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Collaboration Inventory System with Limited Resources and Weibull Distribution Deterioration

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Abstract. The objective of this study is to develop an optimal joint cost from the perspectives of both the manufacturer and the retailer. The integrated production-inventory model with Weibull distribution deteriorating items is assumed to have a constant demand rate. A limited retailer storage space and multiple delivery per order are considered in this model. A numerical example including the sensitivity analysis is given to validate the results of the production-inventory model.

Keywords: Integrated, Weibull Distribution, Deteriorating Items, Limited Storage Capacity.

1. INTRODUCTION

The problem of deteriorating inventory has received considerable attention in recent years. Deterioration is defined as change, damage, decay, evaporation, pilferage, spoilage, obsolescence and loss of utility or loss of marginal value of a commodity that results in decreasing usefulness from the original one. Products such as vegetables, fish, medicine, blood, alcohol, gasoline and radioactive chemicals have finite shelf life, and start to deteriorate once they are produced. Most researches in deteriorating inventory assumed constant rate of deterioration. However, the Weibull distribution is used to represent the product in stock deteriorates with time. The deterioration rate increases with age. The longer the items remain unused, the higher the rate at which they fail is.

Ghare and Schrader (1963) were two of the first authors who studied inventory problems considering deteriorating of items. Covert and Philip (1973) assumed a two-parameter Weibull distribution deterioration to consider varying deterioration rate of deterioration. Specially, the Weibull distribution is used to represent the distribution of the time to deterioration. The rate of deterioration increased with age, or the longer the items remained unused, the higher the rate at which they failed. Wee (1999) developed a deterministic inventory model with quantity discount, pricing and partial backordering when the product in stock deteriorates with times. Wee and Law (2001) applied the discounted cash-flow approach to a deterministic inventory model of an item that deteriorates over time at varying rate and with a price-dependent demand. Wang (2002) studied the inventory problem for deteriorating items with time-varying demands and shortages over a finite planning horizon. He assumed the backlogging rate to be time-dependent. Skouri and Papachristos (2003) proposed an algorithm to find the optimal stopping and restarting production times for an EOQ model with deteriorating items and time-dependent partial backlogging. Papachristos and Skouri (2003) considered a model where the demand rate is a decreasing function of the selling price and the

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backlogging rate is a time-dependent function. Abad (2003) considered the pricing and lot-sizing problem for perishable goods under finite production, exponential decay and partial backordering and lost sale. Goyal and Giri (2003) consider the production-inventory problem when the demand, production and deterioration rate are assumed to vary with time. Shortages are allowed to be backlogged partially. Papachristos and Skouri (2003) generalized the work of Wee (1999) and considered a model where the demand rate is a convex decreasing function of the selling price and the backlogging rate is a time-dependent function.

The study on manufacturer-retail cooperation has received a great deal of attention from researchers in recent years. This is because decision made independently by one player will not result in global optimum. Traditionally, inventory models considered the different sub-systems in the supply chain independently. Since the last decade, several authors have studied the integrated inventory models in which the manufacturers and the retailers coordinate their production and ordering policies in order to lower the joint inventory costs. Communication and information technology advancement play an important role in improving the performance of inventory control. Due to increasing competition and market globalization, enterprises must develop supply chain that can respond quickly to customers' needs. Global optimality will only be realized if the perspectives of all players are considered. In the supply chain, the cooperation should take account of the limited resources. Whenever the storage capacity of the company warehouse is insufficient, excess stock is held in a rented warehouse (which is more expensive).

Several authors have studied the integrated inventory models in which the manufacturers and the retailers coordinate their production and ordering policies, in order to lower the joint inventory costs. Goyal (1976) were among the first authors who studied manufacturer-retailer inventory in the integrated supply chain. Since then, a number of studies on manufacturer-retailer inventory models have been done. Hill (1997) incorporated the effect of finite replenishment rate and multiple delivery to respond quality to demand. Goyal and Nebebe (2000) developed an economic production and shipment policy from vendors' perspective. Woo et al (2001) developed an integrated vendor-buyer inventory policy to minimize joint total cost. Rau *et al.* (2003) developed a multi-echelon inventory model for a deteriorating item and derived an optimal joint total cost from the perspectives of the supplier, the producer, and the buyer. Yang and Wee (2003) developed an integrated inventory model for single-vendor and single-buyer with constant deterioration rate of deterioration and multiple delivery per order. Sana *et al.* (2004) developed a production-inventory model for a deteriorating item with trended demand and shortages. Banerjee (2005) developed a model to simultaneously determine the inventory policies of the supplier and the buyer with contractual agreement.

Most of the previous works on inventory models did not consider simultaneously the above-mentioned factors. This is not true in real life since the above factors are significant. For this reason, we incorporated manufacturer-retailer cooperation to develop an integrated production-inventory model. The main contribution of our study is to consider the factors such as Weibull distribution, multiple delivery and limited resources simultaneously in a model. To the best of our knowledge, this type of model has not been developed before. Our objective is to minimize the total cost per unit time. The classical optimization technique is used to derive the optimal solution. A numerical example including the sensitivity analysis is given to validate the results of the production-inventory model.

