Dynamic COQ Model for Different Quality Levels*

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Abstract

A COQ model plays an important role in the total quality cost survey. Based on the methodology of continuous quality improvement, a dynamic COQ model for different quality level is developed in this paper. A quality level is defined by Six Sigma level that can be measured by two indicators. The relationships among the four major quality costs are analyzed. Finally, the curves of total quality costs for different quality level are presented.

Key Words: Quality Cost, Quality Level, Six Sigma, COQ Model

1. Introduction

With the development of market economy and buyers' market, a company needs to improve its quality level and reduce its cost of poor quality at the same time. Analysis of quality costs has always played a major role in the discussing of the economic effects of quality, and formulating quality improvement strategies. Traditional practices of quality control and assurance have been based on accepting the fundamental premise 'tolerance' or a certain quality level (QL) in production and service, that is, the optimal quality cost is reached to an acceptable quality cost. However, this view is in conflict with current trends in industry to strive for best possible quality. Six Sigma view asserts that the most economical quality level is very close to perfection, and that the objective of companies should be at a zero-defect level. The total quality cost continues to fall as the zero-defect level is approached. As we known, product perfection or zero defects should be done by continuous quality improvement step by step. How to measure total quality cost at different quality level, and what's the relationship among the quality cost components should be discussed in quality cost issues.

Since the cost of quality (COQ) model was constructed, a number of quality cost literatures have been presented (Plunkett and Dale, 1988; Hwang and Aspinwall, 1996; Shah and Fitzroy, 1998; Freiesleben, 2004a). From the review of the literatures, the investigations of

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quality cost are concentrated on the two aspects. The first aspect constructs the COQ Model and discusses its implementing methods, which are related to categories of quality cost (Tannock, 1997; Krishnan, Agus, and Husain, 2000; Giakatis, Enkawa, and Washitani, 2001), modification and extension for the COQ Model (Hwang and Aspinwall, 1999; Miller and Morris, 2000; Freiesleben, 2004b), and the models' comparison and application (Mandall and Shah, 2002; Malchi, 2003; Navee, 2003). The second aspect deals with quality cost surveys, which involves in collection, measurement and reporting of quality cost data (Tkaczyk, and Jagøa, 2001; Lina and Johnson, 2004), and provides empirical evidence of relationships between major quality cost components (Can and Erdal, 2000; Aoieong, Tang, and Ahmed, 2002; Oppermann, Sauer and Wohlrabe, 2003).

This paper focus on the first aspect of the quality cost issues, that is, the COQ Model and its development. Some key observations can be made from the literature review. Juran's old COQ model or the PAF (prevention, appraisal and failure) model is contradictory with the Six Sigma thinking (Shah and Mandal, 1999), the latter claims continuous quality improvement and the best quality level, which fits with realities very well. Using the thinking of Six Sigma, Juran and Gryna (1993) improved the old PAF model and got the modified COQ model, however, which neglected an important parameter-time and still was a static model. Freiesleben (2004b) presented a developed COQ model, which is a dynamic model for quality level, but only described the decreasing prevention and appraisal costs (PAC) on a certain failure cost (FC). In fact, the COQ models should be to depict the development of the total costs of quality for changing quality levels.

The main purpose of this paper is to set up a COQ model at different quality level and analyze the relationships among the total quality cost, prevention, appraisal and failure cost for changing quality level. Firstly, the old quality cost model, the modified quality cost model and the developed quality cost model are compared, and a quality level is defined by Six Sigma level. Secondly, based on the methodology of continuous quality improvement, a dynamic COQ model for different quality level is developed in this paper. A total quality cost model is formulated, and a total quality curve is plotted for different quality level. Thirdly, the curves of prevention and appraisal costs for different quality level on a certain failure cost are depicted respectively. In virtue of these curves, the relationship between prevention, appraisal cost and failure cost is revealed. Finally, the curves of failure costs for different quality level on certain prevention and appraisal cost are presented, and the situation of failure cost changing for different quality level is shown.

2. COQ Models versus QL

Juran and Lundvall first proposed the COQ Model or the PAF Model in 1951, seen

Figure 1, which shows that the costs of poor quality (internal and external failure costs) decrease with higher quality levels, while the costs of achieving good quality (appraisal and prevention costs) increase. The total cost function, representing the sum of both cost categories, has a parabolic shape (Plunkett and Dale 1988). The economically optimal level of quality is regarded as an acceptable quality level (APL)(Love, Guo, and Irwin 1995). The model's inherent quality cost trade off has widely shaped the perception that the optimal level of quality would become the objective of a company's efforts for quality activities.

