

A Study on Mount Performance for Structure-Borne Noise Reduction in Resiliently Mounted System

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Abstract

SBN (Structure-Borne Noise) reduction in resiliently mounted machineries are predicted by using mass-spring model and wave model. In mass-spring model, mount is modeled as a spring, while in wave model, mount is considered as an equivalent elastic rod for taking account into longitudinal wave propagation. The predictions for SBN reduction through mounts are compared to the measurements for four different pumps. It is found that the mass-spring model is valid only in low frequency range below few hundred Hz, while for high frequency ranges longitudinal wave propagation in the mount must be considered to explain the measurements. It is also shown that impedance of the floor slightly affects low frequency behaviour in mass-spring and wave model below 50 Hz - 80 Hz, so that in engineering practice the effect of floor impedance may be neglected in computing mount performance.

Keywords: resilient mount, floor impedance, wave model

1. Introduction

Resilient mounts are widely used in reduction of structure-borne noise (SBN), where SBN means vibration in the frequency range between 10 Hz and 10000 Hz. In many circumstances single resilient system is enough to meet the designed SBN reduction, or vibration isolation. However, in naval applications where very strict requirements on SBN levels must be fulfilled [1], double resilient system is frequently used. Although numerous references [2] may be found on the mount performance, a few works are available on the double resilient systems. Gaul [3] studied substructure behaviour of resilient mounts for single and double resilient systems by using CAD compatible BEM. Kim et al [4] investigated vibration isolation of resiliently mounted system. A basic vibration theory of compound system may be found in the book by

Snowdon [5], in which general transmissibility and phase equations are derived. In general, one usually models a complicated machinery supported by resilient mounts as mass-spring system, and predicts transmissibility, which is a measure of the reduction of transmitted force or motion afforded by resilient mount [6]. However, the predicted transmissibility shows large discrepancy from the measurement [4].

In this paper, we study the reduction of SBN in doubly resilient mounted pump/motor assemblies mainly used in Korean naval ships. We model pump/motor assemblies as two DOF systems with masses and springs. We also consider the mount as an equivalent cylindrical block and include wave propagation in the mount. In addition, we study the effect of floor impedance. The predictions for SBN reduction are compared to the measurements done for four pump/motor assemblies.

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II. Mass-Spring Model

We show a typical measurement setup of the double resiliently mounted system in Fig. 1. The pump/motor assembly is on the bed plate lying on the fixture structure. In Fig. 2, we show a simplified three DOF mass-spring model lying on an elastic floor, in which motor/pump/upper bed is modeled as a mass M_1 , lower bed as M_2 , and upper and lower mounts are modeled as springs with constant k_1 and k_2 . If we assume a harmonic excitation $F = F_0 e^{j\omega t}$, and displacements of M_1 , M_2 , and floor are X_1 , X_2 , and X_B respectively, the equations of motion are given by

$$(k_1 - \omega^2 M_1)X_1 - k_1 X_2 = F_0 \quad (1)$$

$$-k_1 X_1 + (k_1 + k_2 - \omega^2 M_2)X_2 - k_2 X_B = 0 \quad (2)$$

$$-k_2 X_2 + (k_2 + j\omega Z)X_B = 0 \quad (3)$$

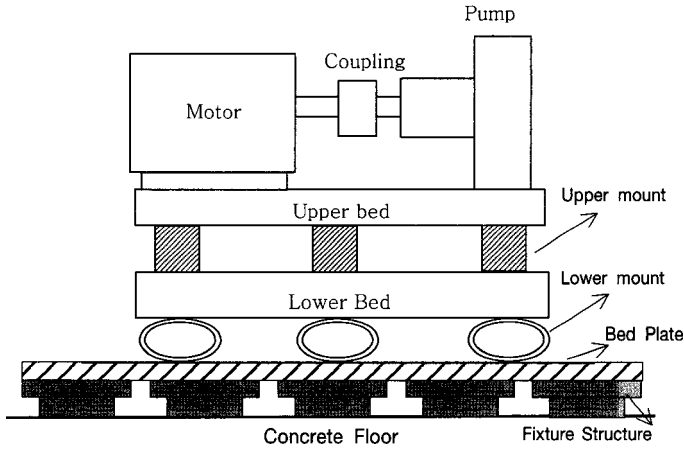


Fig. 1. Resiliently mounted pump/motor assembly on the bed plate lying on fixture structure.