2. ASSUMPTION AND NOTATION

The mathematical model in this paper is developed based on the following assumptions:

1. Customer's demand rate is known and constant.
2. The production rate is finite and constant, larger than the demand rate and is unaffected by the lot size.
3. The study considered manufacturer-retailer cooperation. Shortages are not allowed.
4. Deterioration of the item follows a two-parameter Weibull distribution, and the deteriorated units are not replaced.
5. Deterioration occurs as soon as items are received into inventory.
6. Carrying cost applies to good units only.
7. Single producer and single distributor are considered.
8. Multiple delivery per order is considered.
9. There is only one production cycle per order.
10. The planning horizon is infinite.

The following notation is used throughout the paper:

p	Production rate (units/unit time)
d	Consumer's demand rate (units/unit time), $p > d$
g	Raw material's scale parameter for the deterioration rate
h	Raw material's shape parameter for the deterioration rate
α	Finished goods' scale parameter for the deterioration rate
β	Finished goods' shape parameter for the deterioration rate
Q_w	Manufacturer's raw materials order quantity per order
Q_m	Manufacturer's finished goods production quantity per production
Q_r	Retailer's received quantity per delivery from

the manufacturer
I_m Manufacturer's finished goods maximum inventory level
n The number of delivery per order
T_1 Production time when inventory builds up
T_2 Time period when there is no production and inventory depletes
T_3 Time with positive inventory in the retailer's inventory system, $T_3 = T/n$
T Length of the cycle, $T = T_1 + T_2$
$I_w(t_1)$ Manufacturer's raw materials inventory level at any time t_1, $0 \le t_1 \le T_1$
$I_{mi}(t_i)$ Manufacturer's inventory level at any time t, $0 \le t_i \le T_i$, $i = 1,2$
$I_r(t_3)$ Retailer's inventory level at any time t_3, $0 \le t_3 \le T_3$
C_{1w} Manufacturer's ordering cost per order cycle ($/cycle)
C_{1m} Manufacturer's setup cost per production cycle ($/cycle)
C_{1r} Retailer's ordering cost per order cycle ($/order)
C_{2w} Manufacturer's raw materials per unit holding cost per unit time ($/unit/unit time)
C_{2m} Manufacturer's per unit holding cost per unit time ($/unit/unit time)
C_{2r} Retailer's per unit holding cost per unit time ($/unit/unit time)
C_w Manufacturer's raw materials per unit cost ($/unit)
C_m Manufacturer's finished goods per unit cost ($/unit)
C_r Retailer's finished goods per unit cost ($/unit)
K_m Manufacturer's transportation cost per delivery ($/delivery)
K_r Retailer's incoming quality control cost per delivery ($/delivery)
TUC_w Raw material's total cost per unit time ($/unit time)
TUC_m Manufacturer's total cost per unit time ($/unit time)
TUC_r Retailer's total cost per unit time ($/unit time)
TUC Integrated total cost function including TUC_w, TUC_m and TUC_r per unit time ($/unit time)
F Retailer's maximum limited storage space
f Retailer's storage requirement for each unit

3. MODEL DEVELOPMENT

This integrated material flow is shown in Figure 1. Because we focused on manufacturer-retailer cooperation, there are two stages in our model. The first stage is the manufacturer's production system. The manufacturer procures raw materials from outside suppliers and delivers the fixed quantities to the manufacturers' warehouse at a fixed-time interval. The manufacturer withdraws raw materials from the warehouse to produce the finished goods. The second stage is the retailer's inventory system. Fixed quantities of finished goods with multiple delivery are delivered to the retailer at a fixed-time interval.

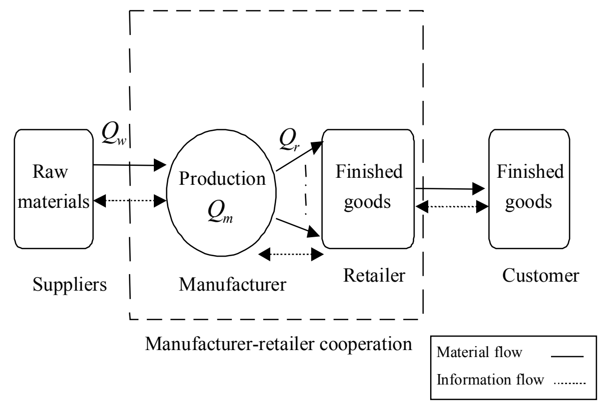

Figure 1. The integrated material flow

The raw material's inventory system is shown in Figure 2a. A supplier procures the raw materials and delivers the fixed quantities, Q_w, to the manufacturer's warehouse at a fixed-time interval. The manufacturer withdraws raw materials from the warehouse. During the T_1 time period, the inventory level decreases due to both the manufacturer's demand and deterioration.

The manufacturer's inventory system in Figure 2b can be divided into two independent phases depicted by T_1 to T_2. This methodology reduces the complexity in our problem derivation and analysis. Each phase has its own time unit, t_i, which starts from the beginning of the phase, T_i. During T_1 time period, there is an inventory buildup. At $t_1 = T_1$, the production stops and the inventory level increases to its maximum, I_m. There is no production during T_2 time period and the inventory level decreases due to demand and deterioration. The inventory level becomes zero at $t_2 = T_2$.

The change in raw material's inventory level is depicted in Figure 2c. At $t_3 = 0$, the initial replenishment is made in the retailer's inventory system. During T_3 time period, the inventory level decreases due to both demand and deterioration. At $t_3 = T_3$, the inventory level is zero. There are n delivery in T time period.

