Kangweon-Kyungki Math. Jour. **15** (2007), No. 1, pp. 27–37

R-SEMI-GENERALIZED FUZZY CONTINUOUS MAPS

Won Keun Min and Chun-Kee Park*

Abstract. In this paper, we introduce the concepts of r-semi-generalized fuzzy closed sets, r-semi-generalized fuzzy open sets, r-semi-generalized fuzzy continuous maps in fuzzy topological spaces and investigate some of their properties.

1. Introduction

R. Badard [1] introduced the concept of the fuzzy topological space which is an extension of Chang's fuzzy topological space [4]. Many mathematical structures in fuzzy topological spaces were introduced and studied. In particular, K. C. Chattopadhyay and S. K. Samanta [5] and S. J. Lee and E. P. Lee [7] introduced the concepts of fuzzy r-closure and fuzzy r-interior in fuzzy topological spaces and obtained some of their properties. S. J. Lee and E. P. Lee [7] also introduced the concepts of fuzzy r-semi-open sets and fuzzy r-semi-continuous maps in fuzzy topological spaces which are generalizations of fuzzy semi-open sets and fuzzy semi-continuous maps in Chang's fuzzy topological space and obtained some of their properties. P. Bhattacharya and B. K. Lahiri [3] introduced the concepts of semi-generalized open sets and semi-generalized closed sets in topological spaces.

In this paper, we introduce the concepts of r-semi-generalized fuzzy closed sets, r-semi-generalized fuzzy open sets, r-semi-generalized fuzzy continuous maps in fuzzy topological spaces and investigate some of their properties.

Received April 12, 2007.

2000 Mathematics Subject Classification: 54A40, 54A05, 54C08.

Key words and phrases: r-semi-generalized fuzzy closed sets, r-semi-generalized fuzzy open sets, r-semi-generalized fuzzy continuous maps.

* Corresponding author

2. Preliminaries

Throughout this paper, let X be a nonempty set, $I = [0,1]$ and $I_0 = (0,1]$. The family of all fuzzy sets of X will be denoted by I^X. By $\tilde{0}$ and $\tilde{1}$ we denote the characteristic functions of ϕ and X, respectively. For any $\mu \in I^X$, μ^c denotes the complement of μ, i.e., $\mu^c = \tilde{1} - \mu$.

A *fuzzy topology* [1, 10], which is also called a *smooth topology,* on X is a map $\tau : I^X \to I$ satisfying the following conditions:

(O1) $\tau(\tilde{0}) = \tau(\tilde{1}) = 1$;

(O2) $\forall \mu_1, \mu_2 \in I^X,\ \tau(\mu_1 \wedge \mu_2) \geq \tau(\mu_1) \wedge \tau(\mu_2)$;

(O3) for each subfamily $\{\mu_i : i \in \Gamma\} \subseteq I^X,\ \tau(\cup_{i\in\Gamma}\, \mu_i) \geq \wedge_{i\in\Gamma}\, \tau(\mu_i)$.

The pair (X, τ) is called a *fuzzy topological space* (for short, fts), which is also called a *smooth topological space.*

Definition 2.1. [5, 7] Let (X, τ) be a fts. For $\mu \in I^X$ and $r \in I_0$, the *fuzzy r-closure* of μ is defined by

$$cl(\mu, r) = \wedge\{\rho \in I^X \mid \mu \leq \rho,\ \tau(\rho^c) \geq r\}$$

and the *fuzzy r-interior* of μ is defined by

$$int(\mu, r) = \vee\{\rho \in I^X \mid \mu \geq \rho,\ \tau(\rho) \geq r\}.$$

For $r \in I_0$, we call μ a *fuzzy r-open set* of X if $\tau(\mu) \geq r$ and μ a *fuzzy r-closed set* of X if $\tau(\mu^c) \geq r$.

Theorem 2.2. *[5, 7] Let (X, τ) be a fts. Then for $\mu, \lambda \in I^X$ and $r, s \in I_0$,*

(1) $cl(\tilde{0}, r) = \tilde{0}$, $int(\tilde{1}, r) = \tilde{1}$,

(2) $\mu \leq cl(\mu, r)$, $int(\mu, r) \leq \mu$,

(3) $cl(\mu, r) \leq cl(\mu, s)$, $int(\mu, r) \geq int(\mu, s)$ *if* $r \leq s$,

(4) $cl(\mu \vee \lambda, r) = cl(\mu, r) \vee cl(\lambda, r)$, $int(\mu \wedge \lambda, r) = int(\mu, r) \wedge int(\lambda, r)$,

(5) $cl(cl(\mu, r), r) = cl(\mu, r)$, $int(int(\mu, r), r) = int(\mu, r)$.