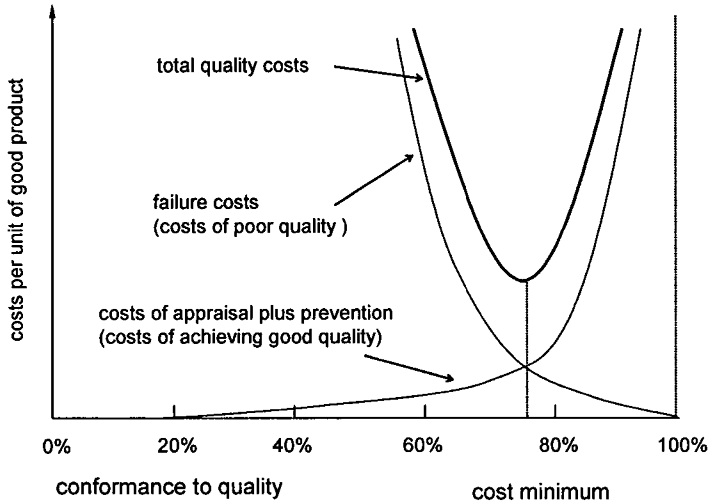

Figure 1. The old COQ model

However, such a perspective ignores the possibility of significant quality improvements and the possibility of maintaining low defect levels by process monitoring. Juran and Gryna (1993) modified the above model, and a new COQ model is displayed in Figure 2. This model shows that a weaker increase in prevention and appraisal costs, accounting for a higher prioritization of prevention and new technological solutions which reduce the failure rate and make process monitoring feasible. The total cost curve is negatively sloped and the cost optimum reaches to the perfect quality level. Although the new COQ model reflects Deming's viewpoint that we do not need a COQ model to determine an optimal level of quality (Deming, 1986), it is still a static model indeed, which only describes the relationship among the total cost, the prevention plus appraisal cost and failure cost on a certain quality level.

With the increasing successes of Six Sigma, which show that quality perfection is a desirable objective (Linderman et al., 2003), it can be achieved by the successive quality improvements activities. Shah and Mandal(1999) pointed out that the investigation of quality costs should reflect to the changes of quality improvement stages. The changeable and dynamic

COQ models should be paid more attention to in the area of quality costs. Freiesleben (2004) first discussed the dynamism of quality cost, when he re-examined the above two COQ Models traded off quality-cost. By scrutinizing the models' assumptions, he detected that they present static 'snapshots', yet neglect the dynamism of continuous improvement. Additionally, the COQ Models are not adequate for determining an economically optimal quality level. Base on the both of COQ Models, Freiesleben developed a dynamic COQ model, which can display different levels of 'optimal' prevention and appraisal cost depending on the current quality level, seen Figure 3.

