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Can the Skewed Student-t Distribution Assumption Provide Accurate Estimates of Value-at-Risk?

Sang Hoon Kang* · Seong-Min Yoon**

〈abstract〉

It is well known that the distributional properties of financial asset returns exhibit fatter-tails and skewer-mean than the assumption of normal distribution. The correct assumption of return distribution might improve the estimated performance of the Value-at-Risk (VaR) models in financial markets. In this paper, we estimate and compare the VaR performance using the RiskMetrics, GARCH and FIGARCH models based on the normal and skewed-Student-t distributions in two daily returns of the Korean Composite Stock Index (KOSPI) and Korean Won-US Dollar (KRW-USD) exchange rate. We also perform the expected shortfall to assess the size of expected loss in terms of the estimation of the empirical failure rate. From the results of empirical VaR analysis, it is found that the presence of long memory in the volatility of sample returns is not an important in estimating an accurate VaR performance. However, it is more important to consider a model with skewed-Student-t distribution innovation in determining better VaR. In short, the appropriate assumption of return distribution provides more accurate VaR models for the portfolio managers and investors.

Keywords : Value-at-Risk(VaR), Skewed Student-t, RiskMetrics, GARCH, FIGARCH, Expected Shortfall

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* Ph.D. Candidate, School of Commerce, University of South Australia.

** Professor, Division of Economics, Pukyong National University. Corresponding author, E-mail : smyoon@pknu.ac.kr.

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Ⅰ. Introduction

Over three decades, identification of the distributional property for financial asset returns has received greater attention from financial analysts and academic researchers (Bollerslev, 1987; Hansen, 1994; Lambert and Laurent, 2001). With no doubt, the financial asset returns suffer from excess skewness and kurtosis, implying that assumption of Gaussian error innovation is inappropriate for explaining the skewed and fat-tailed characteristics of returns (Fang and Lai, 1997; Harvey and Siddique, 2000; Smith, 2006; Theodossiou, 1998).

However, most of studies have mainly focused on the fat-tail distribution and have ignored the skewness of a return distribution. As far as probability theory is concerned, the skewness is measure of the asymmetry of the probability distribution. So a negative (positive) skewness reflects price changes in such a way that the probability of positive price changes is higher (lower) than that of negative price changes. By understanding the appropriate shape of the return distribution, investors or risk managers are able to measure investment risk exposure in the financial markets (Premaratne and Bera, 2005).

In recent years, a central issue in risk management has been to determine the amount of capital requirement for banking and securities sectors to meet their exposure to market risk. Indeed, most financial institutions seek to avoid their exposure to losses or minimize possible investment risk and put great emphasis for the development and adoption of accurate measures of market risk (Stambaugh, 1996; Duffie and Pan, 1997). A primary tool of financial risk assessment is Value-at-Risk (VaR), which simply calculates the maximum loss (or worst case scenario) of an investment, over a given time period and a given significance level.[1] That is, the estimation of VaR mainly concerns the tail properties of distribution for measuring the risk (Duffie and Pan, 1997).

However, such a popular VaR model, RiskMetrics model (RiskMetrics Group, 1996), completely ignores the presence of asymmetric and fat-tailed characteristics

1) The Bank for International Settlements (BIS) imposes the confidence level at 99% and the time horizon at 10 days for the purpose of measuring the adequacy of bank capital.

in the return distribution due to its strict normality assumption (Alexander, 1996; Pafka and Kondor, 2001). Hence, the VaR models based on the normal distribution may lead to spurious forecasting results at high quantiles for skewed and fat-tailed distributions. Bali and Theodossiou (2007) empirically advocated the relevance of skewed Student-t distribution of Hansen (1994), which provides a flexible tool for modeling the empirical distribution of financial data exhibiting skewness and fat-tails.

Despite VaR's conceptual simplicity, many critiques point out the problems of VaR models. For example, Artzner et al. (1999) have cited the following short-comings of VaR: (1) VaR only measures distribution quantiles, and thus disregards extreme loss beyond the VaR level. (2) VaR models do not satisfy the subadditivity property which is part of necessary requirements to be coherent measure of risk.[2) To alleviate these problems inherent in VaR, more recent empirical studies prefer the use of expected shortfall as a supplementary measure of VaR (Kim and You, 2006; Oh and Moon, 2006; Scaillet, 2004; Yamai and Yoshiba, 2002, 2005).

Another important issue in the VaR literature is that the volatility of financial asset returns often exhibits the long memory property where the autocorrelations of the absolute and squared returns of many financial time series are characterized by a very slow decay (Baillie, 1996). This property is an important component for market risk management, investment portfolio and the pricing of derivative securities (Poon and Granger, 2003). The fractionally integrated GARCH (FIGARCH) model of Baillie, Bollerslev and Mikkelsen (1996) takes into consideration the fractional integrated (long memory) process of the conditional variance, which dates back to Granger (1980) and Hosking (1981). The FIGARCH specification provides us greater flexibility in modeling the conditional variance and estimating VaR in contrast to the standard GARCH model.

The primary aim of this article is to investigate volatility persistence for daily returns of the Korean Composite Stock Index (KOSPI) and Korean Won-US Dollar

2) The subadditivity property specifies that the total risk on a portfolio should not be greater than the sum of the individual risk. This means that aggregating individual risks does not increase overall risk.

(KRW-USD) exchange rate using the RiskMetrics, GARCH and FIGARCH models. To further enhance the robustness of the estimation results, we compare the performance of the various VaR models with the normal and skewed Student-t distribution innovations.

The contribution of this article is three fold. First, most empirical studies dealing with VaR have paid little attention to a long memory volatility process in the Korean financial markets (see literature reviews at the next section). In this reason, we identified a long memory volatility process in the Korean financial markets.

The second contribution is that the VaR analysis evidences the relevance of asymmetry and tail-fatness in the return distribution of Korean financial data. For instance, the distribution of KOSPI returns is left-skewed while the distribution of KRW-USD exchange rate returns is right-skewed. Thus, the models with skewed Student-t distribution provide more accurate VaR results than the normal distribution VaR models.

Finally, we also estimate the expected shortfall to assess the size of expected loss in terms of the estimation of the empirical failure rate. The measures of expected shortfall provide the determination of requirement capital corresponding to the catastrophic market risk in the Korean financial markets.

The reminder of this paper is organized as follows. Section 2 briefly reviews previous studies regarding to the cases of Korean financial markets. Section 3 describes the theoretical properties of long memory VaR models under different distribution innovations. Section 4 provides the statistical characteristics of sample data and empirical results. The concluding section summarizes the outcomes of applying the VaR analysis to the two Korean financial data.

Ⅱ. Literature Reviews

The topic of VaR is relatively a new genre in the Korean financial economics. Since the October 1997 currency crisis in Korea, many academics and practicers have made great efforts to build reliable risk measurement methods and risk management techniques. And many empirical studies have focused on the cases of VaR for the Korean financial markets.

Initially, empirical VaR studies attempted to capture the characteristics of fat-tails distribution using the GARCH class models under the normal distribution. Moon et al. (2003) assessed the volatility models for extreme events in the fat-tailed distribution such as RiskMetrics and GARCH class models using the KRW-USD exchange rates after the currency crisis. Their results seem to support the effectiveness of GARCH models rather than the RiskMetrics. Cho (2004) also compared the backtesting performance of several traditional volatility models using daily returns of KOSPI 200 index.[3] However, even though the unconditional distribution of a GARCH process reveal fatter tails, its standardized residuals is still not normal. Thus these studies totally disregard the tail characteristics of return distribution such as fat-tails because their volatility measures assume conditional normal errors.