(6) $cl(\mu, r)^c = int(\mu^c, r)$, $int(\mu, r)^c = cl(\mu^c, r)$.

Definition 2.3. [7] Let (X, τ) and (Y, σ) be fts's and $r \in I_0$. Then a map $f : (X, \tau) \to (Y, \sigma)$ is called

(1) a *fuzzy r-continuous map* if $f^{-1}(\mu)$ is a fuzzy r-open set of X for each fuzzy r-open set μ of Y, or equivalently, $f^{-1}(\mu)$ is a fuzzy r-closed set of X for each fuzzy r-closed set μ of Y.

(2) a *fuzzy r-open map* if $f(\mu)$ is a fuzzy r-open set of Y for each fuzzy r-open set μ of X.
(3) a *fuzzy r-closed map* if $f(\mu)$ is a fuzzy r-closed set of Y for each fuzzy r-closed set μ of X.

DEFINITION 2.4. [7] Let (X, τ) be a fts, $\mu \in I^X$ and $r \in I_0$.

(1) A fuzzy set μ is called *fuzzy r-semi-open* if there is a fuzzy r-open set ρ of X such that $\rho \leq \mu \leq cl(\rho, r)$.
(2) A fuzzy set μ is called *fuzzy r-semi-closed* if there is a fuzzy r-closed set ρ of X such that $int(\rho, r) \leq \mu \leq \rho$.

THEOREM 2.5. *[7] Let (X, τ) be a fts, $\mu \in I^X$ and $r \in I_0$. Then the following are equivalent:*

(1) *μ is a fuzzy r-semi-open set.*
(2) *μ^c is a fuzzy r-semi-closed set.*
(3) *$cl(int(\mu, r), r) \geq \mu$.*
(4) *$int(cl(\mu^c, r), r) \leq \mu^c$.*

THEOREM 2.6. *[7] Let (X, τ) be a fts and $r \in I_0$. Then*

(1) *Any union of fuzzy r-semi-open sets is fuzzy r-semi-open .*
(2) *Any intersection of fuzzy r-semi-closed sets is fuzzy r-semi-closed .*

DEFINITION 2.7. [7] Let (X, τ) be a fts. For $\mu \in I^X$ and $r \in I_0$, the *fuzzy r-semi-closure of* μ is defined by

$$scl(\mu, r) = \wedge\{\rho \in I^X | \ \mu \leq \rho, \ \rho \text{ is fuzzy r-semi-closed}\}.$$

and the *fuzzy r-semi-interior of* μ is defined by

$$sint(\mu, r) = \vee\{\rho \in I^X | \ \mu \geq \rho, \ \rho \text{ is fuzzy r-semi-open}\}.$$

THEOREM 2.8. *[7] Let (X, τ) be a fts. Then for $\mu, \lambda \in I^X$ and $r, s \in I_0$,*

(1) $scl(\tilde{0}, r) = \tilde{0}$, $sint(\tilde{1}, r) = \tilde{1}$,
(2) $\mu \leq scl(\mu, r)$, $sint(\mu, r) \leq \mu$,
(3) $scl(\mu, r) \leq scl(\mu, s)$, $sint(\mu, r) \geq sint(\mu, s)$ *if* $r \leq s$,

(4) $scl(\mu \vee \lambda, r) \geq scl(\mu, r) \vee scl(\lambda, r)$, $sint(\mu \wedge \lambda, r) \leq int(\mu, r) \wedge int(\lambda, r)$,
(5) $int(\mu, r) \leq sint(\mu, r) \leq \mu \leq scl(\mu, r) \leq cl(\mu, r)$,
(6) $scl(scl(\mu, r), r) = scl(\mu, r)$, $sint(sint(\mu, r), r) = sint(\mu, r)$.
(7) $scl(\mu, r)^c = sint(\mu^c, r)$, $sint(\mu, r)^c = scl(\mu^c, r)$.

