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ON STRONGLY REGULAR NEAR-SUBTRACTION SEMIGROUPS

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Abstract. In this paper we introduce the notion of strongly regular near-subtraction semigroups (right). We have shown that a near-subtraction semigroup X is strongly regular if and only if it is regular and without non zero nilpotent elements. We have also shown that in a strongly regular near-subtraction semigroup X, the following holds: (i) Xa is an ideal for every $a \in X$ (ii) If P is a prime ideal of X, then there exists no proper k-ideal M such that $P \subset M$ (iii) Every ideal I of X fulfills $I = I^2$.

1. Introduction

B. M. Schein [9] considered systems of the form $(\phi; \circ, \backslash)$, where ϕ is a set of functions closed under the composition " $\circ$ " of functions (and hence $(\phi; \circ)$ is a function semigroup) and the set theoretic subtraction "$\backslash$"(and hence $(\phi; \backslash)$ is a subtraction algebra in the sense of [1]). B. Zelinka [10] discussed a problem proposed by B. M. Schein concerning the structure of multiplication in a subtraction semigroup. E. H. Roh, K. H. Kim and J. G. Lee [8] obtained significant results in subtraction semigroups.

From Ring-Theory, Near-ring (right) theory has been developed by Pilz [7], Mason [6], Meldrum [5] and Clay [3]. In this paper we introduce near-subtraction semigroup (right) which is not a subtraction semigroup. Similar to Near-ring (right), we have obtained significant results in near-subtraction semigroups (right).

2. Preliminaries

A non empty set X together with a binary operation " $-$ " is said to be a subtraction algebra if it satisfies the following:

(1) $x - (y - x) = x$.
(2) $x - (x - y) = y - (y - x)$.
(3) $(x - y) - z = (x - z) - y$, for every $x, y, z \in X$.

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Example 1. Let A be any non empty set. Then $(P(A), \setminus)$ is a subtraction algebra, where "$P(A)$" denotes the power set of A and "$\setminus$" denotes the set theoretic subtraction.

Example 2. Let $X = \{0, a, b, 1\}$ in which " $-$ " is defined by

$-$	0	a	b	1
0	0	0	0	0
a	a	0	a	0
b	b	b	0	0
1	1	b	a	0

Then $(X, -)$ is a subtraction algebra.

In a subtraction algebra the following holds:

(1) $x - 0 = x$ and $0 - x = 0$.
(2) $(x - y) - x = 0$.
(3) $(x - y) - y = x - y$.
(4) $(x - y) - (y - x) = x - y$, where $0 = x - x$ is an element that does not depend on the choice of $x \in X$.

Following [4], we have the following definition of subtraction semigroup.

Definition 3. A nonempty set X together with two binary operations " $-$ " and "." is said to be a subtraction semigroup if it satisfies the following:

(1) $(X; -)$ is a subtraction algebra.
(2) $(X; \cdot)$ is a semigroup.
(3) $x(y - z) = xy - xz$ and $(x - y)z = xz - yz$ for every $x, y, z \in X$.

Example 4. Let $X = \{0, a, b, 1\}$ in which "$-$" and "." are defined as follows:

$-$	0	a	b	1
0	0	0	0	0
a	a	0	a	0
b	b	b	0	0
1	1	b	a	0

.	0	a	b	1
0	0	0	0	0
a	0	a	0	a
b	0	0	b	b
1	0	a	b	1

Then $(X, -, \cdot)$ is a subtraction semigroup.

3. Near-subtraction semigroup

Here we introduce the notion of near-subtraction semigroup.

Definition 5. A nonempty set X together with two binary operations " $-$ " and "." is said to be a near-subtraction semigroup (right) if

(1) $(X; -)$ is a subtraction algebra.
(2) $(X; \cdot)$ is a semigroup and
(3) $(x - y)z = xz - yz$, for every $x, y, z \in X$.

Note: It is clear that $0x = 0$, for every $x \in X$

Similarly we can define a near-subtraction semigroup (left). Hereafter a near-subtraction semigroup means it is a near-subtraction semigroup (right) only.