For estimating the extreme events of fat-tail distribution, some applications introduced an alternative VaR methodology, the extreme value theory (EVT) which models only the tails of distribution. Yu and Lee (2004) found the EVT approach provide a good VaR performance both in-sample and out-of-sample backtesting for the KOSPI returns. Yeo (2006a, 2006b) also confirmed the superiority of the EVT to measure VaR in the Korean financial markets. Although the EVT approach provides robustness of VaR estimation in contrast to the RiskMetrics model, it focuses on extreme events in the financial markets and thus it is unable to represent the stylized factors of volatility such as volatility clustering, asymmetry volatility and volatility persistence. In addition, it also requires the choice of threshold point which is vaguely defined in the literature and demands the complicate mathematical terms (Yeo, 2006a, 2006b). Moon et al. (2003) empirically compared the VaR performance between the GARCH class models (including asymmetry models) and the EVT model. Their evidence indicated that both VaR models produce a similar forecasting in the Korean exchange market.

Subsequently, empirical studies have developed a GARCH type framework with different distribution innovations. For example, Yu (2005) developed a Student-t

3) This study employs several volatility measures : the historical standard deviations, the exponentially weighted moving average (EWMA), the standard deviations from GARCH(1, 1).

distribution VaR approach capturing the characteristics of fat-tail distribution for the KRW-USD exchange rate returns using five variants of GARCH class models. The out-of-sample results of this study indicated that the volatility models based on the Student-t distribution provide more accurate VaR values in contrast to the models based on the normal distribution.

However, the Student-t distribution VaR approach does not provide any information on the skewness of conditional distribution for the Korean financial returns. Kim and Kim (2006) first considered the VaR for daily returns of KRW-USD exchange rate using the APARCH model based on the normal, Student-t and skewed Student-t distribution. Their results provide evidence that the APARCH(1, 1) model with the skewed Student-t distribution seems to capture the characteristics of the skewness and excess kurtosis in the return distribution.

As mentioned earlier, the presence of long memory is an important issue for modeling the accuracy of volatility forecasting in the financial markets (Degiannakis, 2004; Vilasuso, 2002). Although several empirical studies have reached the conclusion that the presence of long memory volatility exists in the Korean stock market, little interest has been given to the VaR analysis in modeling the long memory volatility process in the case of Korean financial markets (Kang and Yoon, 2006; Lee, Kim and Lee, 2002). In this perspective, we investigate the validity of long memory property in modeling the volatility dynamics of Korean financial data with the VaR analysis.

Ⅲ. Methodology

1. Lo's R/S Analysis

The classical rescaled range (R/S) statistic of Hurst (1951) is a popular and robust tool for detecting long memory property in the time series with large skewness and excess kurtosis (Mandelbrot, 1971). Despite the strength of the R/S statistic, many econometric studies argued the weakness of the statistic (Anis and Lloyd, 1976; Lo, 1991). The most well known drawback is that the results of the analysis are extremely sensitive to the presence of short-term dependence (Lo, 1991 ; Jacobsen, 1996).

To account for the potential effect of short-term dependence on the R/S statistic, Lo (1991) proposed the modified R/S analysis which examines the null hypothesis of short memory process against the long memory alternatives. Let $\bar{y}$ be the sample mean of a return series $\{y_1, y_2, y_3, \cdots, y_n\}$. The modified R/S statistic, denoted as Q_n, is given by the range of partial sums of deviations from the mean rescaled by its standard deviation:

$$Q_n = \frac{1}{(\sigma_n(q))} \left\{ \max_{1 \le j \le N} \sum_{i=1} (y_i - \bar{y}_n) - \min_{1 \le j \le N} \sum_{i=1}^{j} (y_n - \bar{y}_n) \right\}, \tag{1}$$

where the denominator $\sigma_n(q)$ includes the sample variance, $\hat{\sigma}_y^2$, and autocovariance, $\hat{v}_i$, estimators:

$$\sigma_n(q) = \frac{1}{n} \sum_{i=1}^{n} (y_i - \bar{y}_n)^2 + \frac{2}{n} \sum_{i=1}^{q} w_i(q) \left\{ \sum_{t=i+1}^{n} (y_t - \bar{y})(y_{t-i} - \bar{y}_n) \right\}$$

$$= \hat{\sigma}_y^2 + 2 \sum_{i=1}^{q} w_i(q)\, \hat{v}_i, \tag{2}$$

where q is number of periods lagged and the weight is defined as $w_i(q) = 1 - i/q + 1$, for $q < n$. If the series of sample returns are subject to short-term dependence, the variance term in the denominator includes some autocovariance terms which are weighted according to their lag values(q).

Under the null hypothesis of short memory, Lo (1991) defined the modified R/S statistic, $V_n(q)$, by setting :

$$V_n(q) = \frac{1}{\sqrt{n}} Q_n \overset{a}{\sim} V, \tag{3}$$

in which the distribution function F_V of v is given by :

$$F_V(v) = 1 + 2\sum_{j=1}^{\infty} (1 - 4j^2 v^2) \exp(-2j^2 v^2). \tag{4}$$

The critical values of significant levels are computed from equation (4) and tabulated by Lo (1991 : 1288) for the purpose of the hypothesis test under the null hypothesis of short-term dependence against long-term dependence alternatives.

2. RiskMetrics Model

The RiskMetrics model relies on the specification of the variance equation of the portfolio returns and the assumption of a Gaussian error distribution. Generally, the RiskMetrics model is equivalent to a normal Integrated GARCH (IGARCH) specification where the autoregressive parameter is set at a pre-specified value λ, whereas the coefficient of ε_{t-1}^2 equals to $1-\lambda$ (Giot and Laurent, 2003). The basic RiskMetrics model can be defined as follows :

$$\varepsilon_t = z_t \sigma_t, \qquad z_t \sim N(0, 1) \tag{5}$$

$$\sigma_t^2 = (1-\lambda)\, \varepsilon_{t-1}^2 + \lambda \sigma_{t-1}^2, \tag{6}$$

where $0 \le \lambda \le 1$. The RiskMetrics Group (1996) suggests $\lambda = 0.94$ for the best backtesting results. Therefore, the RiskMetrics specification does not require estimation of unknown parameters in the volatility equation as all parameters are already present at given values.

3. GARCH Model

The GARCH model of Bollerslev (1986) has an autoregressive moving average (ARMA) form for the conditional variance ${\sigma_t}^2$. The GARCH (p, q) is expressed as :

$$y_t = \mu + \varepsilon_t \tag{7}$$

$$\varepsilon_t = z_t \sigma_t, \;\; z_t \sim N(0, 1) \tag{8}$$

$$\sigma_t^2 = \omega + \alpha(L)\, \varepsilon_t^2 + \beta(L) \sigma_t^2 \tag{9}$$

where $\omega > 0$ and $\alpha_i \ge 0$ for all i, $\beta_j \ge 0$ for all j, L denotes the lag or backshift operator, and $\alpha(L) \equiv \alpha_1 L + \alpha_2 L^2 + \cdots + \alpha_q L^q$ and $\beta(L) \equiv \beta_1 L + B_2 L^2 + \cdots + \beta_p L^p$. In the

GARCH (p, q) model, current conditional variance is parameterised to depend upon q lags of the squared error and $p$lags of conditional variance. Assuming that $v_t \equiv \varepsilon_t^2 - \sigma_t^2$, the GARCH$(p, q)$ can be rewritten as an ARMA process in ε_t^2:

$$[1-\alpha(L)-\beta(L)]\varepsilon_t^2 = \omega + [1-\beta(L)]v_t \tag{10}$$

The $\{v_t\}$ process interpreted as the innovations for the conditional variance has zero mean serially uncorrelated. If all the roots of the polynomial $[1-\beta(L)]$ and $[1-\alpha(L)-\beta(L)]$ lie outside the unit circle, then ε_t^2 exhibits covariance stationarity, and volatility shocks decay at a geometric rate referring to short memory in the conditional variance.