DEFINITION 2.9. [7] Let (X, τ) and (Y, σ) be fts's and $r \in I_0$. Then a map $f : (X, \tau) \to (Y, \sigma)$ is called

(1) a *fuzzy r-semi-continuous map* if $f^{-1}(\mu)$ is a fuzzy r-semi-open set of X for each fuzzy r-open set μ of Y, or equivalently, $f^{-1}(\mu)$ is a fuzzy r-semi-closed set of X for each fuzzy r-closed set μ of Y.
(2) a *fuzzy r-semi-open map* if $f(\mu)$ is a fuzzy r-semi-open set of Y for each fuzzy r-open set μ of X.
(3) a *fuzzy r-semi-closed map* if $f(\mu)$ is a fuzzy r-semi-closed set of Y for each fuzzy r-closed set μ of X.

3. r-semi-generalized fuzzy closed sets

DEFINITION 3.1. [9] Let (X, τ) be a fts, $\mu, \rho \in I^X$ and $r \in I_0$.

(1) A fuzzy set μ is called *r-generalized fuzzy closed* (for short, r-gfc) if $cl(\mu, r) \leq \rho$ whenever $\mu \leq \rho$ and $\tau(\rho) \geq r$.
(2) A fuzzy set μ is called *r-generalized fuzzy open* (for short, r-gfo) if μ^c is r-gfc.

DEFINITION 3.2. Let (X, τ) be a fts, $\mu, \rho \in I^X$ and $r \in I_0$.

(1) A fuzzy set μ is called *r-generalized fuzzy semi-closed* (for short, r-gfsc) if $scl(\mu, r) \leq \rho$ whenever $\mu \leq \rho$ and $\tau(\rho) \geq r$.
(2) A fuzzy set μ is called *r-generalized fuzzy semi-open* (for short, r-gfso) if μ^c is r-gfsc.

DEFINITION 3.3. Let (X, τ) be a fts, $\mu, \rho \in I^X$ and $r \in I_0$.

(1) A fuzzy set μ is called *r-semi-generalized fuzzy closed* (for short, r-sgfc) if $scl(\mu, r) \leq \rho$ whenever $\mu \leq \rho$ and ρ is r-semi-open.
(2) A fuzzy set μ is called *r-semi-generalized fuzzy open* (for short, r-sgfo) if μ^c is r-sgfc.

Clearly, μ is r-gfc $\Rightarrow \mu$ is r-gfsc, μ is r-gfo $\Rightarrow \mu$ is r-gfso, μ is r-sgfc $\Rightarrow \mu$ is r-gfsc, μ is r-sgfo $\Rightarrow \mu$ is r-gfso.

THEOREM 3.4. *Let (X, τ) be a fts and $r \in I_0$.*

(1) *If μ is a r-sgfc set and $\mu \leq \lambda \leq scl(\mu, r)$, then λ is a r-sgfc set.*
(2) *If μ is a fuzzy r-semi-closed set, then μ is a r-sgfc set.*
(3) *μ is a r-sgfo set if and only if $\rho \leq sint(\mu, r)$ whenever $\rho \leq \mu$ and ρ is r-semi-closed.*
(4) *If μ is a r-sgfo set and $sint(\mu, r) \leq \lambda \leq \mu$, then λ is a r-sgfo set.*
(5) *If μ is a fuzzy r-semi-open set, then μ is a r-sgfo set.*

Proof. (1) Let $\lambda \leq \rho$ and ρ be r-semi-open. Then $\mu \leq \rho$. Since μ is r-sgfc, $scl(\mu, r) \leq \rho$. Since $\lambda \leq scl(\mu, r)$, $scl(\lambda, r) \leq scl(scl(\mu, r), r) = scl(\mu, r) \leq \rho$. Hence λ is a r-sgfc set.

(2), (3) and (5) are obvious.

(4) Since $sint(\mu, r) \leq \lambda \leq \mu$, $\mu^c \leq \lambda^c \leq sint(\mu, r)^c = scl(\mu^c, r)$. Since μ is a r-sgfo set, μ^c is a r-sgfc set. By (1), λ^c is a r-sgfc set. Hence λ is a r-sgfo set.