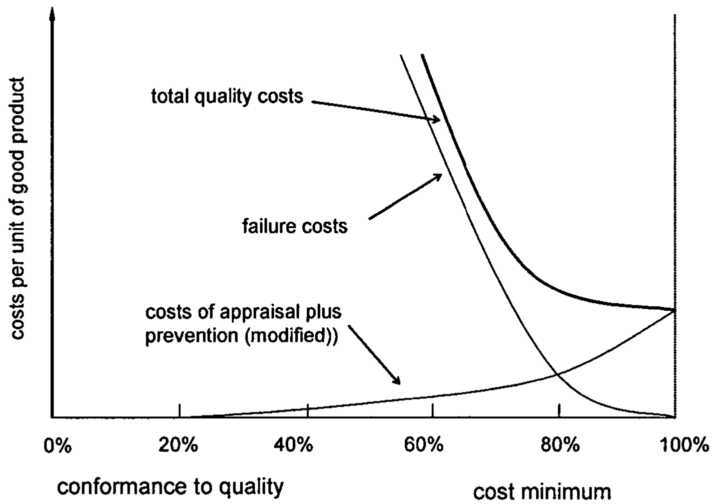

Figure 2. The modified COQ model

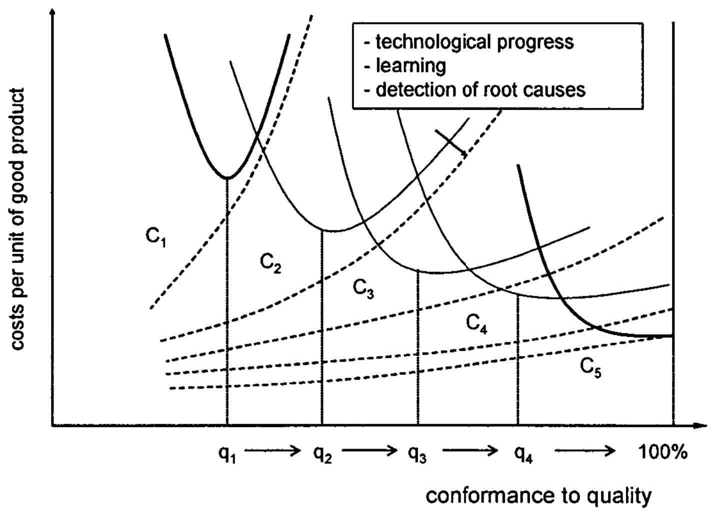

Figure 3. The developed quality cost model

The procedure of the total costs of achieving good quality is shown in Figure 3, where the curve of prevention and appraisal cost C_1 shift to C_2, when the quality level increases from q_1 to q_2 by detecting and eliminating quality problems. The failure cost curve is not affected by this continuous improvement progress in this procedure. In other words, Freiesleben' developed COQ model is not really dynamic, which is only shown that the changes of prevention and appraisal costs, when failure cost curves are in the same shapes. Meanwhile, the relationship between four major quality costs and the quality level has not yet formulated.

In order to determine the relationship between quality cost and quality level, a quality level should be measured depending on a company's quality improvement activities. As well known, the objective of quality improvement activities is reached to best possible perfection or zero defects, which is a hard and long way, but it can be divided into several sequential quality improvement stages. Six Sigma plays a very important role in continuous quality improvement. A Six Sigma level can be introduced to represent as a quality level in the different improvement stages.

Quality level, which is also called conformance to quality, is usually taken as the rate of good product. In Six Sigma management, an efficiency of quality improvement activities can be measured by sigma level, that is, the rate of poor quality products or the rate of defects. Table 1.shows the sigma level related to the rate of defects and rate of good products.

Table 1. Sigma level versus the efficiency of quality improvement

Sigma Level	Rate of Defects (DPMO)	Rate of Good Products (%)
1σ	691,462	30%
2σ	308,537	70%
3σ	66,807	93.3%
4σ	6,210	99.4%
5σ	233	99.98%
6σ	3.4	99.9997%

For the convenience of description, the whole procedure of quality improvement in a company can be divided into n stages. The efficiency of the kth quality improvement stage is represented by $k\sigma$, which is called as the kth quality level, where $k=1, 2, \cdots, n$. According to Table 1, the kth quality level can be obtained by $k\sigma$ as soon as the efficiency of the kth quality improvement is determined. Usually, when σ is given, the k is bigger, the higher quality level is, which are positive correlation.

3. COQ Model for Different Quality Level

In this session, a dynamic COQ model for different quality level will be developed. A total quality cost model is first formulated, which's curve will be plotted for different quality level. By means of the COQ model, the curves of prevention and appraisal costs for different quality level on a certain failure cost are depicted respectively, which can be used for analyzing the relationship between prevention, appraisal cost and failure. At the same time, the curves of failure costs for different quality level on a certain prevention and appraisal cost are drawn in such as to show the failure costs changing depending on the quality level of prevention and appraisal costs.