Example 6. Let Γ be a subtraction algebra. Then the set $M(\Gamma)$ of all mappings of Γ into Γ is a near-subtraction semigroup under pointwise subtraction and composition of mappings. $M(\Gamma)$ is not a subtraction semigroup.

Example 7. Let $\Gamma = \{0, 1\}$ in which " $-$ " is defined by

$-$	0	1
0	0	0
1	1	0

Then Γ is a subtraction algebra. Now $M(\Gamma) = \{0,\ a,\ b,\ 1\}$, where 0, a, b, 1 are all functions from Γ to Γ. $M(\Gamma)$ is a near-subtraction semigroup under pointwise subtraction and composition and we have

$-$	0	a	b	1
0	0	0	0	0
a	a	0	1	b
b	b	0	0	b
1	1	0	1	0

.	0	a	b	1
0	0	0	0	0
a	a	a	a	a
b	a	0	1	b
1	0	a	b	1

Definition 8. A near-subtraction semigroup X is said to be zero-symmetric if $x0 = 0$ for every $x \in X$.

Example 9. Let Γ be a subtraction algebra. Then $M_0(\Gamma) = \{f : \Gamma \to \Gamma | f(0) = 0\}$ is a zero-symmetric near-subtraction semigroup under pointwise subtraction and composition of mappings.

Example 10. Let $X = \{0,\ 1\}$ in which " $-$ " and " $\cdot$ " are defined by

$-$	0	1
0	0	0
1	1	0

.	0	1
0	0	0
1	0	1

Then $(X, -, \cdot)$ is a zero-symmetric near-subtraction semigroup.

Definition 11. A near-subtraction semigroup X is said to have an identity if there exists an element $1 \in X$ such that $1.x = x.1 = x$, for every $x \in X$.

Definition 12. A non empty subset S of a subtraction algebra X is said to be a subalgebra of X, if $x - y \in S$, whenever $x, y \in S$.

Definition 13. Let $(X, -, \cdot)$ be a near-subtraction semigroup. A nonempty subset I of X is called

(1) a left ideal if I is a sub algebra of $(X, -)$ and $xi - x(x' - i) \in I$ for all $x,\ x' \in X$ and $i \in I$

(2) a right ideal if I is a subalgebra of $(X, -)$ and $IX \subseteq I$

(3) an ideal if I is both a left and right ideal.

Note:

(1) Suppose if X is a subtraction semigroup and I is a left ideal of X, then for $i \in I$ and $x,\ x' \in X$, we have $xi - x(x' - i) = xi - (xx' - xi) = xi \in I$ by Property 1 of subtraction algebra. Thus we have $XI \subseteq I$.

(2) If X is a zero symmetric near-subtraction semigroup, then for $i \in I$ and $x \in X$, we have $xi - x(0 - i) = xi - 0 = xi \in I$.

Definition 14. An ideal I of X is said to be a k-ideal if $x - y \in I$ and $y \in I$ implies $x \in I$.

Example 15. Consider the following near-subtraction semigroup

$-$	0	1	2	3	4	5
0	0	0	0	0	0	0
1	1	0	3	4	3	1
2	2	5	0	2	5	4
3	3	0	3	0	3	3
4	4	0	0	4	0	4
5	5	5	0	5	5	0

.	0	1	2	3	4	5
0	0	0	0	0	0	0
1	0	1	4	3	4	0
2	0	4	2	0	4	5
3	0	3	0	3	0	0
4	0	4	4	0	4	0
5	0	0	5	0	0	5

Here $\{0,\ 1,\ 3,\ 4\}$ is a k-ideal. $\{0,\ 3,\ 4,\ 5\}$ is an ideal but not a k-ideal, since $2 - 4 = 5 \in \{0,\ 3,\ 4,\ 5\}$ and $2 \notin \{0,\ 3,\ 4,\ 5\}$.