□

DEFINITION 3.5. Let (X, τ) be a fts. For $\mu \in I^X$ and $r \in I_0$, the *r-semi-generalized fuzzy closure of* μ is defined by

$$sgcl(\mu, r) = \wedge\{\rho \in I^X \mid \mu \leq \rho,\ \rho \text{ is r-sgfc}\}.$$

and the *r-semi-generalized fuzzy interior of* μ is defined by

$$sgint(\mu, r) = \vee\{\rho \in I^X \mid \mu \geq \rho,\ \rho \text{ is r-sgfo}\}.$$

THEOREM 3.6. *Let (X, τ) be a fts. Then for $\mu, \lambda \in I^X$ and $r, s \in I_0$,*

(1) $sgcl(\tilde{0}, r) = \tilde{0}$,
(2) $\mu \leq sgcl(\mu, r)$,
(3) $sgcl(\mu, r) \leq sgcl(\mu, s)$ *if* $r \leq s$,
(4) $sgcl(\mu, r) \leq sgcl(\lambda, r)$ *if* $\mu \leq \lambda$,
(5) $sgcl(\mu \vee \lambda, r) \geq sgcl(\mu, r) \vee sgcl(\lambda, r)$,
(6) $sgcl(sgcl(\mu, r), r) = sgcl(\mu, r)$,
(7) $gscl(\mu, r) \leq sgcl(\mu, r)$.

Proof. (1), (2),(3) and (4) are easily obtained from Definition 3.5.

(5) Since $\mu \leq \mu \vee \lambda$ and $\lambda \leq \mu \vee \lambda$, $sgcl(\mu, r) \leq sgcl(\mu \vee \lambda, r)$ and $sgcl(\lambda, r) \leq sgcl(\mu \vee \lambda, r)$ by (4). Hence $sgcl(\mu, r) \vee sgcl(\lambda, r) \leq sgcl(\mu \vee \lambda, r)$.

(6) $sgcl(\mu, r) \leq sgcl(sgcl(\mu, r), r)$ by (2) and (4). Suppose that $sgcl(\mu, r) \not\geq sgcl(sgcl(\mu, r), r)$. Then there exist $x \in X$ and $t \in (0, 1)$ such that $sgcl(\mu, r)(x) < t < sgcl(sgcl(\mu, r), r)(x)$.
Since $sgcl(\mu, r)(x) < t$, there exists a r-sgfc set μ_1 with $\mu \leq \mu_1$ such that $sgcl(\mu, r)(x) \leq \mu_1(x) < t$. Since $\mu \leq \mu_1$, $sgcl(\mu, r) \leq \mu_1$ and $sgcl(sgcl(\mu, r), r) \leq \mu_1$ by (4). Hence $sgcl(sgcl(\mu, r), r)(x) \leq \mu_1(x) < t$. This is a contraction. Hence $sgcl(\mu, r) \geq sgcl(sgcl(\mu, r), r)$. Thus $sgcl(sgcl(\mu,\ r), r) = sgcl(\mu, r)$.

(7) Since every r-sgfc set is r-gsfc set, $gscl(\mu, r) \leq sgcl(\mu, r)$ by Definition 3.5.

□

THEOREM 3.7. *Let (X, τ) be a fts. Then for $\mu, \lambda \in I^X$ and $r, s \in I_0$,*

(1) $sgint(\tilde{1}, r) = \tilde{1}$,
(2) $sgint(\mu, r) \leq \mu$,
(3) $sgint(\mu, r) \geq sgint(\mu, s)$ *if* $r \leq s$,
(4) $sgint(\mu, r) \leq sgint(\lambda, r)$ *if* $\mu \leq \lambda$,
(5) $sgint(\mu \wedge \lambda, r) \leq sgint(\mu, r) \wedge sgint(\lambda, r)$,
(6) $sgint(sgint(\mu, r), r) = sgint(\mu, r)$,
(7) $gsint(\mu, r) \leq sgint(\mu, r)$.

Proof. The proof is similar to Theorem 3.6.

□

THEOREM 3.8. *Let (X, τ) be a fts. Then for $\mu \in I^X$ and $r \in I_0$,*

(1) $sgcl(\mu, r)^c = sgint(\mu^c, r)$,
(2) $sgint(\mu, r)^c = sgcl(\mu^c, r)$.