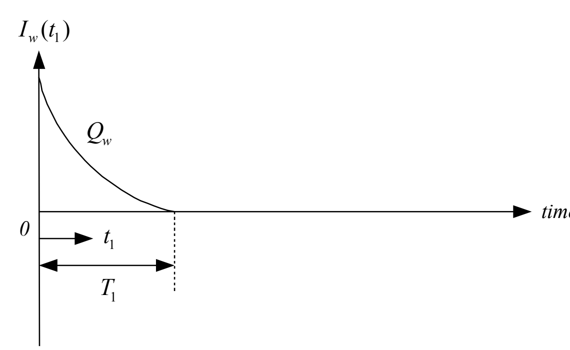

Figure 2a. The raw materials' inventory system

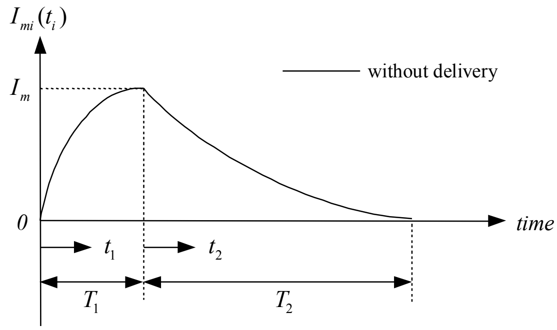

Figure 2b. The manufacturer's inventory system

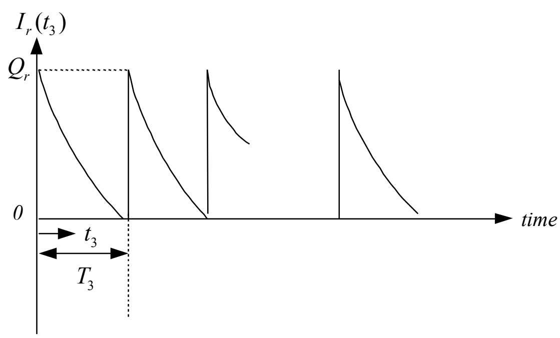

Figure 2c. The retailer's inventory system

The derivations of cost are provided in this section. It is difficult to formulate an exact solution when we used Weibull distribution in this model. To derive an approximation, we make the following assumptions. The cost can be computed by applying the Taylor's series expansion. For very small g and α values (g, $\alpha << 1$), the second and higher order terms of g and α are neglected. The derivation of each term is as follows:

3.1 Manufacturer's Raw Materials Inventory System

The manufacturer's raw materials inventory system at any time t can be represented by the following differential equation:

$$\frac{dI_w(t_1)}{dt_1} = -p - ght_1^{h-1} I_w(t_1), \quad 0 \le t_1 \le T_1 \tag{1}$$

Using the boundary condition, $I_w(T_1) = 0$, the first-order differential equation can be solved is

$$I_w(t_1) = pe^{-gt_1^h} \int_{t_1}^{T_1} e^{gu^h} du, \qquad 0 \le t_1 \le T_1 \tag{2}$$

Based on Figure 2a and $I_w(0) = Q_w$, the maximum inventory level of the raw materials, i.e., the order quantity per order from outside suppliers is

$$Q_w = p\int_0^{T_1} e^{gu^h} du = p\int_0^{T_1} \sum_{m=0}^{\infty} \frac{(gu^h)^m}{m!} du \approx p\left(T_1 + \frac{gT_1^{h+1}}{h+1} \right) \tag{3}$$

(1) At the start of the cycle, the cycle has an initial replenishment ordering cost. The ordering cost is

$$OR_w = C_{1w} \tag{4}$$

(2) Inventory occurs during T_1 time period. The holding cost is

$$\begin{aligned} HD_w &= C_{2w} \int_0^{T_1} I_w(t_1) dt \\ &= C_{2w} \int_0^{T_1} \{ pe^{-gt_1^h} \int_{t_1}^{T_1} e^{gu^h} du \} dt_1 \\ &= C_{2w} p \int_0^{T_1} \left\{ \left[\sum_{m=0}^{\infty} \frac{(-gt_1^h)^m}{m!} \right] \int_{t_1}^{T_1} \left[\sum_{m=0}^{\infty} \frac{(gu^h)^m}{m!} \right] du \right\} dt_1 \\ &\approx C_{2w} p \left\{ \frac{T_1^2}{2} + \frac{ghT_1^{h+2}}{(h+1)(h+2)} \right\} \end{aligned} \tag{5}$$

(3) The item cost includes loss due to deterioration as well as the cost of the item sold. The item cost is

$$IT_w = C_w Q_w \approx C_w p \left(T_1 + \frac{gT_1^{h+1}}{h+1} \right) \tag{6}$$

The total cost during the cycle is the sum of the ordering cost (OR_w), the holding cost (HD_w) and the item cost (IT_w). For raw materials, the total cost per unit time is

$$TUC_w = \frac{OR_w + HD_w + IT_w}{T} \tag{7}$$

3.2 Manufacturer's Finished Goods Inventory System

In Figure 2b, the inventory system without delivery can be represented by the following differential equation:

$$\frac{dI_{m1}(t_1)}{dt_1} = p - d - \alpha\beta t_1^{\beta-1} I_{m1}(t_1) \qquad 0 \le t_1 \le T_1 \tag{8}$$

$$\frac{dI_{m2}(t_2)}{dt_2} = -d - \alpha\beta t_2^{\beta-1} I_{m2}(t_2) \qquad 0 \le t_2 \le T_2 \tag{9}$$