A near-subtraction semigroup X is said to be regular if given $a \in X$, there is $x \in X$ such that $axa = a$. Following Ring Theory, X is called strongly regular when for each $a \in X$, $a = xa^2$, for some $x \in X$. For any nonempty subsets A and B of X, $AB = \{ab | a \in A,\ b \in B\}$. An ideal P of X is said to be a prime ideal if for ideals A, B of X, $AB \subseteq P$ implies $A \subseteq P$ or $B \subseteq P$. A proper ideal P of X is called completelyprime (semicompletely prime) if $ab \in P$ implies either $a \in P$ or $b \in P(a^2 \in P$ implies $a \in P)$. For any $x \in X$, $\langle x \rangle$ stands for the principal ideal generated by x, which is the intersection of all ideals of X containing x and $\langle x \rangle_k$ is the principal k-ideal generated by x which is the intersection of all k-ideals of X containing x. If B and C are subsets of X, we denote the set $\{x \in X | xC \subseteq B\}$ by $(B : C)$ and if $B = \{0\}$ we write $(B : C)$ by $l(C)$ and $r(C) = \{x \in X | Cx = 0\}$. An element $x \in X$ is said to be nilpotent if there exist a positive integer n such that $x^n = 0$. A near-subtraction semigroup X is said to have IFP (insertion of factors property) if for $a,\ b$ in X if $ab = 0$ implies $axb = 0$ for all $x \in X$. Unless otherwise stated, throughout this paper X stands for a zero-symmetric near-subtraction semigroup.

Lemma 16. *Let X be a near-subtraction semigroup. For any $x,\ y \in X$, $x = y$ if and only if $x - y = 0$ and $y - x = 0$.*

Proof. Suppose that $x - y = 0$ and $y - x = 0$. Then $x = x - 0 = x - (x - y) = y - (y - x) = y - 0 = y$. The converse part is obvious. □

Lemma 17. *If X is a near-subtraction semigroup, then the following assertions are equivalent:*

(*a*) *X has the IFP.*
(*b*) *For each $x \in X : (0 : x)$ is a k-ideal of X.*
(*c*) *For each subset S of X : $(0 : S)$ is a k-ideal of X.*

Proof. (*a*) $\Rightarrow$ (*b*) For $r_1,\ r_2 \in (0 : x)$, $(r_1 - r_2)x = r_1x - r_2x = 0$, showing that $r_1 - r_2 \in (0 : x)$. Let $y,\ y' \in X$ and $i \in (0 : x)$. Then $(yi - y(y' - i))x = yix - y(y' - i)x = 0$. And $iyx = 0$ by IFP. Thus $(0 : x)$ is an ideal of X, for every $x \in X$. Suppose $r_1 - r_2 \in (0 : x)$ and $r_2 \in (0 : x)$, then $(r_1 - r_2)x = 0$ and $r_2x = 0$. Hence $r_1x - r_2x = 0$ implies $r_1x = 0$. Thus $r_1 \in (0 : x)$ showing that $(0 : x)$ is a k-ideal of X for every $x \in X$.

(*b*) $\Rightarrow$ (*c*) is obvious.

(*c*) $\Rightarrow$ (*a*) Let $a,\ b \in X$ such that $ab = 0$. Then $a \in (0 : b)$. Hence by (c), $ax \in (0 : b)$ for every $x \in X$. Thus $axb = 0$, for every $x \in X$. □

Note: If X has no non zero nilpotent elements then $ab = 0$ implies $ba = 0$ and hence $l(S) = r(S)$ for any subset S of X. In this case we denote the set by $A(S)$.

Lemma 18. *If X has no non zero nilpotent elements then for any $0 \neq x \in X$,*
1) *$A(x)$ is a semicompletely prime ideal*
2) *$ab \in A(x)$ implies $ba \in A(x)$*
3) *$x_1\, x_2 \cdots x_n \in A(x)$ implies $\langle x_1\rangle_k\, \langle x_2\rangle_k\, \ldots\, \langle x_n\rangle_k \subseteq A(x)$ for all $x_1, x_2, \ldots,$ x_n in X.*

Proof. 1) Now $ab = 0$ implies $ba = 0$ since $(ba)^2 = b(ab)a$. Again for any x in X, $(axb)^2 = ax(ba)xb = 0$ as $ba = 0$. Hence $axb = 0$. Thus X has IFP. By Lemma 17, $A(x)$ is an ideal of X. Suppose $a^2 \in A(x)$. Then $0 = a(ax) = (ax)a$ so that $(ax)^2 = 0$ and thus $ax = 0$. Hence $A(x)$ is a semicompletely prime ideal.