Proof. (1) From Definition 3.5, we have

$$\begin{aligned} sgcl(\mu, r)^c &= (\wedge\{\rho \in I^X \mid \mu \leq \rho,\ \rho \text{ is r-sgfc}\})^c \\ &= \vee\{\rho^c \in I^X \mid \mu^c \geq \rho^c,\ \rho^c \text{ is r-sgfo}\} \\ &= \vee\{\lambda \in I^X \mid \mu^c \geq \lambda,\ \lambda \text{ is r-sgfo}\} \\ &= sgint(\mu^c, r). \end{aligned}$$

(2) The proof is similar to (1).

□

THEOREM 3.9. *Let (X,τ) be a fts. Then for $\mu \in I^X$ and $r \in I_0$,*

(1) $sgint(sgcl(sgint(sgcl(\mu,r),r),r),r) = sgint(sgcl(\mu,r),r)$,
(2) $sgcl(sgint(sgcl(sgint(\mu,r),r),r),r) = sgcl(sgint(\mu,r),r)$.

Proof. (1) Since we have $sgint(sgcl(\mu,r),r) \leq sgcl(\mu,r)$, $sgcl(sgint(sgcl(\mu,r),r),r) \leq sgcl(sgcl(\mu,r),r) = sgcl(\mu,r)$ by Theorem 3.6(6). Hence we have

$$sgint(sgcl(sgint(sgcl(\mu,r),r),r),r) \leq sgint(sgcl(\mu,r),r).$$

Conversely, since $sgint(sgcl(\mu,r),r) \leq sgcl(sgint(sgcl(\mu,r),r),r)$, from Theorem 3.7(6) we have

$$\begin{aligned} sgint(sgcl(\mu,r),r) &= sgint(sgint(sgcl(\mu,r),r),r) \\ &\leq sgint(sgcl(sgint(sgcl(\mu,r),r),r),r). \end{aligned}$$

(2) The proof is similar to (1).

□

4. r-semi-generalized fuzzy continuous maps

DEFINITION 4.1. Let (X,τ) and (Y,σ) be fts's and $r \in I_0$ and let $f : (X,\tau) \to (Y,\sigma)$ be a map.

(1) f is called *r-semi-generalized fuzzy continuous* (for short, r-semi-gf-continuous) if $f^{-1}(\mu)$ is a r-sgfc set of X for each fuzzy r-closed set μ of Y.
(2) f is called *strongly r-semi-generalized fuzzy continuous* (shortly, strongly r-semi-gf-continuous) if $f^{-1}(\mu)$ is a fuzzy r-closed set of X for each r-sgfc set μ of Y.
(3) f is called *r-semi-generalized fuzzy irresolute* (for short, r-semi-gf-irresolute) if $f^{-1}(\mu)$ is a r-sgfc set of X for each r-sgfc set μ of Y.
(4) f is called *r-semi-generalized fuzzy open* (for short, r-semi-gf-open) if $f(\mu)$ is a r-sgfo set of Y for each fuzzy r-open set μ of X.

(5) f is called *strongly r-semi-generalized fuzzy open* (for short, strongly r-semi-gf-open) if $f(\mu)$ is a r-sgfo set of Y for each r-sgfo set μ of X.

REMARK 4.2. Let (X, τ) and (Y, σ) be fts's and $r \in I_0$ and let $f : (X, \tau) \to (Y, \sigma)$ be a map.

(1) If f is fuzzy r-semi-continuous, then f is r-semi-gf-continuous.
(2) If f is r-semi-gf-irresolute, then f is r-semi-gf-continuous.
(3) If f is strong r-semi-gf-continuous, then f is r-semi-gf-irresolute.
(4) If f is fuzzy r-semi-open, then f is r-semi-gf-open.
(5) If f is strongly r-semi-gf-open, then f is r-semi-gf-open.

THEOREM 4.3. *Let (X, τ) and (Y, σ) be fts's and $r \in I_0$ and let $f : (X, \tau) \to (Y, \sigma)$ be a map. Then the following are equivalent:*

(1) *f is r-semi-gf-continuous.*
(2) *$f^{-1}(\mu)$ is a r-sgfo set of X for each fuzzy r-open set μ of Y.*

Proof. It is obvious.