4. FIGARCH Model

The most popular long memory approach in the conditional mean is a fractional integrated ARMA(ARFIMA) process proposed by Granger (1980), Granger and Joyeux (1980), and Hosking (1981). Unlike the knife-edge distinction between $I(0)$ and $I(1)$ processes, the ARFIMA processes distinguish between short memory and long memory in terms of fractional orders of the integration process $I(d)$. The distinction of between $I(0)$ and $I(1)$ processes is too narrow for modeling the long memory property in the conditional mean.

Similar to the ARFIMA process for the condition mean, Baillie, Bollerslev and Mikkelsen (1996) extended the general GARCH model with the fractional integration process $I(d)$ and proposed the FIGARCH model. From Equation (10), the FIGARCH (p, d, q) process for ε_t is defined by:

$$\phi(L)(1-L)^d\varepsilon_t^2 = \omega + [1-\beta(L)]v_t, \tag{11}$$

where, $v_t \equiv \varepsilon_t^2 - \sigma_t^2$. d, ω, ϕ, and β must be non-negativity to ensure the conditional variance is positive. The $\{v_t\}$ process can be interpreted as an innovation for conditional variance and has zero mean serially uncorrelated. All the roots of $\phi(L)$ and $[1-\beta(L)]$ lie outside the unit root circle. Equation (11) can be re-written as:

$$\sigma_t^2 = \omega + \{1 - [1-\beta(L)]^{-1}\phi(L)(1-L)^d\}\varepsilon_t^2$$
$$\equiv \omega + \lambda(L)\varepsilon_t^2, \tag{12}$$

where $\lambda(L) = \lambda_1 L + \lambda_2 L^2 + \cdots$. and $0 \le d \le 1$. For the FIGARCH (p, d, q) process to be well defined and the conditional variance to be positive for all t, all the coefficients in the infinite ARCH representation must be nonnegative; i.e., $\lambda_j \ge 0$ for $j = 1, 2, \cdots$ (Bollerslev and Mikkelsen, 1996).[4)]

The FIGARCH model provides greater flexibility for modeling the conditional variance as it accommodates the covariance stationary GARCH model when $d = 0$ and the IGARCH model when $d = 1$ as special cases. For the FIGARCH model in Equation (11), the persistence of shocks to the conditional variance, or the degree of long memory is measured by the fractional differencing parameter d. Thus, the attraction of the FIGARCH model is that for $0 < d < 1$, it is sufficiently flexible to allow for intermediate range of persistence.

5. Model Densities

The parameters of volatility models can be estimated by using non-linear optimization procedures to maximize the logarithm of the likelihood function. Under the assumption that the innovations follow a normal distribution, i.e. $Z_t \sim N(0, 1)$, the log-likelihood function for Gaussian or normal distribution (L_{Norm}) can be expressed as:

$$L_{Norm} = -\frac{1}{2}\sum_{t=1}^{T}[\text{In}(2\pi) + \text{In}(\sigma_t^2) + z_t^2], \tag{13}$$

where T is number of observations. However, it is widely observed that the residuals suffer from the skewness and excess kurtosis, namely, the distribution of

4) The FIGARCH process has impulse response weights. For high j, $\lambda_k \approx j^{d-1}$ provides a measure of long memory process or a process that hyperbolically decays (Baillie, Bollerslev and Mikkelsen, 1996). For the FIGARCH model, λ_j will persist at the high j and then eventually convergent to zero. This means that shocks to volatility decay at the slow hyperbolic rate.

residuals tends to have skewed mean and fatter tails than a normal distribution.

To incorporate any excess skewness and kurtosis, we consider a skewed Student-t distribution proposed by Lambert and Laurent (2001). If $Z_t \sim SKST(0, 1, k, v)$, the log-likelihood distribution of the skewed Student-t distribution (L_{SkSt}) is as follows:

$$L_{SkSt} = T\left\{\text{In } \Gamma\left(\frac{v+1}{2}\right) - \text{In } \Gamma\left(\frac{v}{2}\right) - \frac{1}{2}\text{In } [\pi(v-2)] + \text{In } \left(\frac{2}{k+\frac{1}{k}}\right) + \text{In } (s)\right\}$$

$$-\frac{1}{2}\sum_{t=1}^{T}\left[\text{In } (\sigma_t^2) + (1+v)\text{In } \left[1 + \frac{(sz_t+m)^2}{v-2}k^{-2I_t}\right]\right], \qquad (14)$$

where $\Gamma(\cdot)$ is the gamma function. $I_t = 1$ if $z_t \geq -m/s$ or $I_t = -1$ if $z_t < -m/s$, k is an asymmetry parameter. The constants $m = m(k, v)$ and $s + \sqrt{s^2(k, v)}$ are the mean and standard deviation of the skewed Student-t distribution as following:

$$m(k, v) = \frac{\Gamma\left(\frac{v-1}{2}\right)\sqrt{v-2}}{\sqrt{\pi}\,\Gamma\left(\frac{v}{2}\right)}\left(k - \frac{1}{k}\right),$$

$$s^2(k, v) = \left(k^2 + \frac{1}{k^2} - 1\right) - m^2, \qquad (15)$$

where $2 < v \leq \infty$, additional parameter v standing for the number of degrees of freedom that measure the degree of fat-tails of the density. For example, the lower values of parameter v indicate the fat-tails of the density. The value of $\text{In}(k)$ can represent the degree of asymmetry of residual distribution. For example, if $\text{In}(k) > 0 (\text{In}(k) < 0)$, the density is right (left) skewed. When $k = 1$, the skewed Student-t distribution equals the general Student-t distribution, i.e. $z_t \sim ST(0, 1, v)$.

6. VaR Models and Tests

1) VaR Models

Nowadays, traders or portfolio managers find that their portfolios change dramatically one day to next, and are concerned with not only long trading positions but also short trading positions. So the performance of each VaR model should be compared on the based of both long trading positions (the left tail of distribution) and short trading positions (the right tail of distribution).

We also compare the performance of the FIGARCH model estimated with the assumption of three distributions including the normal, Student-t, and skewed Student-t distributions discussed above. In addition, the one-step-ahead VaR is computed with the results of estimated volatility models and its given distribution. The VaR of α quantile for long and short trading positions are computed as follows:

Under the assumption of the normal distribution,

$$VaR_{long} = \mu_t - z_\alpha \sigma_t \text{ and } VaR_{short} = \mu_t + z_\alpha \sigma_t, \tag{16}$$

where z_α is the left or right quantile at α% for the normal distribution in equation (13).

Under the assumption of the skewed Student-t distribution,

$$VaR_{long} = \mu_t - skst_{\alpha,v,k}\,\sigma_t \text{ and } VaR_{short} = \mu_t + skst_{\alpha,v,k}\,\sigma_t, \tag{17}$$

where $st_{\alpha,v,k}$ is the left or right quantile at α% for the skewed Student-t distribution in equation (14). If $\text{In}(k) < 0$, the VaR for long trading positions will be larger for the same conditional variance than the VaR for short trading positions. When $\text{In}\ (k) > 0$, vice versa.