2) Suppose $ab \in A(x)$. Then $(ba)^2 = b(ab)a \in A(x)$ and hence $ba \in A(x)$.

3) Let $x_1 \cdots x_n \in A(x)$. It can be easily verified that $(A(x) : S)$ is a k-ideal for any subset S of X. Since $x_1 \in (A(x) : x_2x_3 \cdots x_n)$ we have $\langle x_1\rangle_k \subseteq (A(x) : x_2x_3 \cdots x_n)$ so that $\langle x_1\rangle_k x_2x_3 \cdots x_n \subseteq A(x)$. By the property (2), we have $x_2 \cdots x_n\langle x_1\rangle_k \subseteq A(x)$. Now $x_2 \in (A(x) : x_3 \cdots x_n\langle x_1\rangle_k)$ so that $\langle x_2\rangle_k \subseteq (A(x) : x_3 \cdots x_n\langle x_1\rangle_k)$ and hence $\langle x_2\rangle_k x_3 \cdots x_n\langle x1\rangle_k \subseteq A(x)$. Thus $x_3 \cdots x_n\langle x_1\rangle_k\langle x_2\rangle_k \subseteq A(x)$. Continuing the process we arrive at (3). □

Lemma 19. *Let X be a subtraction semigroup with identity. Then for any two ideals A and P, $(A \cup XPX) = \{x \in X | x - a = 0$ for some $a \in A$ or $x - r_1pr_2 = 0$ for some $r_1,\ r_2 \in X$ and for some $p \in P\}$ is an ideal of X containing both A and P.*

Proof. Let $x,\ y \in (A \cup XPX)$. Then $x - a_1 = 0$ for some $a_1 \in A$ or $x - r_1pr_2 = 0$, for some $r_1, r_2 \in X$ and for some $p \in P$ and $y - a_2 = 0$ for some $a_2 \in A$ or $y - r_3qr_4 = 0$, for some $r_3, r_4 \in X$ and for some $q \in P$. Suppose $x - a_1 = 0$ and $y - a_2 = 0$. Then $(x - y) - a_1 = (x - a_1) - y = 0 - y = 0$ and hence $x - y \in (A \cup XPX)$. Similarly for other cases we can easily verify that $x - y \in (A \cup XPX)$. Let $i \in (A \cup XPX)$. Then $i - a = 0$ for some $a \in A$ or $i - r_1sr_2 = 0$, for some $r_1,\ r_2 \in X$ and for some $s \in P$. For $r \in X$, we have $ir - ar = 0$ or $ir - r_1sr_2r = 0$. Since A is an ideal, $ar \in A$ and hence $ir \in (A \cup XPX)$. Again for $r, r' \in X$, we have $(ri - r(r' - i)) - ra = (ri - ra) - r(r' - i) = 0$ or $(ri - r(r' - i)) - rr_1sr_2 = (ri - rr_1sr_2) = 0$ showing that $ri - r(r' - i) \in (A \cup XPX)$. Clearly $A \subseteq (A \cup XPX)$ and $P \subseteq (A \cup XPX)$. Hence $(A \cup XPX)$ is an ideal containning both A and P. □

Lemma 20. *Let X be a near-subtraction semigroup with identity. Then for any two k- ideals A and P, $(A \cup XPX) = \{x \in X : x - a = 0$ for some $a \in A$ or $x - r_1pr_2 = 0$ for some $r_1,\ r_2 \in X$ and for some $p \in P\}$ is an ideal of X containning both A and P.*