The first-order differential equations can be solved by using the boundary conditions $I_{m1}(0) = 0$, $I_{m2}(T_2) = 0$, one has

$$I_{m1}(t_1) = (p-d)e^{-\alpha t_1^\beta} \int_0^{t_1} e^{\alpha u^\beta} du, \quad 0 \le t_1 \le T_1 \tag{10}$$

$$I_{m2}(t_2) = de^{-\alpha t_2^\beta} \int_{t_2}^{T_2} e^{\alpha u^\beta} du, \qquad 0 \le t_2 \le T_2 \tag{11}$$

Based on Figure 2b and $I_{m2}(0) = I_m$, the maximum inventory level of the finished goods is

$$\begin{aligned} I_m &= d\int_0^{T_2} e^{\alpha u^\beta} du = d\int_0^{T_2} \left(\sum_{m=0}^{\infty} \frac{(\alpha u^\beta)^m}{m!} \right) du \\ &\approx d\left(T_2 + \frac{\alpha T_2^{\beta+1}}{\beta+1} \right) \end{aligned} \tag{12}$$

The production quantity is

$$Q_m = pT_1 \tag{13}$$

(1) The cycle has an initial production set-up cost, C_{1m}, at the start of the cycle. The set-up cost includes the transportation cost is

$$SE = C_{1m} + nK_m \tag{14}$$

(2) Inventory is carried during T_1 and T_2 time periods. The last term in equation (15) is the holding cost of the items that are delivered to the retailer. The present worth of holding cost is

$$\begin{aligned}
& HD_m \\
&= C_{2m}\left\{\int_0^{T_1} I_1(t_1)\,dt_1 + \int_0^{T_2} I_2(t_2)\,dt_2\right\} \\
&\quad -n\left\{C_{2m}\int_0^{T_3} I_r(t_3)\,dt_3\right\} \\
&= C_{2m}\int_0^{T_1}\left\{(p-d)\,e^{-\alpha t_1^\beta}\int_0^{t_1} e^{\alpha u^\beta}du\right\}dt_1 \\
&\quad +C_{2m}\int_0^{T_2}\left\{de^{-\alpha t_2^\beta}\int_{t_2}^{T_2} e^{\alpha u^\beta}du\right\}dt_2 \\
&\quad -n\left\{C_{2m}\int_0^{T_3}\left[de^{-\alpha t_3^\beta}\int_{t_3}^{T_3} e^{\alpha u^\beta}du\right]dt_3\right\} \\
&= C_{2m}(p-d)\int_0^{T_1}\left\{\left[\sum_{m=0}^{\infty}\frac{(-\alpha t_1^\beta)^m}{m!}\right]\int_0^{t_1}\left(\sum_{m=0}^{\infty}\frac{(\alpha u^\beta)^m}{m!}\right)du\right\}dt_1 \\
&\quad +C_{2m}d\left\{\int_0^{T_2}\left[\sum_{m=0}^{\infty}\frac{(-\alpha t_2^\beta)^m}{m!}\right]\left[\int_{t_2}^{T_2}\sum_{m=0}^{\infty}\frac{(\alpha u^\beta)^m}{m!}du\right]dt_2\right\} \\
&\quad -n\left\{C_{2m}\int_0^{T_3}\left[d\left(\sum_{m=0}^{\infty}\frac{(-\alpha t_3^\beta)^m}{m!}\right)\int_{t_3}^{T_3}\left(\sum_{m=0}^{\infty}\frac{(\alpha u^\beta)^m}{m!}\right)du\right]dt_3\right\} \\
&\approx C_{2m}(p-d)\left\{\frac{T_1^2}{2}-\frac{\alpha\beta T_1^{\beta+2}}{(\beta+1)(\beta+2)}\right\} \\
&\quad +C_{2m}d\left\{\frac{T_2^2}{2}+\frac{\alpha\beta T_2^{\beta+2}}{(\beta+1)(\beta+2)}\right\} \\
&\quad -nC_{2m}d\left\{\frac{T_3^2}{2}+\frac{\alpha\beta T_3^{\beta+2}}{(\beta+1)(\beta+2)}\right\}
\end{aligned} \tag{15}$$

(3) Production occurs during T_1 time period. The item cost includes loss due to deterioration as well as the cost of the item sold. The item cost is

$$IT_m = C_m Q_m = C_m pT_1 \tag{16}$$

The total cost during the cycle is the sum of the set-up cost (SE), the holding cost (HD_m) and the item cost (IT_m). The present worth total cost per unit time is

$$TUC_m = \frac{SE + HD_m + IT_m}{T} \tag{17}$$

3.3 Retailer's Inventory System

The retailer's inventory system can be represented by the following differential equation:

$$\frac{dI_r(t_3)}{dt_3} = -d - \alpha\beta t_3^{\beta-1} I_r(t_3), \qquad 0 \le t_3 \le T_3 \tag{18}$$

Using the boundary condition, $I_r(T_3) = 0$, the first-order differential equation can be solved. One has

$$I_r(t_3) = de^{-\alpha t_3^\beta}\int_{t_3}^{T_3} e^{\alpha u^\beta}du, \qquad 0 \le t_3 \le T_3 \tag{19}$$