2) Tests for Accuracy of VaR Estimates

We calculate the VaR at the pre-specified significance level α ranging from 5% to 1% and then evaluate their performance by calculating the failure rate for both the left and right tails of the distribution of the sample return series. The failure rate is defined as the ratio of the number (x) of times which returns exceed the forecasted VaR to the sample size (T). Following Giot and Laurent (2003), testing the accuracy of the model is equivalent to testing the hypothesis $H_0 : f = \alpha$ versus $H_1 : f \neq \alpha$, where f is the failure rate, i.e. if the VaR model is correctly specified the failure rate should be equal to the pre-specified significance level α. This test is also called the Kupiec LR test which tests the hypothesis using likelihood ratio test (Kupiec, 1995). The LR statistic is as follows:

$$LR = -2\ln\left[(1-\alpha)^{T-x}\alpha^{x}\right] + 2\ln\left[(1-\hat{f})^{T-x}(\hat{f})^{x}\right] \sim \chi^2(1), \tag{18}$$

where $\hat{f}$ is the estimated failure rate. Under the null hypothesis, the Kupeic LR statistic has a chi-square distribution with 1 degree of freedom. The critical value of $\chi^2(1)$ at the 5% level is 3.841.[5)]

Ⅳ. Empirical Results

1. The Preliminary Analysis of Data

This study considers two Korean financial market data: KOSPI and KRW-USD exchange rate. Both time-series data consist of daily observations and cover the period from October 1998 to December 2005.[6)] Price series are converted into the percentage logarithmic return series, i.e. the returns are calculated by $y_t = \ln(P_t/P_{t-1}) \times 100$, where P_t is the current price and P_{t-1} is the previous day's price.

Descriptive graphs (level of index (a), daily returns (b), density of the daily returns vs. normal distribution (c), and tail distributions against the normal distribution (d)) for daily KOSPI and KRW-USD exchange rate are given in [Figure 1] and [Figure 2], respectively.

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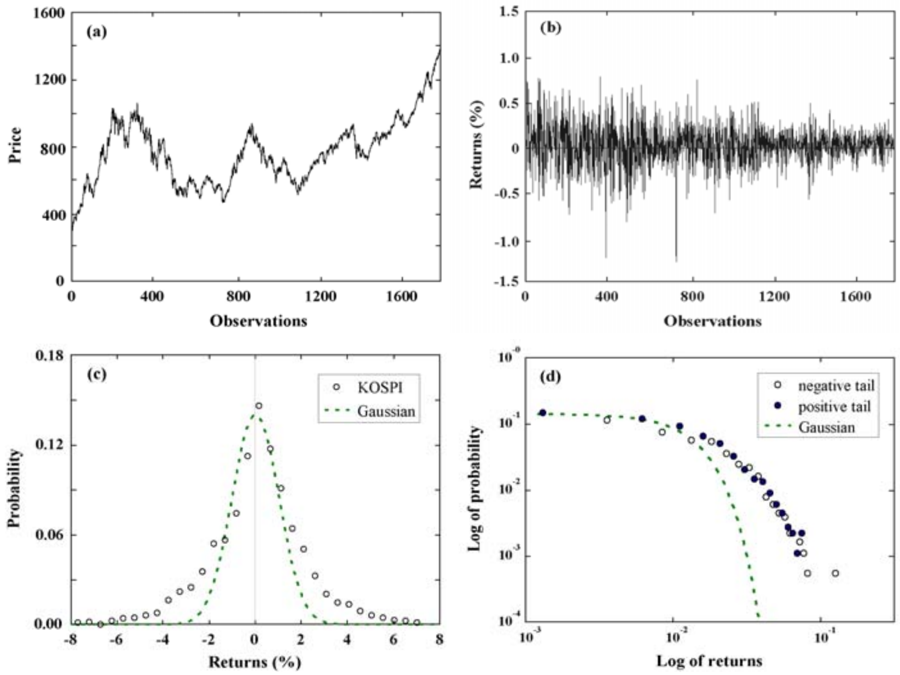
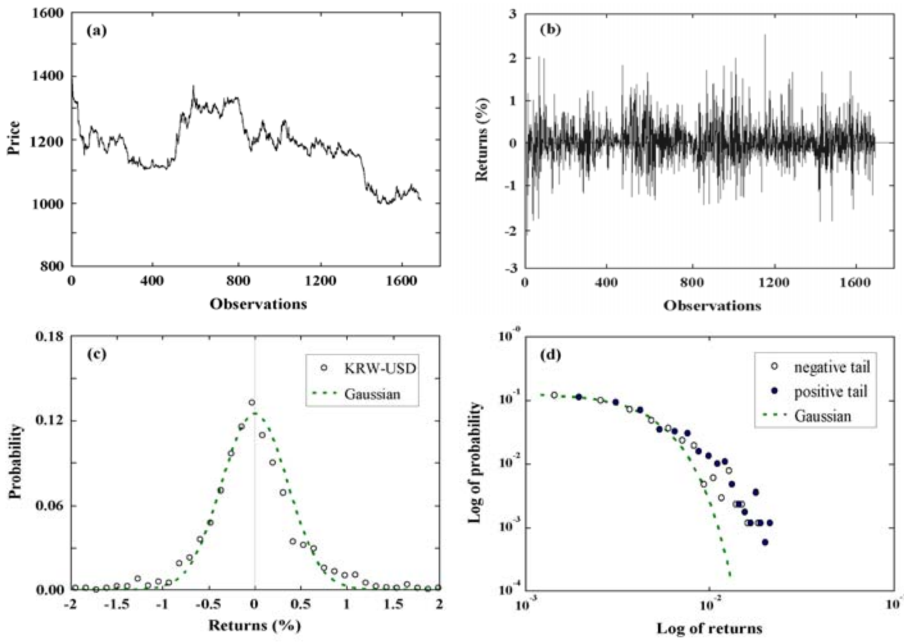
5) Alternative evaluation techniques have been developed in the VaR literature: especially, evaluation based on interval forecast evaluation as proposed by Christoffersen (1998), distribution forecast evaluation as proposed by Crnkovi and Drachman (1996) and probabilityforecasting framework as proposed by Lopez (2001). Among these, the Kupiec LR test provides the simplicity of test framework and the it is widely applied in evaluating the effectiveness of VaR models (Giot and Laurent, 2003; Tang and Shieh, 2006 Wu and Shieh, 2007). We apply the Kupiec LR test in this study.

6) It is well known that a structural break would induce the spurious long memory property. To avoid such structural break problem, we consider the sample data period after the Korean currency crisis.



[Figure 1] Descriptive Graphs for KOSPI Index



[Figure 2] Descriptive Graphs for KRW-USD Exchange Rate

It is clear that volatility clustering has been observed in the graphs of daily returns. The density graphs against the normal distribution show that the return distributions exhibit fat-tails and asymmetry. In particular, the density for the KOSPI returns appears to be left skewed, whereas the KRW-USD exchange rate returns has the right skewed density. In addition, the tail distributions of both cases confirm that the tail distributions are fatter than those of Gaussian distribution and asymmetric because negative tails is unequal to their counterparts. The fat-tailed and asymmetric properties of return distribution motivate the use of non-normality distribution innovations in this study.