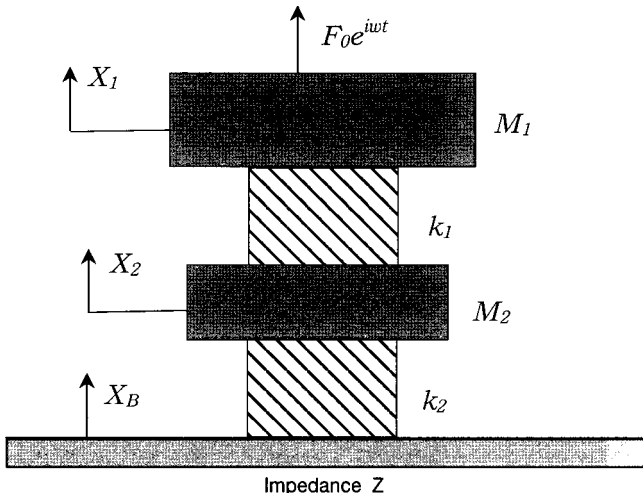


Fig. 2. Mass-spring model on an elastic floor.

where impedance of the floor is Z .

After solving (1)–(3), we obtain the displacements as

$$X_1 = F_0 [(k_1 + k_2 - \omega^2 M_2)(k_2 + j\omega Z) - k_2^2] / \Delta \quad (4)$$

$$X_2 = F_0 k_1 (k_2 + j\omega Z) / \Delta \quad (5)$$

$$X_B = F_0 k_1 k_2 / \Delta \quad (6)$$

where Δ is determinant of the matrix formed by Eqs. (1)–(3). We define the SBN reduction through the upper mount τ_{SPRING} as

$$\tau_{SPRING} = 20 \log(|X_2 / X_1|) \quad (7)$$

If impedance of the floor becomes infinite ($Z \rightarrow \infty$), Eqs. (4)–(6) are simplified as

$$X_1 = F_0 (k_1 + k_2 - \omega^2 M_2) / \Delta_\infty \quad (8)$$

$$X_2 = F_0 k_1 / \Delta_\infty \quad (9)$$

$$X_B = 0 \quad (10)$$

in which

$$\Delta_\infty = (k_1 - \omega^2 M_1)(k_1 + k_2 - \omega^2 M_2) - k_1^2 \quad (11)$$

III. Wave Model

We consider wave model, in which we take account into longitudinal wave propagation in the mount. In Fig. 3, we show the wave model, where $U(x)$, E_1 , A_1 , ρ_1 , L_1 and

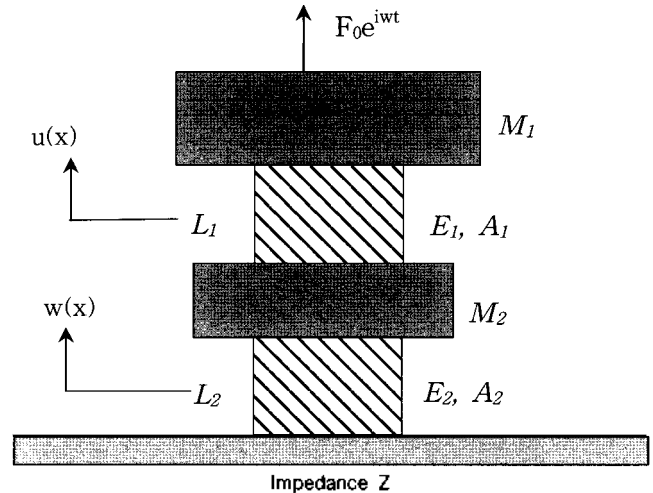


Fig. 3. Wave model on an elastic floor.