Proof. Clearly $A \subseteq (A \cup XPX)$ and $P \subseteq (A \cup XPX)$. Let $y_1,\ y_2 \in (A \cup XPX)$. Then $y_1 - a_1 = 0$ for some $a_1 \in A$ or $y_1 - r_1p_1r_2 = 0$ for some $r_1,\ r_2 \in X$ and for some $p_1 \in P$ and $y_2 - a_2 = 0$ for some $a_2 \in A$ or $y_2 - r_3p_2r_4 = 0$ and for some $r_3, r_4 \in X$ and for some $p_2 \in P$. Suppose $y_1 - a_1 = 0$ for some $a_1 \in A$ and $y_2 - a_2 = 0$ for some $a_2 \in A$. Then $(y_1 - y_2) - a_1 = (y_1 - a_1) - y_2 = 0$. Similarly for other cases we can easily verify that $y_1 - y_2 \in (A \cup XPX)$. Let $i \in (A \cup XPX)$. Then $i - a = 0$ for some $a \in A$ or $i - r_5pr_6 = 0$ for some $r_5,\ r_6 \in X$ and for some $p \in P$. Since A and P are k-ideals, $i \in A$ or P. Hence $ir \in A$ or P for every $r \in X$. Thus $ir \in (A \cup XPX)$ for every $r \in X$. Similarly $xi - x(x' - i) \in A$ or P for every $x,\ x' \in X$. Thus $(A \cup XPX)$ is an ideal containning both A and P. □

Note: In Lemma 20, $(A \cup XPX)$ coincides with $A \cup P$.

Theorem 21. *Let X be a near-subtraction semigroup with identity having no non zero nilpotent elements in which every ideal is a k-ideal. For any $x(\neq 0)$ in X, if P is a minimal prime ideal containing $A(x)$ then P is completely prime.*

Proof. Let M be the multiplicative subsemigroup of X generated by $X \backslash P$. We claim that $A(x) \cap M = \phi$. If not choose an element y in $A(x) \cap M$. Since $y \in M$ there exists $x_1, x_2, \ldots, x_n$ in $X \backslash P$ such that $y = x_1x_2 \cdots x_n \in A(x)$. By Lemma 18, we have $\langle x_1\rangle_k\langle x_2\rangle_k \cdots \langle x_n\rangle_k \subseteq A(x) \subseteq P$. Thus $\langle x_i\rangle_k \subseteq P$ for some i. Hence $x_i \in P$ which contradicts our assumption. Let $K = \{J | J$ is a ideal of X such that $A(x) \subseteq J$ and $J \cap M = \phi\}$. K is non empty as $A(x) \in K$. By Zorns lemma, K contains a maximal element say Q. Hence $Q \subseteq X \backslash M$. Now we show that Q is prime. Otherwise there exists ideals A and B such that $AB \subseteq Q$, $A \nsubseteq Q$ and $B \nsubseteq Q$. Consider the ideals $(Q \cup XAX)$ and $(Q \cup XBX)$. Since Q is maximal, we have $(Q \cup XAX) \cap M \neq \phi$ and $(Q \cup XBX) \cap M \neq \phi$. Let $p \in (Q \cup XAX) \cap M$ and $s \in (Q \cup XBX) \cap M$. Then $ps \in M$. Since $p \in (Q \cup XAX)$ we have $p - q_1 = 0$ for some $q_1 \in Q$ or $p - r_1ar_2 = 0$ for some $r_1,\ r_2 \in X$, for some $a \in A$. And $s - q_2 = 0$ for some $q_2 \in Q$ or $s - r_3br_4 = 0$ for some $r_3,\ r_4 \in X$, for some $b \in B$. Since $Q,\ A,\ B$ are k-ideals and $AB \subseteq Q$, we have $ps \in Q$. Therefore $Q \cap M \neq \phi$ a contradiction. Hence Q is a prime ideal.

Now $A(x) \subseteq Q \subseteq X \backslash M \subseteq P$. By the minimality of P, we have $Q = X \backslash M = P$. Since M is a multiplicative semigroup, P is a completely prime ideal. □

Remark 22. The above theorem fails if X is not zero-symmetric. Consider the near-subtraction semigroup in Example 7, where X is not zero-symmetric and every ideal is a k-ideal. Here 0 is the minimal prime ideal, but it is not completely prime as $ba = 0$.