Descriptive statistics for both sample returns are summarized in <Table 1>. The two return series reveal that they do not correspond to the normal distribution assumption. For example, both skewness and kurtosis statistics in the table reveal that the returns distribution is not normally distributed. Likewise, the Jarque-Bera test (J-B) statistics also reject the null hypothesis of normality in both sample re-

<Table 1> Descriptive Statistics for Sample Returns

	KOSPI	KRW-USD
No. of Obs.	1794	1702
Mean (%)	0.084	-0.018
Standard deviation (%)	2.083	0.485
Minimum	-12.80	-2.22
Maximum	7.697	2.499
Skewness	-0.291	0.254
Excess kurtosis	2.560	2.293
Jarque-Bera	250.64^{***}	205.05^{***}
$Q_s(10)$	208.22^{***}	199.08^{***}
BDS(10)	22.70^{***}	22.12^{***}

Notes : The Jarque-Bera statistic tests for the null hypothesis of normality in sample returns. The $Q_s(10)$ is the Box-Pierce statistic for the squared return series for up to 10th order serial correlation. The BDS (10) corresponds to the t-statistic of the BDS test with the embedding dimension m = 10. *** indicates a rejection of null hypothesis at a 1% significance level.

turn series. We also examine the null hypothesis of a white-noise process using the Box-Pierces test statistic of the squared return residuals ($Q_s(10)$). Under the null hypothesis of independence, the test statistic is distributed asymptotically as a x^2 (chi-square) distribution with 10 degrees of freedom. As shown in the table the squared residuals fail to be an independently and identically distributed (*i.i.d.*) process, since the squared return residuals are highly correlated up to the 10^{th} lag. Likewise, the BDS(10) test statistics of Brock et al. (1996) also reject the null hypothesis that the residuals are pure random processes. Thus, these *i.i.d.*-series tests imply the return residuals exhibit linear dependence, non-linear dependence, or chaos.

Before testing for the long memory property in volatility, both return series are subjected to three unit root tests to determine whether stationarity, integration, or fractional integration should be considered for each daily data : ADF (augmented-Dickey-Fuller), PP (Phillips-Peron) and KPSS (Kwiatkowski, Phillips, Schmidt and Shin) tests. These tests differ in the null hypothesis. The null hypothesis of the ADF and PP tests is that a time series contains a unit root, $I(1)$ process, while the KPSS test has the null hypothesis of stationarity, $I(0)$ process.

The empirical results of stationarity test for both sample returns are presented in <Table 2> Large negative values for the ADF and PP tests for both returns support the rejection of the null hypothesis of a unit root at a 1% significance level.

<Table 2> Unit Root Tests and Lo's R / S Analysis

	KOSPI	KRW-USD
ADF test	-40.27^{***}	-40.05^{***}
PP test	-40.22^{***}	-42.15^{***}
KPSS test	0.201	0.148
Lo's R / S test for R_t	1.640	1.349
Lo's R / S test for R_t^2	3.526^{***}	1.637
Lo's R / S test for $\|R\|$	4.021^{***}	2.142^{***}

Notes : (1) Mackinnon's 1% critical value is -3.435 for ADF and PP tests. (2) A KPSS critical value is 0.739 at the 1% significance level. (3) The critical value of Lo' modified R/S test is 2.098 at a 1% significance level. See <Table 1>.

Thus, both sample returns are stationary and suitable for subsequent tests in this study. Additionally, the statistics of the KPSS test indicate that return series are insignificant to reject the null hypothesis of stationarity, implying that they are stationary processes. Thus both return series are stationary and a short memory process in the level of returns.

Furthermore, the results of Lo's R/S test statistic for daily returns, squared and absolute returns are given at the bottom of <Table 2>. In this paper, squared returns and absolute returns are used as volatility proxies. For the case of returns, the value of modified R/S statistic supports the null hypothesis of short memory, implying that there is little evidence of long memory in the level of returns. However, two proxies of volatility display strong evidence of long memory except for the squared returns of KRW-USD exchange rate. Note that the dynamic dependences in the absolute returns are much stronger than those in the squared returns. Thus, the absolute returns used as the proxy of volatility display a long memory process where the autocorrelation function decays at a hyperbolic rate, instead of an exponential rate, over the longer lags.

2. Empirical Results of Long Memory in Volatility

In this subsection, we estimate GARCH(1, 1) and FIGARCH(1, d, 1) models under the normal (N) and skewed Student-t (skSt) distribution innovations. <Table 3> compares the estimation results of the GARCH(1, 1)-N, GARCH(1, 1)-skSt, FIGARCH(1, d, 1)-N, and FIGARCH(1, d, 1)-skSt models for both the return series.

In order to check the relevance of residuals distribution, this table also provides a set of diagnostic tests: the lowest value of Akaike's information criteria (AIC) indicates the best model amongst the normal, Student-t and skewed Student-t models. Box-Pierce Q_s statistic tests the *i.i.d.* series of squared standardized residuals. If the conditional variance equations are correctly specified, then Q_s statistic should support the null hypothesis of the *i.i.d.* series. Besides, the LM ARCH statistic of Engle (1982) is used to test the presence of remaining ARCH effects in the residuals. The ARCH(10) statistic is to test the joint significance of lagged squared residuals up to 10^{th} order. Finally, we calculate the BDS(10) statistic for checking non-linear dependence or chaos in the standardized residuals.

As shown at <Table 3>, the normal FIGARCH(1, d, 1) model is found to capture the long memory property in the volatility of both cases. For instance, the parameter(d) of normal FIGARCH model significantly rejects the validity of GARCH(1, 1) null hypothesis ($d=0$) at the 1% significance level. From this evidence, the normal FIGARCH model outperforms the GARCH model in modeling the long memory property in the Korean financial markets in contrast to the assumption of the efficient market hypothesis which states that all available information is fully and immediately reflected into prices. This finding is consistent with prior long memory literature including Kang, Kim and Yoon (2006) which found the long memory property in the volatility of KOSPI return series.

According to the calculated values of AIC and the significance of $Q_s(10)$, ARCH (10) and BDS(10) statistics in <Table 3>, the FIGARCH-skSt model outperforms other models (GARCH-N, GARCH-skSt and FIGARCH-N) for both cases. For the case of FIGARCH-skSt model, the estimated values of asymmetric parameter $\ln(k)$ (-0.060 for the KOSPI and 0.115 for the KRW-USD exchange rate) are significantly different from zero, indicating that the densities of both standardized residuals are skewed. For example, the asymmetric parameter of the KOSPI returns is significantly negative so that the density is skewed to the left side, while the density of the KRW-USD exchange rate returns is skewed to the right side, due to its positive asymmetric parameter. In addition, the estimated values of Student-t parameter (v) for both cases are statistically significant, indicating that the densities of all standardized residuals exhibit fat-tails. As a result, we conclude that the FIGARCH-skSt model can capture the asymmetry and fat-tails of return distribution.

3. Empirical Results for VaR Analysis

In this section, we move to compute not only in-sample VaR values for examining the estimated model's goodness-of-fit ability but also out-of-sample VaR values for evaluating the forecasting performance of the estimated models. Under the assumption of different distributions, the long memory model is tested with a VaR significance level α from 5% to 1%, and its performance is assessed by computing the failure rate for sample returns. If the VaR models are correctly specified, the failure rate would be equal to the pre-specified VaR level α. This information provides a more accurate assessment of possible trading losses.