$W(x)$, E_2 , A_2 , ρ_2 , L_2 denote displacement, Young's modulus, cross sectional area, density and height of the upper and lower mount respectively. Although the real mounts are of complicated shapes, we simplify the mount as an equivalent cylindrical rod of constant cross sectional area. The longitudinal wave propagation in mounts is governed by

$$U''(x) + k_1^2 U(x) = 0 \quad (12a)$$

$$W''(x) + k_2^2 W(x) = 0 \quad (12b)$$

where $k_1^2 = \omega^2 \rho_1 / E_1$ and $k_2^2 = \omega^2 \rho_2 / E_2$. The boundary equations are

$$E_1 A_1 U'(L_1) = F_0 + \omega^2 M_1 U(L_1) \quad (13)$$

$$-E_1 A_1 U'(0) + E_2 A_2 W'(L_2) = \omega^2 M_2 U(0) \quad (14)$$

$$E_2 A_2 W'(0) = Z_j \omega W(0) \quad (15)$$

$$U(0) = W(L_2) \quad (16)$$

in which Eqs. (13) and (14) are force equilibrium at $x_1 = L_1$, $x_1 = 0$, and Eq. (15) is impedance relation at $x_2 = 0$, while Eq. (16) is continuity of displacements. If floor is rigid, Eq. (15) is replaced by

$$W(0) = 0 \quad (17)$$

The solutions of Eqs. (12) are

$$U = C_1 \cos k_1 x_1 + C_2 \sin k_1 x_1 \quad (18a)$$

$$W = D_1 \cos k_2 x_2 + D_2 \sin k_2 x_2 \quad (18b)$$

After solving Eqs. (18) for boundary conditions (13)–(17), we can determine unknowns C_1 , C_2 , D_1 , D_2 . The SBN reduction is given by

$$\tau_{WAVE} = 20 \log \left| \frac{U(0)}{U(L_1)} \right| = 20 \log \left| \frac{C_1}{C_1 \cos k_1 L_1 + C_2 \sin k_1 L_1} \right| \quad (19)$$

IV. Comparisons of Predictions and Measurements

For four different double resilient mounted pumps, we

Table 1. Main parameters of the test pumps.

	Pump A	Pump B	Pump C	Pump D	
Usage	Fuel Oil X-fer	Fresh water	Sea Water Cooling	Fire pump	
rpm	1800	3600	1200	3600	
M_1 (kg)	189	248	1046	1685	
M_2 (kg)	35	52	140	240	
Upper mount	Number	4	4	4	6
	k_1 (105N/m)	6.5	5.5	17.8	1.96
	Height L_1 (m)	0.04	0.04	0.05	0.065
	Area A_1 (m ²)	0.00636	0.00636	0.0154	0.0127
	dynamic factor α_1	1.4	1.4	1.4	2
Lower mount	Number	4	6	4	6
	k_2 (105N/m)	0.9	0.9	5.66	8.25
	Height L_2 (m)	0.04	0.04	0.05	0.05
	Area A_2 (m ²)	0.01	0.01	0.03	0.03
	dynamic factor α_2	2.28	2.28	1.24	1.43

measured SBN reductions through upper mounts. The main parameters are summarized in Table 1.

Although the real mount is of complicated shape, we assume the mount as an equivalent cylindrical rod with height L and cross-sectional area A , for which Young's modulus E and spring constant k are related as

$$E = kL/A \quad (20)$$

If dynamic factor of the mount, the ratio of the dynamic and static Young's modulus, is α and damping is η , total