As the above the discussion, a long journey for achieving product perfection or zero- defect can be partitioned into n quality improvement stages, each of which's improvement efficiency is denoted by different σ level, that is, $k\sigma$, $k=1, 2, \cdots, n$. In the kth stage, the total quality cost model will be set up by used of the old COQ model. As seen as Figure 1, the curve of prevention and appraisal cost is increasing exponentially with increasing conformance to quality. Oppositely, the curve of failure cost is decreasing exponentially with increasing conformance to quality. Usually, two exponential functions are introduced to describe these relations (Zhao, 2000), which total quality cost model is as follows:

$$C_T = C_P + C_F = e^{a+bq} + e^{c-dq} \tag{1}$$

The total quality cost, prevention, appraisal cost, and failure cost are represented by C_T, C_P and C_F respectively, where a, b, c and d are all positive constants. This COQ model is a static model that shows the relations among C_T, C_P and C_F at a certain quality level. In order to set up the dynamic COQ model at different quality level, the constants, a, b, c and d in the equation (1) should be connected with some quality level or sigma level. Denoted $k\sigma$ as the quality level in the kth improvement stage, the total quality cost model corresponding to the kσ quality level is formulated as the following, that is,

$$C_T(k\sigma, k\sigma) = C_P(k\sigma) + C_F(k\sigma) = P e^{k\sigma} + F e^{-k\sigma} \tag{2}$$

Where $C_T(k\sigma, k\sigma)$, $C_P(k\sigma)$ and $C_F(k\sigma)$ represent the total quality cost, prevention, appraisal cost, and failure cost at the $k\sigma$ quality level respectively. The optimal total quality cost is obtained at a $k\sigma$ quality level, where $k\sigma = \frac{\ln F - \ln P}{2}$. As seen as, a new dynamic model for total quality cost is constructed in equation (2). According to the model, the curves of total quality costs can be drawn in Figure 4, which describe the relationships among the total quality cost, prevention, appraisal cost, and failure cost at the $k\sigma$ $(k=1, 2, \cdots, n)$ quality level.

Now let us analyze the relationships among the four major costs at different quality levels. With the modern technology, looking for quality problems, and then detecting the main root sources and eliminating these problems, the $k_1\sigma$ quality level is improved to $k_2\sigma$, that is, $k_2\sigma > k_1\sigma$. From the equation (2), the prevention, appraisal cost at $k_1\sigma$ is greater than the one at $k_2\sigma$, that is, when $k_2\sigma > k_1\sigma$,

$$C_P(k_1\sigma) = P_1 e^{k_1\sigma} \geq P_2 e^{k_2\sigma} = C_P(k_2\sigma) \tag{3}$$

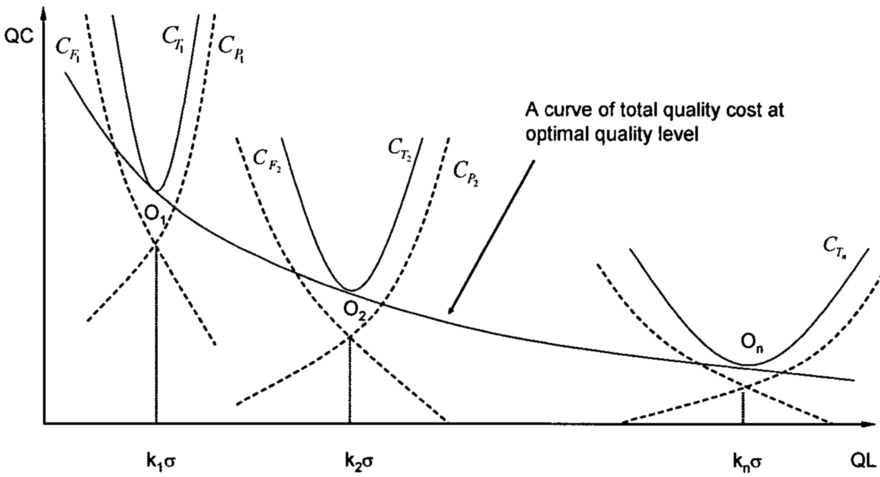

Figure 4. The COQ model on different levels

As shown in Figure 4, the curve of prevention and appraisal cost at $k_1\sigma$, C_{P_1}, shift to C_{P_2} which is the curve of prevention and appraisal cost at $k_2\sigma$ for the quality level changing from $k_1\sigma$ to $k_2\sigma$. The greater quality improvement is, the flatter curve of prevention and appraisal cost is. Similarly, when $k_1\sigma$ is improved to $k_2\sigma$, for the failure quality costs, we have

$$C_F(k_1\sigma) = F_1 e^{-k_1\sigma} \geq F_2 e^{-k_2\sigma} = C_F(k_2\sigma) \tag{4}$$

The curve C_{F_1} shift to C_{F_2} with the quality level from $k_1\sigma$ to $k_2\sigma$. The latter curve is flatter than the former curve.