<Table 3> Estimation Results from GARCH and FIGARCH models

	KOSPI				KRW-USD			
Models	GARCH-N	GARCH-skSt	FIGARCH-N	FIGARCH-skSt	GARCH-N	GARCH-skSt	FIGARCH-N	FIGARCH-skSt
μ	0.131 (0.044)***	0.118 (0.039)***	0.139 (0.038)***	0.139 (0.037)***	−0.013 (0.010)	−0.015 (0.009)	−0.012 (0.009)	−0.016 (0.009)*
ω	0.015 (0.015)	0.011 (0.009)	0.053 (0.037)	0.051 (0.048)	0.006 (0.004)	0.007 (0.006)	0.005 (0.002)**	0.008 (0.004)**
α_1	0.055 (0.029)*	0.045 (0.015)***	0.195 (0.047)***	0.148 (0.059)**	0.091 (0.028)***	0.166 (0.059)***	0.478 (0.081)***	0.342 (0.098)***
β_1	0.942 (0.029)***	0.952 (0.015)***	0.585 (0.066)***	0.545 (0.094)***	0.885 (0.039)***	0.827 (0.068)***	0.716 (0.089)***	0.651 (0.109)***
d	–	–	0.416 (0.060)***	0.410 (0.071)***	–	–	0.484 (0.110)***	0.563 (0.120)***
v	–	7.518 (1.464)***	–	8.160 (1.390)***	–	4.627 (0.513)***	–	5.079 (0.566)***
$\text{In}(k)$	–	−0.067 (0.027)**	–	−0.060 (0.030)**	–	0.117 (0.033)***	–	0.115 (0.032)***
$\text{In}(L)$	−3704.61	−3662.27	−3694.31	−3656.03	−1085.35	−999.72	−1075.27	−993.74
AIC	4.13446	4.08949	4.12410	4.08365	1.28009	1.18181	1.26942	1.17597
$Q_s(10)$	5.306	5.943	9.061	9.327	6.908	6.847	3.715	4.364
ARCH (10)	0.491	0.546	0.408	0.445	0.533	0.656	0.368	0.452
BDS (10)	0.790	1.455	−0.346	−0.550	4.812***	1.201	2.761***	1.004

Notes : Standard errors are in parentheses below corresponding parameter estimates. In (L) is the value of the maximized Gaussian log likelihood. *, ** and *** indicate a rejection of the null hypothesis at the 10%, 5% and 1% significance levels, respectively. See Table 1.

<Table 4> In-Sample VaR Estimation for KOSPI Index

Short positions				Long positions			
α	Failure rate	Kupiec LR	P-value	α	Failure rate	Kupiec LR	P-value
RiskMetrics model							
0.05	0.046	0.389	0.532	0.05	0.063	6.406**	0.011
0.04	0.039	0.045	0.831	0.04	0.052	6.562**	0.010
0.03	0.030	0.026	0.870	0.03	0.039	5.149**	0.023
0.02	0.021	0.125	0.723	0.02	0.028	5.758**	0.016
0.01	0.012	0.865	0.352	0.01	0.018	10.23**	0.001
GARCH-N model							
0.05	0.039	4.313**	0.037	0.05	0.054	0.770	0.380
0.04	0.033	1.957	0.161	0.04	0.045	1.191	0.275
0.03	0.027	0.522	0.469	0.03	0.032	0.315	0.574
0.02	0.016	1.148	0.283	0.02	0.024	1.788	0.181
0.01	0.010	0.000	1.000	0.01	0.016	6.015**	0.014
GARCH-St model							
0.05	0.052	0.213	0.643	0.05	0.057	1.985	1.985
0.04	0.039	0.008	0.926	0.04	0.045	1.191	1.191
0.03	0.029	0.012	0.909	0.03	0.028	0.064	0.064
0.02	0.018	0.242	0.622	0.02	0.020	0.000	0.000
0.01	0.006	2.248	0.133	0.01	0.008	0.515	0.515
FIGARCH-N model							
0.05	0.044	1.677	0.195	0.05	0.060	4.102**	0.042
0.04	0.034	2.122	0.145	0.04	0.046	2.065	0.150
0.03	0.028	0.458	0.498	0.03	0.038	4.060**	0.043
0.02	0.019	0.242	0.622	0.02	0.027	4.398**	0.035
0.01	0.011	0.062	0.803	0.01	0.015	4.866**	0.027
FIGARCH-skSt model							
0.05	0.053	0.213	0.643	0.05	0.060	3.699	0.054
0.04	0.041	0.000	0.976	0.04	0.045	1.191	0.275
0.03	0.030	0.012	0.909	0.03	0.035	1.874	0.171
0.02	0.021	0.000	0.983	0.02	0.021	0.125	0.723
0.01	0.009	0.219	0.639	0.01	0.009	0.050	0.821

Note : ** indicates a rejection of the null hypothesis at the 5% significance level.

<Table 5> In-Sample VaR Estimation for the KRW-USD Exchange Rate

Short positions				Long positions			
α	Failure rate	Kupiec LR	P-value	α	Failure rate	Kupiec LR	P-value
RiskMetrics model							
0.05	0.058	2.275	0.131	0.05	0.049	0.000	0.991
0.04	0.050	4.547**	0.032	0.04	0.038	0.147	0.701
0.03	0.039	4.681**	0.030	0.03	0.031	0.171	0.678
0.02	0.029	7.490**	0.006	0.02	0.023	1.008	0.315
0.01	0.022	21.00**	0.000	0.01	0.018	10.58**	0.001
GARCH-N model							
0.05	0.050	0.009	0.920	0.05	0.039	3.872**	0.049
0.04	0.042	0.362	0.547	0.04	0.032	2.371	0.123
0.03	0.034	1.213	0.270	0.03	0.024	2.188	0.139
0.02	0.027	3.867**	0.049	0.02	0.021	0.255	0.613
0.01	0.019	11.89**	0.000	0.01	0.014	3.302	0.069
GARCH-St model							
0.05	0.046	0.471	0.492	0.05	0.049	0.015	0.902
0.04	0.039	0.017	0.893	0.04	0.042	0.230	0.630
0.03	0.031	0.075	0.784	0.03	0.031	0.075	0.784
0.02	0.019	0.000	0.994	0.02	0.021	0.255	0.613
0.01	0.008	0.576	0.447	0.01	0.012	1.347	0.245
FIGARCH-N model							
0.05	0.052	0.102	0.748	0.05	0.038	5.421**	0.019
0.04	0.044	0.521	0.470	0.04	0.032	2.371	0.123
0.03	0.035	1.213	0.270	0.03	0.028	0.192	0.660
0.02	0.028	4.506**	0.033	0.02	0.021	0.255	0.613
0.01	0.021	14.70**	0.000	0.01	0.013	1.912	0.166
FIGARCH-skSt model							
0.05	0.050	0.000	0.991	0.05	0.049	0.015	0.902
0.04	0.042	0.128	0.719	0.04	0.042	0.362	0.547
0.03	0.033	0.305	0.580	0.03	0.033	0.687	0.407
0.02	0.024	1.008	0.315	0.02	0.023	1.008	0.315
0.01	0.010	0.063	0.801	0.01	0.012	1.347	0.245

Note : See <Table 4>.

1) In-sample VaR Analysis

The empirical results for the in-sample VaR analysis for the KOSPI and KRW-USD exchange rate are summarized in <Table 4> and <Table 5>, respectively. These tables contain the failure rate, Kupiec LR statistics and P-values. According to these tables, we observe that the models (RiskMatrics, GARCH-N and FIGARCH-N) based on the normal distribution innovation have a poor performance for both long and short positions of the KOSPI and KRW-USD exchange rate.