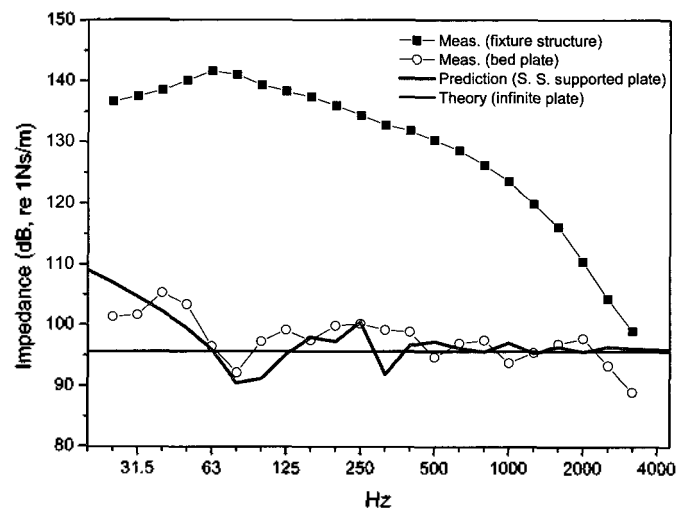


Fig. 4. Comparison of measured impedances and theoretical values for bed plate and fixture structure.

equivalent Young's modulus is given by

$$E_{total} = \alpha E(1 + i\eta) \quad (21)$$

As shown in Fig. 1, the pumps are fixed on the steel bed plate whose size is 2 m (length) x 0.9 m (width) x 25.4 mm (thickness), which is in turn on the fixture structure. We measured impedance of the bed plate and of fixture structure by using impact hammer. We used small hammer (B&K model 8202, weight: 0.434 kg) for 400 – 5000 Hz, and large hammer (ENDEVCO model 2305, weight: 6.8 kg) for 20 – 630 Hz, where averaged impedances are taken for overlapped frequency range. We also computed impedance of a simply supported plate with the same size of the bed plate, which is given by

$$\frac{1}{Z} = \frac{4j\omega}{M_p} \sum \sum \frac{\sin^2(n\pi x_0/L_x)\sin^2(m\pi y_0/L_y)}{\omega_{nm}^2(1 + j\eta)^2 - \omega^2} \quad (22)$$

where M_p is mass of the plate, L_x , L_y are length and width, x_0 , y_0 are excitation point, and ω_{nm} , η are natural frequency and loss factor of (n, m) mode. We performed average of Eq. (22) over the area covered by the pump. The impedance of an infinite plate with thickness h , density ρ , and flexural rigidity D is given by

$$Z_\infty = 8\sqrt{D\rho h} \quad (23)$$

In Fig. 4, we compared measured impedances of the bed plate with theoretical results (22) and (23), in which $\eta = 0.1$. It is found that impedance of the bed plate approaches that of an infinite plate and simply supported plate as

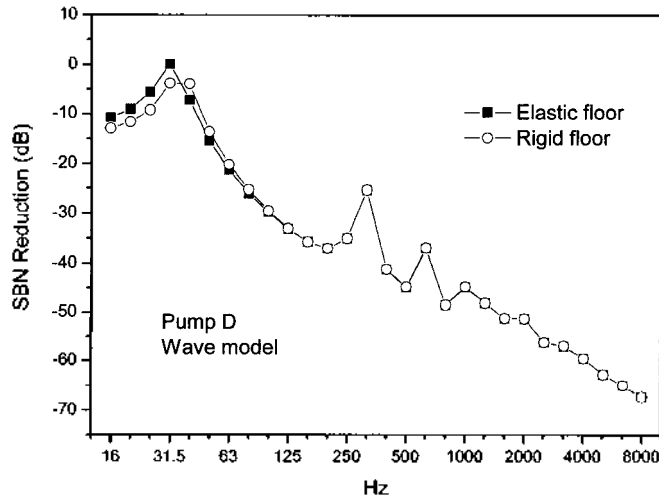


Fig. 5. Effect of floor impedance on prediction (19) by wave model.