□

THEOREM 4.4. *Let (X, τ) and (Y, σ) be fts's and $r \in I_0$ and let $f : (X, \tau) \to (Y, \sigma)$ be a map. Then the following are equivalent:*

(1) *f is strongly r-semi-gf-continuous.*
(2) *$f^{-1}(\mu)$ is a fuzzy r-open set of X for each r-sgfo set μ of Y.*

Proof. It is obvious.

□

THEOREM 4.5. *Let (X, τ) and (Y, σ) be fts's and $r \in I_0$. If $f : (X, \tau) \to (Y, \sigma)$ is a r-semi-gf-continuous map, then $f(sgcl(\mu, r)) \leq cl(f(\mu), r)$ for each $\mu \in I^X$.*

Proof. For each $\mu \in I^X$, $cl(f(\mu), r)$ is a fuzzy r-closed set of Y. Since f is r-semi-gf-continuous, $f^{-1}(cl(f(\mu), r))$ is a r-sgfc set of X. $\mu \leq f^{-1}(cl(f(\mu), r))$ and so $sgcl(\mu, r) \leq f^{-1}(cl(f(\mu), r))$. Hence $f(sgcl(\mu, r)) \leq cl(f(\mu), r)$.

□

THEOREM 4.6. *Let (X, τ) and (Y, σ) be fts's and $r \in I_0$. Then $f : (X, \tau) \to (Y, \sigma)$ is a r-semi-gf-irresolute map if and only if $f^{-1}(\mu)$ is a r-sgfo set of X for each r-sgfo set μ of Y.*

Proof. It is obvious. □

THEOREM 4.7. *Let (X, τ), (Y, σ) and (Z, ν) be fts's and $r \in I_0$. If $f : (X, \tau) \to (Y, \sigma)$ is a r-semi-gf-irresolute map and $g : (Y, \sigma) \to (Z, \nu)$ is a r-semi-gf-continuous map, then $g \circ f : (X, \tau) \to (Z, \nu)$ is a r-semi-gf-continuous map.*

Proof. It is obvious. □

THEOREM 4.8. *Let (X, τ) and (Y, σ) be fts's and $r \in I_0$. If $f : (X, \tau) \to (Y, \sigma)$ is a r-semi-gf-irresolute map, then*

(1) $f(sgcl(\mu, r)) \leq sgcl(f(\mu), r)$ *for each* $\mu \in I^X$,
(2) $sgcl(f^{-1}(\mu), r) \leq f^{-1}(sgcl(\mu, r))$ *for each* $\mu \in I^Y$,
(3) $f^{-1}(sgint(\mu, r)) \leq sgint(f^{-1}(\mu), r)$ *for each* $\mu \in I^Y$.

Proof. (1) For each $\mu \in I^X$, we have

$$\begin{aligned} f^{-1}(sgcl(f(\mu), r)) &= f^{-1}(\wedge\{\rho \in I^Y \mid f(\mu) \leq \rho,\ \rho \text{ is r-sgfc}\}) \\ &\geq f^{-1}(\wedge\{\rho \in I^Y \mid \mu \leq f^{-1}(\rho),\ \rho \text{ is r-sgfc}\}) \\ &\geq \wedge\{f^{-1}(\rho) \in I^X \mid \mu \leq f^{-1}(\rho),\ f^{-1}(\rho) \text{ is r-sgfc}\} \\ &\geq \wedge\{\lambda \in I^X \mid \mu \leq \lambda,\ \lambda \text{ is r-sgfc}\} \\ &= sgcl(\mu, r). \end{aligned}$$

Hence $f(sgcl(\mu, r)) \leq sgcl(f(\mu), r)$.

(2) For each $\mu \in I^Y$, we have

$$\begin{aligned} f^{-1}(sgcl(\mu, r)) &= f^{-1}(\wedge\{\rho \in I^Y \mid \mu \leq \rho,\ \rho \text{ is r-sgfc}\}) \\ &\geq f^{-1}(\wedge\{\rho \in I^Y \mid f^{-1}(\mu) \leq f^{-1}(\rho),\ \rho \text{ is r-sgfc}\}) \\ &\geq \wedge\{f^{-1}(\rho) \in I^X \mid f^{-1}(\mu) \leq f^{-1}(\rho),\ f^{-1}(\rho) \text{ is r-sgfc}\} \\ &\geq \wedge\{\lambda \in I^X \mid f^{-1}(\mu) \leq \lambda,\ \lambda \text{ is r-sgfc}\} \\ &= sgcl(f^{-1}(\mu), r). \end{aligned}$$

Hence $sgcl(f^{-1}(\mu), r) \leq f^{-1}(sgcl(\mu, r))$.