For example, the failure rates of normal distribution models significantly exceed the prescribed quantiles and then the null hypothesis ($f=\alpha$) of the Kupiec test is often rejected for the long positions of KOSPI (the short positions of KRW-USD exchange rate). This finding indicates that the normal distribution models tend to underestimate the in-sample VaR values. In addition, as α ranges from 0.05 and 0.01, the normal distribution models for the long positions of KOSPI (the short positions of KRW-USD exchange rate) have the great values of Kupiec LR test and reject the null hypothesis of($f=\alpha$). From this evidence, the normal models can not explain the fat-tailed distribution of Korean financial data. In addition, the performance of RiskMetrics performs worst in the in-sample VaR calculations.

The GARCH-skSt and FIGARCH-skSt models significantly improve on the in-sample VaR performance for both long and short trading positions. They outperform the normal distribution models (RiskMetrics, GARCH-N and FIGARCH-N) in capturing the asymmetric and fat-tailed distribution of the KOSPI returns as well as the KRW-USD exchange rate returns. In addition, the normal distribution models provide a relatively good performance for most of α levels for the short position (long position) for KOSPI (KRW-USD exchange rate) returns. This gives strong evidence that the returns of KOSPI (KRW-USD exchange rate) are skewed to the left (right) rather than to the right (left). Thus, the skewed Student-t distribution models predict crucial loss more accurately than the models with the normal distribution innovation in the in-sample VaR analysis.

Comparing the FIGARCH model with the GARCH model, it has reached the conclusion that the performance of FIGARCH model cannot outperform that of GARCH model in the in-sample VaR analysis. This finding indicates that the long memory volatility feature is not very crucial in determining a proper value of VaR.

<Table 6> Out-of-Sample VaR Analysis for the KOSPI Index

Short positions				Long positions			
α	Failure rate	Kupiec LR	P-value	α	Failure rate	Kupiec LR	P-value
RiskMetrics model							
0.05	0.058	0.642	0.422	0.05	0.066	2.459	0.116
0.04	0.048	0.784	0.375	0.04	0.058	3.720	0.053
0.03	0.032	0.067	0.795	0.03	0.042	2.206	0.137
0.02	0.026	0.839	0.359	0.02	0.032	3.113	0.077
0.01	0.010	0.000	1.000	0.01	0.020	3.913**	0.047
GARCH-N model							
0.05	0.030	3.907**	0.048	0.05	0.042	0.497	0.480
0.04	0.015	8.488**	0.003	0.04	0.032	0.624	0.429
0.03	0.012	5.370**	0.020	0.03	0.025	0.363	0.546
0.02	0.010	2.495	0.114	0.02	0.012	1.322	0.250
0.01	0.002	3.250	0.071	0.01	0.002	3.250	0.071
GARCH-skSt model							
0.05	0.048	0.042	0.836	0.05	0.056	0.365	0.545
0.04	0.040	0.000	1.000	0.04	0.044	0.201	0.653
0.03	0.024	0.663	0.415	0.03	0.036	0.582	0.445
0.02	0.016	0.437	0.508	0.02	0.018	0.105	0.745
0.01	0.004	2.353	0.125	0.01	0.006	0.943	0.331
FIGARCH-N model							
0.05	0.028	6.017**	0.014	0.05	0.050	0.000	1.000
0.04	0.022	5.015**	0.025	0.04	0.040	0.000	1.000
0.03	0.012	7.170**	0.007	0.03	0.030	0.000	1.000
0.02	0.008	4.742**	0.029	0.02	0.022	0.098	0.753
0.01	0.006	0.943	0.331	0.01	0.006	0.943	0.331
FIGARCH-skSt model							
0.05	0.038	1.646	0.199	0.05	0.050	0.000	1.000
0.04	0.030	1.421	0.233	0.04	0.042	0.051	0.820
0.03	0.016	4.042**	0.044	0.03	0.024	0.663	0.415
0.02	0.012	1.902	0.167	0.02	0.016	0.437	0.508
0.01	0.006	0.943	0.331	0.01	0.004	2.353	0.125

Note : See <Table 4>.

<Table 7> Out-of-Sample VaR Analysis for the KRW-USD Exchange Rate

Short positions				Long positions			
α	Failure rate	Kupiec LR	P-value	α	Failure rate	Kupiec LR	P-value
RiskMetrics model							
0.05	0.952	0.042	0.836	0.05	0.052	0.041	0.838
0.04	0.956	0.201	0.653	0.04	0.044	0.201	0.653
0.03	0.962	1.015	0.313	0.03	0.032	0.067	0.795
0.02	0.972	1.454	0.227	0.02	0.026	0.839	0.359
0.01	0.976	7.110**	0.007	0.01	0.024	7.110**	0.007
GARCH-N							
0.05	0.046	0.172	0.677	0.05	0.044	0.394	0.530
0.04	0.040	0.000	1.000	0.04	0.034	0.493	0.482
0.03	0.038	1.015	0.313	0.03	0.026	0.287	0.591
0.02	0.028	1.454	0.227	0.02	0.022	0.098	0.753
0.01	0.024	7.110**	0.007	0.01	0.012	0.189	0.663
GARCH-skSt							
0.05	0.046	0.172	0.677	0.05	0.06	0.992	0.319
0.04	0.046	0.447	0.503	0.04	0.05	1.209	0.271
0.03	0.038	1.015	0.313	0.03	0.036	0.582	0.445
0.02	0.022	0.098	0.753	0.02	0.022	0.098	0.753
0.01	0.010	0.000	1.000	0.01	0.010	0.000	1.000
FIGARCH-N model							
0.05	0.048	0.042	0.836	0.05	0.046	0.172	0.677
0.04	0.048	0.784	0.375	0.04	0.040	0.000	1.000
0.03	0.036	0.582	0.445	0.03	0.034	0.263	0.607
0.02	0.028	1.454	0.227	0.02	0.022	0.098	0.753
0.01	0.024	7.110**	0.007	0.01	0.014	0.718	0.396
FIGARCH-skSt model							
0.05	0.050	0.000	1.000	0.05	0.056	0.365	0.545
0.04	0.046	0.447	0.503	0.04	0.050	1.209	0.271
0.03	0.036	0.582	0.445	0.03	0.038	1.015	0.313
0.02	0.026	0.839	0.359	0.02	0.028	1.454	0.227
0.01	0.014	0.718	0.396	0.01	0.012	0.189	0.663

Note : See <Table 4>.

Consequently, we conjecture scenarios that there is no evidence of long memory in the in-sample VaR and that the performance of GARCH model is moderately equal to that of FIGARCH model in assessing the accuracy of in-sample VaR. However, the appropriateness of distributional assumption is a critical indicator for the assessment of in-sample VaR estimates

2) Out-of-Sample VaR Analysis

We further assess the performance of the model with the normal, Student-t and skewed Student-t innovations by computing the out-of-sample VaR forecasts. Following the analysis of Tang and Shieh (2006), we apply an iterative procedure in which the estimated model for the whole sample is estimated and then compare the predicted one-day-ahead VaR for both the long and short positions.[7)]

The empirical results for the out-of-sample VaR analysis for the KOSPI and KRW-USD exchange rates are reported in <Table 6> and <Table 7>, respectively. The results of this analysis are quite similar to those of the in-sample VaR analysis. Generally, the skewed Student-t distribution models (GARCH-skSt and FIGARCH-skSt) outperform normal distribution models (RiskMetrics, GARCH-N and FIGARCH-N). Interestingly, the RiskMetrics model provides more accurate volatility forecasting results than the GARCH-N and FIGARCH-N models. This can be interpreted as no evidence of long memory in the out-of-sample for the KOSPI and KRW-USD. Thus we conclude that it is not important to consider long memory models in estimating VaR, but the correct assumption of return distribution might improve the accuracy of VaR assessment in the Korean financial markets.