From Figure 2c and $I_r(T_3) = 0$, the retailer's maximum inventory level is the received quantity per delivery from the manufacturer. One has

$$\begin{aligned}
Q_r &= d\int_0^{T_3} e^{\alpha u^\beta}du = d\int_0^{T_3}\left(\sum_{m=0}^{\infty}\frac{(\alpha u^\beta)^m}{m!}\right)du \\
&\approx d\left(T_3 + \frac{\alpha T_3^{\beta+1}}{\beta+1}\right)
\end{aligned} \tag{20}$$

(1) The delivery has an initial ordering cost, C_{1r}, at the start of the delivery. The ordering cost includes incoming quality cost is

$$OR_r = C_{1r} + K_r \tag{21}$$

(2) Inventory occurs during T_3 time period. The holding cost is

$$\begin{aligned}
HD_r &= C_{2r}\left\{\int_0^{T_3} I_{r1}(t_3)dt_3\right\} \\
&= C_{2r}\int_0^{T_3}\left\{de^{-\alpha t_3^\beta}\int_{t_3}^{T_3} e^{\alpha u^\beta}du\right\}dt_3 \\
&= C_{2r}d\int_0^{T_3}\left\{\left[\sum_{m=0}^{\infty}\frac{(-\alpha t_3^\beta)^m}{m!}\right]\int_{t_3}^{T_3}\left[\sum_{m=0}^{\infty}\frac{(\alpha u^\beta)^m}{m!}\right]du\right\}dt_3 \\
&\approx C_{2r}d\left\{\frac{T_3^2}{2}+\frac{\alpha\beta T_3^{\beta+2}}{(\beta+1)(\beta+2)}\right\}
\end{aligned} \tag{22}$$

(3) The item cost includes loss due to deterioration as well as the cost of the item sold. The item cost is

$$IT_r = C_r Q_r \approx C_r d\left(T_3 + \frac{\alpha T_3^{\beta+1}}{\beta+1}\right) \tag{23}$$

The total cost during the delivery is the sum of the ordering cost (OR_r), the holding cost (HD_r) and the item cost (IT_r). The total cost per delivery is

$$TUC_r' = \frac{OR_r + HD_r + IT_r}{T} \tag{24}$$

There are n delivery per cycle. The fixed-time interval between the delivery is $T_3 = T/n$. Therefore, the total cost per cycle is

$$TUC_r = nTUC_r' = n\left(\frac{OR_r + HD_r + IT_r}{T}\right) \quad (25)$$

The time periods can be computed by applying the Taylor's series expansion. In order to solve the objective function, represent T_1 by T_2. From $I_m = I_{m1}(T_1) = I_{m2}(0)$, one has

$$(p-d)\, e^{-\alpha T_1^\beta} \int_0^{T_1} e^{\alpha u^\beta} du = d\int_0^{T_2} e^{\alpha u^\beta} du \quad (26)$$

For a very small α value, the second and higher order terms of α are neglected. Equation (26) can be simplified as

$$(p-d)(1-\alpha T_1^\beta)\left(T_1 + \frac{\alpha T_1^{\beta+1}}{\beta+1}\right) \approx d\left(T_2 + \frac{\alpha T_2^{\beta+1}}{\beta+1}\right) \quad (27)$$

Since T_1 in (27) is a high-power equation, it is difficult to solve analytically for the value of T_1. When $\alpha T_1 << 1$, the method used in Misra (1975), where $\alpha\beta T_1^{\beta+1}/\beta+1$ is neglected, results in the following approximate value for T_1

$$T_1 \approx \frac{d}{p-d}\left(T_2 + \frac{\alpha T_2^{\beta+1}}{\beta+1}\right) \quad (28)$$

The objective of this study is to develop an integrated production-inventory model from the perspectives of both the manufacturer and the retailer. For very small g and α values ($g, \alpha << 1$), an approximate model with multiple delivery is developed to derive the optimal production policy and lot-size. The total cost per unit time of the manufacturer and the retailer is the sum of TUC_w, TUC_m and TUC_r. Since $T_3 = T/n$ and $T = T_1 + T_2$, the objective function can be stated as $TUC(n, T_2) = TUC_w + TUC_m + TUC_r$. If the retailer's storage space limited to a maximum inventory size of F, the problem can be stated as

Minimize: $TUC(n, T_2) = TUC_w + TUC_m + TUC_r$ (29)

Subject to: $fQ_r \le F$ (30)

The optimal inventory levels can be derived by using Lagrange multipliers, one has

$$H(n, \lambda, T_2) = TUC(n, T_2) + \lambda(fQ_r - F) \quad (31)$$

4. SOLUTION PROCEDURE

The following technique is used to derive the optimal value of n and T_2;

Step 1 Since the number of delivery per order, n, is an integer value, start by choosing an integer value of $n \ge 1$.

Step 2 Take the partial derivatives of $H(n, \lambda, T_2)$ with respect to λ and T_2, and equate the results to zero. The necessary conditions for optimality are

$$\frac{\partial H(n, \lambda, T_2)}{\partial \lambda} = 0 \text{ and } \frac{\partial H(n, \lambda, T_2)}{\partial T_2} = 0$$

The simultaneous equations above can be solved for λ and T_2.

Step 3 Using λ and T_2 found at step 2, substitute into equation (31) and derive $H(n, \lambda, T_2)$.

