, Iz, t), M^{\nabla}(IIx_{n-1}, T_n I x_{n-1}, t), \\
& M^{\nabla}(Iz, T_m z, t), M^{\nabla}(IIx_{n-1}, T_m z, t), M^{\nabla}(Iz, T_n I x_{n-1}, t)\} \\
\geq\ & \min\{M(IIx_{n-1}, Iz, t), M(IIx_{n-1}, IIx_n, t), \\
& M^{\nabla}(Iz, T_m z, t), M^{\nabla}(IIx_{n-1}, T_m z, t), M(Iz, IIx_n, t)\},
\end{aligned}$$

$$\begin{aligned}
& N^{\triangle}(IIx_n, T_m z, qt) \\
\leq\ & N_{\triangle}(T_n I x_{n-1}, T_m z, qt) \\
\leq\ & \max\{N(IIx_{n-1}, Iz, t), N^{\triangle}(IIx_{n-1}, T_n I x_{n-1}, t), \\
& N^{\triangle}(Iz, T_m z, t), N^{\triangle}(IIx_{n-1}, T_m z, t), N^{\triangle}(Iz, T_n I x_{n-1}, t)\} \\
\leq\ & \max\{N(IIx_{n-1}, Iz, t), N(IIx_{n-1}, IIx_n, t), \\
& N^{\triangle}(Iz, T_m z, t), N^{\triangle}(IIx_{n-1}, T_m z, t), N(Iz, IIx_n, t)\}
\end{aligned}$$

respectively. Since $\lim_{n\to\infty} M(IIx_{n-1}, Iz, t) = 1$, $\lim_{n\to\infty} M(IIx_{n-1}, IIx_n, t) = 1$, $\lim_{n\to\infty} N(IIx_{n-1}, Iz, t) = 0$ and $\lim_{n\to\infty} N(IIx_{n-1}, IIx_n, t) = 0$, we have

$$\lim_{n\to\infty} M^{\nabla}(IIx_{n-1}, T_m z, t) = M^{\nabla}(Iz, T_m z, t),$$

$$\lim_{n\to\infty} N^{\triangle}(IIx_{n-1}, T_m z, t) = N^{\triangle}(Iz, T_m z, t).$$

Hence for any $m \in N$, we write

$$M^{\nabla}(Iz, T_m z, qt) \geq M^{\nabla}(Iz, T_m z, t) \text{ and } N^{\triangle}(Iz, T_m z, qt) \leq N^{\triangle}(Iz, T_m z, t).$$

This implies by Lemma 2, that $Iz \in T_m z$ and therefore $Iz \in \bigcap T_n z$ for $n \in N$. This completes the proof. □

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