From this point, our empirical results of out-of-sample VaR analysis are consistent with those of So and Yu (2006) who found that long memory models do not necessarily lead to better VaR estimates. They also pointed out that fat-tailed distribution has played an important role in calculating better VaR performance in the financial data.

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7) To conduct out-of sample forecasting analysis, the last two years (500 observations) of our sample are used in this study. The models are re-estimated every 50 observations in the out-of-sample. For more details, see Tang and Shieh (2006).

3) Expected Shortfall

Although the above VaR analyses provide useful information to rank the VaR models, they refer nothing about the potential size of the loss that exceeds VaR. In this sub-section, we compute the expected loss of the above VaR models using the expected shortfall measure. The expected shortfall is defined as the conditional expectation of loss given that the loss is larger than the VaR level. That is, the

<Table 8> Expected Shortfall for KOSPI and KRW-USD(In-sample)

α	5%	4%	3%	2%	1%
Expected Shortfall for Long Position(KOSPI)					
RiskMetrics	-4.012	-4.174	-4.424	-4.758	-5.141
GARCH-N	-4.160	-4.483	-4.688	-5.110	-5.387
GARCH-skSt	-4.181	-4.490	-4.827	-5.266	-6.497
FIGARCH-N	-4.180	-4.509	-4.736	-5.007	-5.341
FIGARCH-skSt	-4.185	-4.514	-4.783	-5.209	-6.423
Expected Shortfall for Short Position(KOSPI)					
RiskMetrics	4.018	4.223	4.458	4.604	5.264
GARCH-N	4.370	4.545	4.731	5.236	5.902
GARCH-skSt	4.176	4.407	4.636	4.985	6.257
FIGARCH-N	4.361	4.541	4.859	5.133	5.513
FIGARCH-skSt	4.230	4.462	4.732	5.150	5.445
Expected Shortfall for Long Position(KRW-USD exchange rate)					
RiskMetrics	-0.984	-1.049	-1.097	-1.183	-1.247
GARCH-N	-1.068	-1.150	-1.242	-1.278	-1.355
GARCH-skSt	-0.977	-1.034	-1.105	-1.241	-1.301
FIGARCH-N	-1.057	-1.115	-1.160	-1.241	-1.310
FIGARCH-skSt	-0.969	-1.009	-1.088	-1.230	-1.310
Expected Shortfall for Short Position(KRW-USD exchange rate)					
RiskMetrics	1.018	1.060	1.098	1.173	1.259
GARCH-N	1.079	1.108	1.167	1.248	1.378
GARCH-skSt	1.062	1.096	1.168	1.300	1.499
FIGARCH-N	1.047	1.102	1.164	1.241	1.335
FIGARCH-skSt	1.050	1.083	1.161	1.278	1.526

expected shortfall measures how much one can lose on average beyond the VaR level (Scaillet, 2004; Yamai and Yoshiba, 2002, 2005). The measure is computed for the above in-sample and out-of-sample estimations of the long and short VaR.[8] The results from the expected shortfall for the KOSPI and KRW-USD exchange rate returns are summarized in <Table 8> and <Table 9>.

<Table 9> Expected Shortfall for KOSPI and KRW-USD (Out-of-sample)

α	5%	4%	3%	2%	1%
Expected Shortfall for Long Position(KOSPI)					
RiskMetrics	-2.621	-2.585	-2.829	-2.957	-3.340
GARCH-N	-2.664	-2.758	-2.936	-3.046	-3.479
GARCH-skSt	-2.750	-2.893	-2.962	-3.237	-4.095
FIGARCH-N	-2.838	-2.972	-3.089	-3.414	-3.985
FIGARCH-skSt	-2.838	-2.947	-3.347	-3.542	-4.365
Expected Shortfall for Short Position(KOSPI)					
RiskMetrics	2.317	2.402	2.42	2.481	2.789
GARCH-N	2.523	2.703	2.795	3.133	3.421
GARCH-skSt	2.584	2.651	2.976	3.118	4.034
FIGARCH-N	2.702	2.776	2.782	2.999	3.189
FIGARCH-skSt	2.610	2.691	2.674	2.782	3.189
Expected Shortfall for Long Position(KRW-USD exchange rate)					
RiskMetrics	-0.938	-0.984	-1.083	-1.166	-1.174
GARCH-N	-0.992	-1.073	-1.166	-1.269	-1.337
GARCH-skSt	-0.883	-0.952	-1.042	-1.204	-1.355
FIGARCH-N	-0.979	-1.032	-1.080	-1.235	-1.323
FIGARCH-skSt	-0.904	-0.947	-1.036	-1.124	-1.329
Expected Shortfall for Short Position(KRW-USD exchange rate)					
RiskMetrics	0.979	1.001	1.028	1.116	1.173
GARCH-N	1.003	1.026	1.045	1.148	1.173
GARCH-skSt	0.975	0.975	1.045	1.182	1.337
FIGARCH-N	1.008	1.008	1.059	1.130	1.173
FIGARCH-skSt	0.985	0.975	1.059	1.148	1.358

8) For the case of the long trading position, the expected shortfall is calculated as the average of the sample returns which are smaller than the long VaR, while for the case of the short trading position, the expected shortfall is computed as the average of the sample returns which are larger than the short VaR (Giot and Laurent, 2003). Note that the expected shortfall measure does not provide the criteria of model selection in calculated VaR models.

The results of expected shortfall are larger (in absolute value) for the models (GARCH-skSt and FIGARCH-skSt) based on the skewed Student-t distributions than for the models (RiskMetrics, GARCH-N and FIGARCH-N) based on the normal distribution. This finding indicates that the skewed Student-t distribution models fail less than normal distribution models, but when they fail, it happens for large (in absolute) returns : the average of these returns is correspondingly large. Although the expected shortfall is not important in determining to rank VaR models or assess models' performance, it is very useful for risk managers to answer the following question : 'when my model fails, how much do I lose on average' (Giot and Laurent, 2003). In short, the use of skewed Student-t distribution models assesses the accuracy of VaR performance, but they require larger capital reservations corresponding to exposure risks.

Ⅴ. Conclusion

Recent econometric literature has focused the distributional properties of financial asset which exhibit fatter-tails and skewer-mean than the normal distribution. The correct assumption of return distribution might improve the estimated performance of the Value-at-Risk models in financial markets. In addition, it is well known that the volatility of financial time series exhibits the long memory property. For these reasons, we investigate two Korean financial data (KOSPI and KRW-USD exchange rate) using the RiskMetrics, GARCH-N, GARCH-skSt, FIGARCH-N and FIGARCH-skSt models.

From the estimation results, the FIGARCH model outperforms GARCH model in capturing the long memory property in the volatility of Korean financial data. However, our in-sample and out-of-sample analyses report that the presence of long memory is not an important indicator in determining a proper value of VaR as the FIGARCH-skSt model cannot outperform the GARCH-skSt model. In addition, our VaR analysis results support that the correct assumption of return distribution improve the VaR performance since the skewed Student-t distribution models (GARCH-skSt and FIGARCH-skSt) are preferred due to their significant values of skewness.

Overall, it is concluded that considering the long memory property does not secure the accuracy of VaR performance, but accounting for skewness and excess kurtosis provides an accurate model selection of VaR models for the portfolio managers and investors.

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