Step 4 Repeat steps 2 and 3 for all possible n values until the minimum $H(n^*, \lambda^*, T_1^*)$ is found. The $H(n^*, \lambda^*, T_1^*)$ values constitute the optimal solution that satisfy the following conditions:

$H(n^* - 1, \lambda, T_2) \ge H(n^*, \lambda^*, T_2^*)$, and
$H(n^*, \lambda^*, T_2^*) \le H(n^* + 1, \lambda, T_2)$.

Step 5 Derive the T_1^*, T_3^*, T^*, Q_w, Q_m, Q_r, TUC_w, TUC_m, TUC_r and TUC^*.

5. NUMERICAL EXAMPLE

Optimal production and replenishment policy to minimize the total cost per unit time may be obtained by using the methodology proposed in the preceding sections. The following numerical example is illustrated the model.

Let production rate $p = 2{,}000{,}000$; demand rate $d = 500{,}000$; ordering cost $C_{1w} = \$1{,}000$, $C_{1r} = \$2{,}000$; set-up cost $C_{1m} = \$100{,}000$; transportation cost $K_m = 1{,}000$; quality-control cost $K_r = 500$; carrying cost $C_{2w} = \$20$, $C_{2m} = \$40$, $C_{2r} = \$60$; item cost $C_w = \$200$, $C_m = \$400$, $C_r = \$600$; deterioration rate $g = 0.1$, $h = 2$, $\alpha = 0.2$, $\beta = 2$; maximum limited storage space $F = 2{,}000$; storage requirement for each unit $f = 2$.

The computational results are shown in Table 1. Table 2 is the comparison of results for the above special conditions

The major conclusions and the special conditions drawn from the numerical example are as follows:

(1) In this example, TUC^* is $\$6037.31 \times 10^5$ while the optimal values of n^*, T_1^*, T_2^* and T_3^* are 49, 245.07×10^{-4}, 734.93×10^{-4} and 20.00×10^{-4} respectively. For the raw materials, Q_w is 49.14.20 units. As for the finished goods, Q_m is 49013.22 units and Q_r is 1000.00 units.
(2) Since TUC is a very complicated function, it is not possible to show analytically the validity of the above sufficient conditions. A graphical representation and numerical analysis are used to show the convexity of the TUC. Based on the above analysis and the graphical representation of Figure 3, one can say that TUC is a convex function.
(3) When n increases, T_1 and T_2 will increase. At this time, the time period between delivery, T_3, will

Table 1. The numerical results for illustrate example

n	λ	$T_1(10^{-4})$	$T_2(10^{-4})$	$T_3(10^{-4})$	$TUC_w(10^5)$	$TUC_m(10^5)$	$TUC_r(10^5)$	$TUC(10^5)$
10	3303.96	50.00	150.00	20.00	1000.76	2056.32	3012.80	6069.89
12	5864.10	60.00	180.00	20.00	1000.74	2048.30	3012.80	6061.83
13	2692.63	65.00	195.00	20.00	1000.73&	2045.25	3012.80	6058.78
14	2544.20	70.00	210.00	20.00	1000.74	2042.66	3012.80	6056.19
20	1943.56	100.00	300.00	20.00	1000.80	2032.89	3012.80	6046.49
30	1417.88	150.02	449.98	20.00	1001.03	2026.17	3012.80	6040.00
40	1095.64	200.04	599.96	20.00	1001.32	2023.66	3012.80	6037.78
48	895.95	240.06	719.94	20.00	1001.58	2022.94	3012.80	6037.32
49	***873.09***	***245.07***	***734.93***	***20.00***	***1001.62***	***2022.90***	***3012.80***	***6037.31****
50	850.59	250.07	749.93	20.00	1001.65	2022.86	3012.80	6037.32
53	785.00	265.08	794.92	20.00	1001.76	2022.82	3012.80	6037.38
54	763.71	270.09	809.91	20.00	1001.80	2088.81#	3012.80	6037.41
55	742.66	275.09	824.91	20.00	1001.83	2022.82	3012.80	6037.46
60	640.63	300.12	899.87	20.00	1002.02	2022.95	3012.80	6037.77
70	447.71	350.19	1049.81	20.00	1002.42	2023.55	3012.80	6038.77
80	262.81	400.29	119.97	20.00	1002.84	2024.50	3012.80	6040.14

& : The optimal solution from the raw material's view (TUC_w)
\# : The optimal solution from the manufacturer's view (TUC_m)
* : The optimal solution from the integrated view (TUC)

Table 2. Comparison of results for special conditions

	n	λ	$T_1(10^{-4})$	$T_2(10^{-4})$	$T_3(10^{-4})$	$TUC_w(10^5)$	$TUC_m(10^5)$	$TUC_r(10^5)$	$TUC(10^5)$
Our example	49	873.09	245.07	734.93	20.00	1001.62	2022.90	3012.80	6037.31
Single delivery	1	26108.60	5.00	15.00	20.00	1005.02	2504.90	3012.80	6522.73
$g=0$, $\alpha=0$	54	865.09	270.00	810.00	20.00	1001.44	2022.16	3012.80	6036.40
$h=1$, $\beta=1$	31	822.08	155.51	464.37	20.00	1005.20	2032.55	3013.40	6051.15

remain unchanged. When the production quantity increases, the number of the delivery increases.