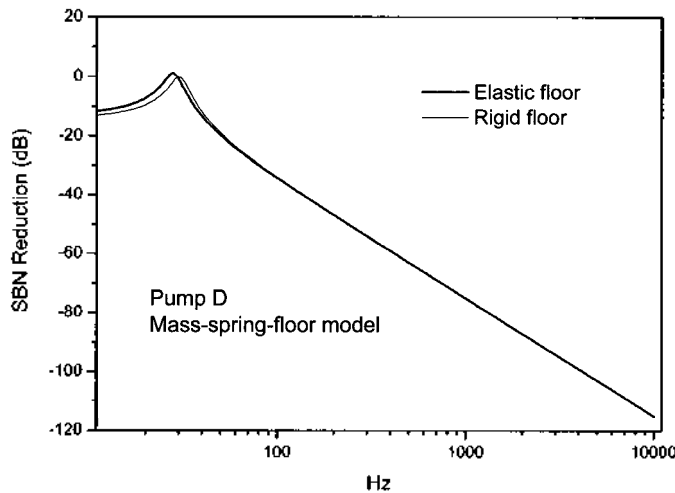


Fig. 6. Effect of floor impedance on prediction (7) by mass-spring model.

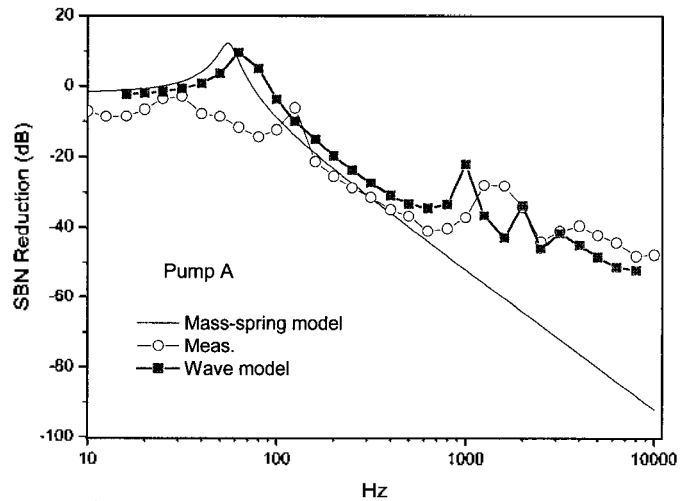


Fig. 7. Comparison of SBN reduction by predictions and measurement for pump A.

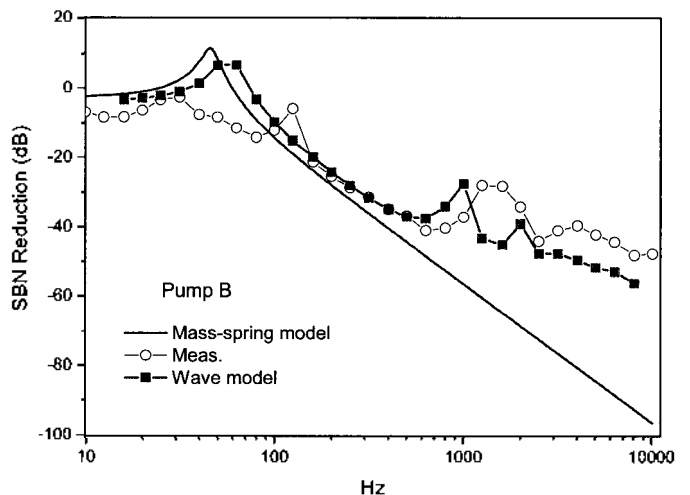


Fig. 8. Comparison of SBN reduction by predictions and measurement for pump B.

frequency becomes higher, while impedance of the fixture structure is much greater than that of the bed plate. In Figs. 5 and 6, we showed effect of the floor impedance on τ_{SPRING} in Eq. (7) and τ_{WAVE} in Eq. (19) for pump D. It is shown that τ_{SPRING} and τ_{WAVE} are affected by the floor impedance for low frequency range, say below 100 Hz.