Lemma 23. *Let X be a near-subtraction semigroup without nonzero nilpotent elements. For any a, b in X if e is an idempotent in X then $abe = aeb$.*

Proof. Clearly X has IFP and $xy = 0$ implies $yx = 0$ for every x, y in X. Let e be an idempotent in X. For every a, b in X, since $(a - ae)e = 0$ we have $(a - ae)be = 0$ so that $abe - aebe = 0$. And $(ae - a)e = 0$, implies $aebe - abe = 0$. Hence $abe = aebe$, by Lemma 16. Since $(eb - ebe)e = 0$, we have $eb(eb - ebe) = 0$ and $ebe(eb - ebe) = 0$. So that $(eb - ebe)^2 = 0$. Hence $eb - ebe = 0$. Similarly $ebe - eb = 0$. Thus $eb = ebe$ and hence $abe = aeb$. □

Theorem 24. *A near-subtraction semigroup X is strongly regular if and only if it is regular and without nonzero nilpotent elements.*

Proof. Let X be strongly regular. Suppose $a \in X$ such that $a^2 = 0$. Since X is strongly regular there exists some $x \in X$ such that $a = xa^2 = 0$. Thus $a^2 = 0$ implies $a = 0$ for every a in X. Hence X is without nonzero nilpotent elements. Now let us show that X is regular. Let $a \in X$. Then $a = xa^2$, for some $x \in X$. Hence $(a - axa)a = 0$. Since X is without nonzero nilpotent elements, X has IFP and $ab = 0$ implies $ba = 0$. So $a(a - axa) = 0$ and $axa(a - axa) = 0$, so that $(a - axa)^2 = 0$ and hence $(a - axa) = 0$.Thus $a - axa = 0$. Similarly we have $(axa - a) = 0$ and hence $a = axa$, by Lemma 16.

Conversely, let X be a regular near-subtraction semigroup without nonzero nilpotent elements. Let $a \in X$. Since X is regular $a = aya$, for some $y \in X$. Since ya is an idempotent, by Lemma 23, $a = aya = ayaya = ayyaa = ay^2a^2 = xa^2$, where $x = ay^2$. Thus X is strongly regular. □

Theorem 25. *Let X be a strongly regular near-subtraction semigroup. Then*

(*a*) *Xa is an ideal for all $a \in X$.*

(*b*) *For every prime ideal P of X there exists no proper k-ideal M such that $P \subset M$.*

(*c*) *Every ideal I of X fulfills $I = I^2$.*

Proof. (*a*) It is obvious that X has no nonzero nilpotent elements so that $ab = 0$ implies $ba = 0$ and X has IFP. Let $a(\neq 0) \in X$ such that $a = xa^2$ for some $x \in X$. Then $(a - axa)a = 0$. Hence $a(a - axa) = 0$ and $axa(a - axa) = 0$, so that $(a - axa)^2 = 0$. Thus $a - axa = 0$. Similarly $axa - a = 0$. Hence $a = axa$. Let $xa = e$. Then e is an idempotent and $Xa = Xe$. Denoting the set $\{n - ne | n \in X\}$ by S we claim that $A(S) = Xe$. Since $(n - ne)e = 0$ for any $n \in X$ using IFP $(n - ne)Xe = 0$. Hence $Xe \subseteq A(S)$. Suppose $y \in A(S)$.

Since X is strongly regular there exists some $z \in X$ such that $y = zy^2$. Now $(zy - zye)y = 0$. Thus $y - ye = 0$. Also $((ye - y) - (ye - y)e)y = 0$, so that $(ye - y)y = 0$. Then $y(ye - y) = 0$ and $ye(ye - y) = 0$. Now $(ye - y)^2 = 0$ and hence $ye - y = 0$. Thus $y = ye \in Xe$ and it follows that $Xa = Xe = A(S)$. Since $A(S)$ is an ideal, Xa is an ideal.

(*b*) Let P be a prime ideal and suppose $P \subset M$ where M is a proper k-ideal. Let $a \in M \backslash P$. Now $a = xa^2$ for some $x \in X$. For any $n \in X$, $na = nxa^2$. Hence $(n - nxa)a = 0$. Since X has IFP, $X(n - nxa)Xa = 0$. Thus $Xa \subseteq P$ or $X(n - nxa) \subseteq P$. Suppose $Xa \subseteq P$. Since $a = xa^2 \in Xa$, we have $a \in P$ a contradiction. Suppose $X(n - nxa) \subseteq P$. Then $(n - nxa) \in P \subset M$. Since $a \in M$ and M is a k-ideal, $n \in M$. Thus $M = X$ a contradiction.

(*c*) The proof is obvious. □

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