(4) If the optimal solution is derived solely from the raw material's view, n^* is 13 and TUC^* is $\$6058.78\times$ $\times 10^5$, an increase of $\$21.47\times 10^5$ per unit time as compared with our example. If the optimal solution is derived solely from the manufacturer's view, n^* is 54 and TUC^* is $\$6037.41\times 10^5$, an increase of $\$0.1\times 10^5$ per unit time as compared with our example. The decision to include the viewpoints of both of the retailer and the manufacturer results in a lower TUC.

(5) When there is a single delivery (i.e. $n=1$), TUC^* is $\$6522.73\times 10^5$. An increase of $\$485.42\times 10^5$ per unit time as compared with our example.

(6) When deterioration is not considered (i.e. g, $\alpha=0$), n^* is 54 and TUC^* is $\$6036.40\times 10^5$. A decrease of $\$0.91\times 10^5$ per unit time as compared with our example.

(7) When the item deteriorates exponentially (i.e. h, $\beta=1$), n^* is 31 and TUC^* is $\$6051.15\times 10^5$. An increase of $\$13.84\times 10^5$ per unit time as compared with our example.

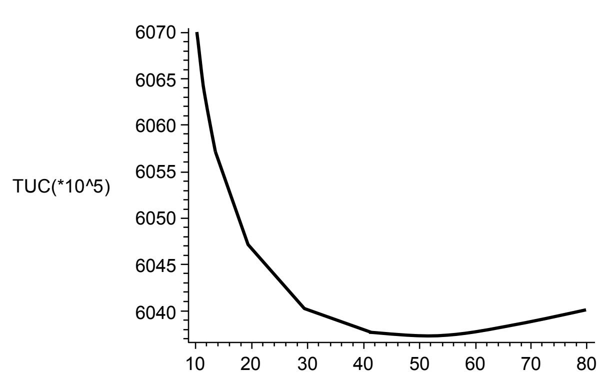

Figure 3. Graphical representation of a convex TUC

6. SENSITIVITY ANALYSIS

In order to study how the parameters affect the optimal solution, we conduct the sensitivity analysis for all parameters. For the numerical example given above, we derive the optimal values of n^* and TC^* for a fixed subset $S = \{p, d, C_{1w}, C_{1m}, C_{1r}, K_m, K_r, C_{2w}, C_{2m}, C_{2r}, C_w, C_m, C_r, g, h, \alpha, \beta, f, F\}$. The base column of S is $S = \{2{,}000{,}000,\ 500{,}000,\ 1{,}000,\ 100{,}000,\ 2{,}000,\ 1{,}000,\ 500,\ 20,\ 40,\ 60,\ 200,\ 400,\ 600,\ 0.1,\ 2,\ 0.2,\ 2,\ 2{,}000,\ 2\}$. The optimal values of n^0, T_1^0, T_2^0 and TUC^0 are derived when one of the parameters in the subset S increases or decreases by 5% while all the other parameters remain unchanged. The results of T_1^0 and T_2^0 are presented in Table 2. The results of the sensitivity analysis are shown in Table 3. The percentage of cost increase index (PCI) is defined as

$$PCI = \frac{TUC^0 - TUC^*}{TUC^*} \times 100\%$$

The main conclusions draws from the sensitivity analysis are as follow:

(1) Because d affects the two stages of the system directly, the value of PCI is the most sensitive to d. When d increases by 10%, the value of PCI increases by about 10%. The values of PCI are more sensitive to the parameters d, C_w, C_m and C_r.

(2) The values of PCI are least sensitive to the parameters p, C_{1w}, K_r, C_{2w}, C_{2r}, g, h and α. When these parameters increase by 10%, the value of PCI increases by less 0.01%. C_{1w} and K_r influence only the ordering cost. C_{2w} and C_{2r} influence only the holding cost. g and h influence only the raw material.

(3) The parameters p, d, C_{1w}, C_{1m}, C_{1r}, K_m, K_r, C_{2w}, C_{2m}, C_{2r}, C_m, C_r, C_r, α and f influence the value of PCI in the same direction. The parameters h, β and F influence the value of