In Figs. 7-10, we compared τ_{WAVE} and τ_{SPRING} to measurements for four pumps in Table 1, where we used $\eta_1 = \eta_2 = 0.2$. We first computed Eq. (19) in narrow band and then converted τ_{WAVE} into averaged value in 1/3 Octave band. The comparisons show that prediction by mass-spring model (7) significantly underestimates $20\log(|X_2/X_1|)$ in high frequency ranges over few hundred Hz, while prediction by wave model (19) follows measurement comparably for high frequency ranges up to few kilo Hz.

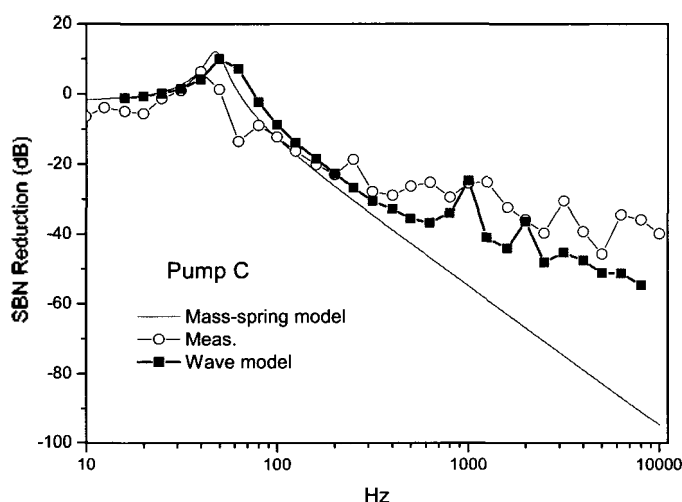


Fig. 9. Comparison of SBN reduction by predictions and measurement for pump C.

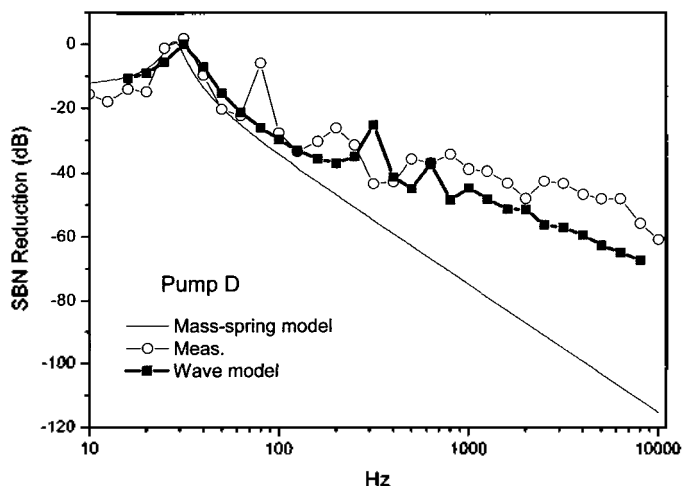


Fig. 10. Comparison of SBN reduction by predictions and measurement for pump D.

It is also shown that predictions for pump A in Fig. 7 and B in Fig. 8 are higher than measurements below 100 Hz. The peaks at 60 - 80 Hz found in pump A and B correspond to the second natural frequencies for two degree of freedom system in Fig. 2, while first natural frequencies are below 10 Hz. In this study, we assumed that pumps and beds are modeled as rigid masses moving only in the vertical direction. However, they may have lateral and rotational motions. Furthermore, there might arise flexural vibrations in lateral directions. Such complexity in reality may have caused discrepancies between predictions and measurements.

V. Discussions and Conclusions

In predicting the SBN reduction through resilient mounts, the mass-spring model is valid only in low frequency range below few hundred Hz, while for high frequency ranges longitudinal wave propagation in the mount must be included. Impedance of the floor affects low frequency behaviour of the mount, but the magnitudes of the differences are found to be small. This means that in practice the effect of floor impedance may be neglected in computing mount performance.

Although the wave model provides more accurate prediction than mass-spring model, both methods have inherent limitations in that they consider only rigid body motion in vertical direction, while in reality the machineries show flexural bending vibrations in lateral directions.

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[Profile]

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