Therefore, the total quality cost at $k_1\sigma$ is greater than the total quality cost at $k_2\sigma$, when $k_2\sigma > k_1\sigma$, that is,

$$C_T(k_1\sigma, k_1\sigma) = P_1 e^{k_1\sigma} + F_1 e^{-k_1\sigma} \geq P_2 e^{k_2\sigma} + F_2 e^{-k_2\sigma} = C_T(k_2\sigma, k_2\sigma) \tag{5}$$

When the quality level $k_1\sigma$ is continuous improved to $k_n\sigma$ by a successive quality activities, the optimal total quality costs are achieved at $k_i\sigma$ $(i = 1, 2, \cdots, n)$ quality levels. The curve of optimal total quality costs can be drawn by linking O_1, O_2 till to O_n, which is a decreasing curve with the quality level increasing.

4. PAC versus FC Model for Different Quality Level

After discussing the situations of total quality cost at different quality level, the relationship among the prevention, appraisal costs depengding on a certain failure cost for different quality levels will be analysed. On the other hand, the transfer trend of the failure cost curves on a certain prevention, appraisal cost for the changing quality levels will be depicted.

Quality level can be measured by different indicators, such as, the rate of defects, the rate of good products, the relations of which are shown as Table 1. For the convenience of statement, the indicators taken as the quality quality measurements will ben selected in the discussion of the COQ models.

4.1 PAC Curves with a FC Curve

Under the current quality management conditions, a company's quality level is keep at a quality level, $k_0\sigma$, which two curves of the prevention, appraisal cost failure cost can be drawn, as seen as C_{P_0} and C_{F_0} in the Figure 5. Here, the rate of good products is taken as a indicator to measure the quality level, which the quality level is improving with the rate of good products increasing. We want to visualize this development by depicting different prevention, appraisal costs curves for the different quality levels with the same failure cost curve, C_{F_0}. With quality improvement effort conducted and new technology developed, the quality level can be improved from $k_0\sigma$ to $k_1\sigma$. For the changing the quality levels, the prevention, appraisal cost curve at $k_0\sigma$, C_{P_0} shifts to C_{P_1}, in turns, to C_{P_n} along the failure cost curve C_{F_0}, shown in Figure 5.

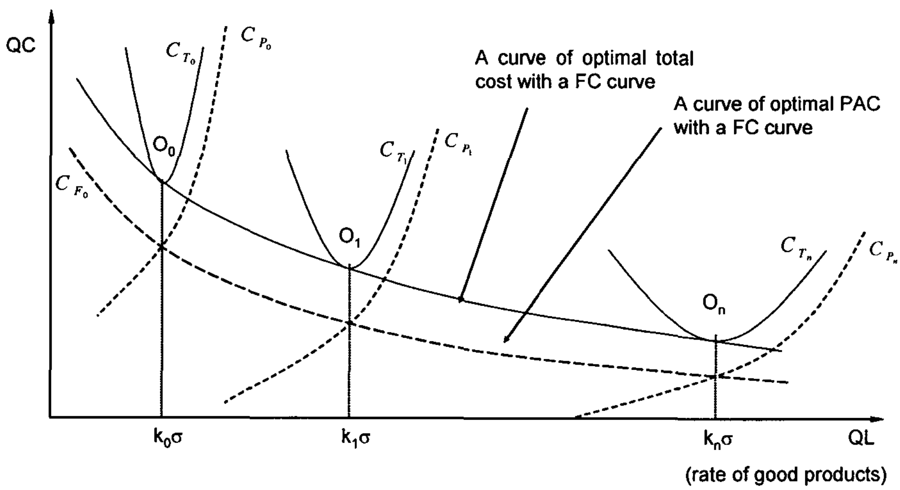

Figure 5. The COQ model with a FC curve on different levels

The relations among optimal prevention, appraisal costs, $C_P(k_i\sigma)$ $i=1,2,\cdots,n$, for different quality levels and the failure cost at $k_0\sigma$, $C_F(k_0\sigma)$ can be represented by the equation (6).

$$C_F(k_0\sigma)=C_P(k_i\sigma)=P_0e^{k_0\sigma}\geq P_ie^{k_i\sigma}\geq P_{i+1}e^{k_{i+1}\sigma}=C_P(k_{i+1}\sigma)=C_F(k_{i+1}\sigma)\quad i=1,2,\cdots,n \qquad (6)$$

Figure 5 shows the COQ model with a failure cost at different levels. With the quality level improving, the curves of prevention, appraisal costs are decreasing along a failure cost curve. The curve $C_{P_{i+1}}$ is flatter than the curve C_{P_i} $(i=1,2,\cdots,n)$. Consequently, the curve of optimal total quality cost with a failure curve is decreasing at different quality level.