(3) For each $\mu \in I^Y$, we have

$$\begin{aligned} f^{-1}(sgint(\mu, r)) &= f^{-1}(\vee\{\rho \in I^Y \mid \rho \leq \mu,\ \rho \text{ is r-sgfo}\}) \\ &\leq f^{-1}(\vee\{\rho \in I^Y \mid f^{-1}(\rho) \leq f^{-1}(\mu),\ \rho \text{ is r-sgfo}\}) \\ &\leq \vee\{f^{-1}(\rho) \in I^X | f^{-1}(\rho) \leq f^{-1}(\mu), f^{-1}(\rho) \text{ is r-sgfo}\} \\ &\leq \vee\{\lambda \in I^X \mid \lambda \leq f^{-1}(\mu),\ \lambda \text{ is r-sgfo}\} \\ &= sgint(f^{-1}(\mu), r). \end{aligned}$$

Hence $f^{-1}(sgint(\mu, r)) \leq sgint(f^{-1}(\mu), r)$.

□

THEOREM 4.9. *Let (X, τ) and (Y, σ) be fts's and $r \in I_0$. If $f : (X, \tau) \to (Y, \sigma)$ is a strongly r-semi-gf-open map, then $f(sgint(\mu, r)) \leq sgint(f(\mu), r)$ for each $\mu \in I^X$.*

Proof. For each $\mu \in I^X$, we have

$$\begin{aligned} f(sgint(\mu, r)) &= f(\vee\{\rho \in I^X \mid \rho \leq \mu,\ \rho \text{ is r-sgfo}\}) \\ &\leq f(\vee\{\rho \in I^X \mid f(\rho) \leq f(\mu),\ \rho \text{ is r-sgfo}\}) \\ &\leq \vee\{f(\rho) \in I^Y \mid f(\rho) \leq f(\mu),\ f(\rho) \text{ is r-sgfo}\}) \\ &\leq \vee\{\lambda \in I^Y \mid \lambda \leq f(\mu),\ \lambda \text{ is r-sgfo}\} \\ &= sgint(f(\mu), r). \end{aligned}$$

Hence $f(sgint(\mu, r)) \leq sgint(f(\mu), r)$.

□

References

1. R. Badard, *Smooth axiomatics*, First IFSA Congress, Palma de Mallorca (July 1986).
2. J. Balasubramanian and P. Sundaram, *On some generalizations of fuzzy continuous functions*, Fuzzy Sets and Systems **86** (1997), 93-100.
3. P. Bhattacharya and B. K. Lahiri, *Semi-generalized closed sets in topology*, Indian J. Math. **29** (1987), 375-382.

4. C. L. Chang, *Fuzzy topological spaces*, J. Math. Anal. Appl. **24** (1968), 182-190.
5. K. C. Chattopadhyay and S. K. Samanta, *Fuzzy topology: fuzzy closure operator, fuzzy compactness and fuzzy connectedness*, Fuzzy Sets and Systems **54** (1993), 207-212.
6. M. Demirci, *On several types of compactness in smooth topological spaces*, Fuzzy Sets and Systems **90** (1997), 83-88.
7. S. J. Lee and E. P. Lee, *Fuzzy r-continuous and fuzzy r-semicontinuous maps*, Int. J. Math. Math. Sci. **27** (2001), no. 1, 53-63.
8. N. Levine, *Generalized closed sets in topological spaces*, Rend. Circ. Mat. Palermo **19** (1970), 89-96.
9. C. K. Park and W. K. Min, *r-generalized fuzzy compactness* (to appear).
10. A. A. Ramadan, *Smooth topological spaces*, Fuzzy Sets and Systems **48** (1992), 371-375.

Department of Mathematics
Kangwon National University
Chuncheon 200–701, Korea
E-mail: `wkmin@kangwon.ac.kr, ckpark@kangwon.ac.kr`