Table 2. The sensitivity analysis of T_1 and T_2

	-10% changed			-5% changed			+5% changed			+10% changed		
	λ	T_1^0	T_2^0	λ	T_1^0	T_2^0	λ	T_1^0	T_2^0	λ	T_1^0	T_2^0
p	866.12	277.85	722.15	880.31	257.96	722.04	866.42	233.40	746.40	860.24	222.79	757.21
d	778.65	230.07	792.15	814.69	240.07	770.5	908.36	255.06	716.36	965.33	260.06	685.40
C_{1w}	872.58	245.07	734.97	872.84	245.07	734.93	873.35	245.07	734.93	873.60	245.07	734.93
C_{1m}	866.01	235.06	704.94	869.91	235.06	719.94	875.59	250.07	749.93	877.44	255.07	764.93
C_{1r}	823.09	245.07	734.93	848.09	245.07	734.93	898.09	245.07	734.93	923.09	245.07	734.93
K_m	848.09	245.07	734.93	860.59	245.07	734.93	885.59	245.07	734.93	898.09	245.07	734.93
K_r	860.59	245.07	734.93	866.84	245.07	734.93	879.34	245.07	734.93	885.59	245.07	734.93
C_{2w}	879.23	245.07	734.93	876.16	245.07	734.93	870.02	245.07	734.93	866.96	245.07	734.93
C_{2m}	865.71	255.07	764.93	868.86	250.07	749.93	878.44	240.06	719.94	860.93	240.06	719.94
C_{2r}	874.59	245.07	734.93	873.84	245.07	734.93	872.34	245.07	734.93	871.59	245.07	734.93
C_w	875.99	245.07	734.93	874.54	245.07	734.93	871.64	245.07	734.93	870.19	245.07	734.93
C_m	878.49	245.07	734.93	875.79	245.07	734.93	870.39	245.07	734.93	867.69	245.07	734.93
C_r	873.10	245.07	734.93	873.10	245.07	734.93	873.09	245.07	734.93	873.08	245.07	734.93
g	873.29	245.07	734.93	873.19	245.07	734.93	872.99	245.07	734.93	872.89	245.07	734.93
h	871.04	245.07	734.93	872.24	245.07	734.93	873.69	245.07	734.93	874.11	245.07	734.93
α	881.23	245.06	734.94	877.16	245.06	734.94	869.02	245.07	734.93	864.95	245.07	734.93
β	871.23	235.11	704.89	874.15	240.08	719.92	868.90	250.05	749.95	862.26	255.04	764.96
f	787.54	244.51	733.27	818.37	247.44	742.04	905.26	247.69	742.79	959.00	245.52	736.30
F	1066.15	247.57	742.43	958.27	247.07	740.93	784.05	246.82	740.18	710.67	247.57	742.43

* : $T_i \times 10^{-4}$, $i = 1, 2$

Table 3. The percentage cost increase (PCI) of the parameters

	-10% changed			-5% changed			+5% changed			+10% changed		
	n	TUC^0	PCI	n	TUC^0	PCI	n	TUC^0	PCI	n	TUC^0	PCI
p	50	6037.09	0.00	49	6037.21	0.00	49	6037.41	0.00	49	6037.50	0.00
d	46	5434.79	-9.98	48	5736.06	-4.99	51	6338.54	4.99	52	6639.78	9.98
C_{1w}	49	6037.30	0.00	49	6037.31	0.00	49	6037.32	0.00	49	6037.32	0.00
C_{1m}	47	6036.27	-0.02	48	6036.80	-0.01	50	6037.82	0.01	51	6038.31	0.02
C_{1r}	49	6036.31	-0.02	49	6036.81	-0.01	49	6037.81	0.01	49	6038.31	0.02
K_m	49	6036.81	-0.01	49	6037.06	0.00	49	6037.56	0.00	49	6037.81	0.01
K_r	49	6037.06	0.00	49	6037.19	0.00	49	6037.44	0.00	49	6037.56	0.00
C_{2w}	49	6037.19	0.00	49	6037.25	0.00	49	6037.38	0.00	49	6037.44	0.00
C_{2m}	51	6036.58	-0.01	50	6036.95	-0.01	48	6037.67	0.01	48	6038.02	0.01
C_{2r}	49	6037.28	0.00	49	6037.30	0.00	49	6037.33	0.00	49	6037.34	0.00
C_w	49	5937.29	-1.66	49	5987.30	-0.83	49	6087.33	0.83	49	6137.34	1.66
C_m	49	5837.26	-3.31	49	5937.29	-1.66	49	6137.34	1.66	49	6237.37	3.31
C_r	49	5737.31	-4.97	49	5887.31	-2.48	49	6187.31	2.48	49	6337.31	4.97
g	49	6037.31	0.00	49	6037.31	0.00	49	6037.32	0.00	49	6037.32	0.00
h	49	6037.34	0.00	49	6037.32	0.00	49	6037.31	0.00	49	6037.30	0.00
α	49	6037.23	0.00	49	6037.27	0.00	49	6037.36	0.00	49	6037.40	0.00
β	47	6037.95	0.01	48	6037.59	0.00	50	6037.10	0.00	51	6036.94	-0.01
f	44	6035.58	-0.03	47	6036.45	-0.01	52	6038.19	0.01	54	6039.06	0.03
F	55	6039.25	0.03	52	6038.23	0.02	47	6036.49	-0.01	45	6035.73	-0.03

* : $TUC^0 \times 10^5$

PCI in the opposite direction.

(4) The value of n is more sensitive to the parameters d, C_{1m}, C_{2m}, β, f and F. The higher parameter value of d, C_{1m}, β and f result in the higher value of n. The value of T_1 and T_2 are more sensitive to the parameters p, d, C_{1m}, C_{2m}, β and F.

7. CONCLUSION

This study develops an optimal joint cost from the perspectives of both the manufacturer and the retailer. The integrated production-inventory model with Weibull distribution deteriorating items is assumed to have a constant demand rate and a limited retailer storage space. A numerical example including the sensitivity analysis is given to validate the results of the production-inventory model.

Multiple delivery is one of the important policies of a successful enterprise. The integrated decision results in lower optimal joint cost compared with an independent decision by the manufacturer. The study is particularly useful for the inventory systems where the manufacturers and their retailers form a strategic alliance with a mutually beneficial objective. To make it acceptable to both parties, the integrated policy should offer some kind of profit sharing policy. The profit sharing policy can be in the form of advanced payment or quantity discounts. As a result of this policy, both the manufacturer and the retailer will benefit in the long run. The application of profit sharing policy is an area worthy of studying for future research. Future research can be done for multi-manufacturer-retailer chain as well as considering a warranty policy in the modeling.

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