4.2 FC Curve with a PAC Curve

Now, we discuss the changing situations of failure cost curves at increasing quality levels in the same way above. In order to show this changing clearly, the quality level can be measured by the rate of defects, which the quality level is improving with the rate of defects decreasing. The development of different failure cost curves for the different quality levels with the same cost prevention, appraisal curve, C_{p_0} can be depicted. With quality improvement effort conducted and other root causes detected, the quality level can be improved from $k_0\sigma$ to $k_1\sigma$. The failure cost curve at $k_0\sigma$, C_{F_0} shifts to C_{F_1} along the prevention, appraisal cost curve C_{P_0}, at the quality level $k_0\sigma$, shown in Figure 6. The curve C_{F_1} is flatter than the curve C_{F_0}, when the quality level from $k_0\sigma$ improved to $k_1\sigma$ in Figure 6,

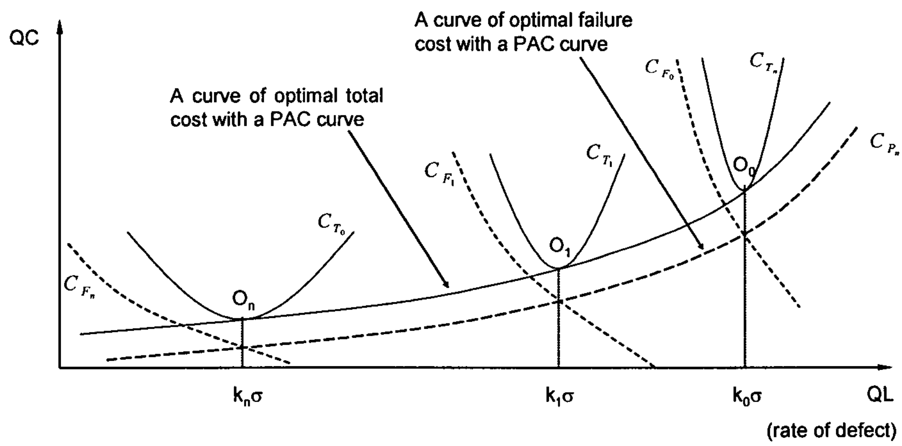

Figure 6. The COQ model with a PAC curve on different levels

The relations among optimal failure costs, $C_F(k_i\sigma)$ $i=1,2,\cdots,n$, for different quality levels and the prevention, appraisal cost at $k_0\sigma$, $C_P(k_0\sigma)$ can be represented by the equation (7).

$$C_P(k_0\sigma) = C_F(k_0\sigma) = F_0 e^{-k_0\sigma} \geq F_i e^{-k_i\sigma} \geq F_{i+1} e^{-k_{i+1}\sigma} = C_F(k_{i+1}\sigma) = C_P(k_{i+1}\sigma)$$
$$i = 1, 2, \cdots, n \tag{7}$$

Figure 6 shows the COQ model with a prevention and appraisal cost at different levels. With the quality level improving, the curves of failure costs are decreasing along a curve of prevention and appraisal cost. The curve $C_{F_{i+1}}$ is flatter than the curve $C_{F_i}(i = 1, 2, \cdots, n)$. Consequently, the curve of optimal total quality cost with a curve of prevention and appraisal cost is decreasing at different quality level.

5. Conclusions

A dynamic COQ model at different quality level is proposed in this paper. In virtue of this model, the relationship among the total quality cost, prevention, appraisal and failure cost for changing quality level can be analyzed. There are some limited aspects in the old quality cost model, the modified quality cost model and the developed quality cost model. A quality level is first defined by Six Sigma level in the new dynamic COQ model. Meanwhile, the two measurements for quality level are used in this paper. Based on the activities of continuous quality improvement, a quality level can be measured by some different indicators separately in practices.

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