# Optimal Electric Energy Subscription Policy for Multiple Plants with Uncertain Demand

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**Abstract.** This paper present a new optimization model to generate aggregate production planning by considering electric cost. The new Time Of Switching (TOS) electric type is introduced by switching over Time Of Day (TOD) and Time Of Use (TOU) electric types to minimize the electric cost. The fuzzy demand and Dynamic inventory tracking with multiple plant capacity are modeled to cover the uncertain demand of customer. The constraint for minimum hour limitation of plant running per one start up event is introduced to minimize plants idle time. Furthermore; the Optimal Weight Moving Average Factor for customer demand forecasting is introduced by monthly factors to reduce forecasting error. Application is illustrated for multiple cement mill plants. The mathematical model was formulated in spreadsheet format. Then the spreadsheet-solver technique was used as a tool to solve the model. A simulation running on part of the system in a test for six months shows the optimal solution could save 60% of the actual cost

Keywords: Production Planning, Stochastic Process, Optimization, TOD, TOU, Fuzzy Demand

#### 1. INTRODUCTION

The energy cost for local operations in the cement industry in Thailand was around 60~64% of production cost during year 2001~2003. The energy cost depended on size and technologies used of the system and 35~38% of energy cost was electric cost. The electric cost for seven cement mill plants to produce three types of cement during first six months in year 2004 was more than 291 million Baht. The increasing of electric cost in cement production and customers demand lead the entrepreneurs to find out a methodology to balance the energy cost and the market demands. A number of researches suggested methods to reduce electric cost. The methods ranged from heuristic method to mathematical model methods. The latter methods have been studied by using several techniques such as linear programming (Jowitt and Germanopoulos, 1992), dynamic programming (Coulback and Orr, 1989), and nonlinear programming (Tsay and Lin, 2000). A mixed-integer programming models is introduced to analyze the customer incentive to install cogeneration system in the peak period of TOU (Tsay and Lin, 2000). An optimization model developed to generate pump schedule for water supply in Pittsburgh could have saved 20% of the actual cost (Vilas *et al.*, 1996).

This paper present a new optimization model to generate aggregate production planning by considering electric cost. The new TOS electricity is introduced by switching over TOD and TOU electricity to minimize the electric cost. The fuzzy demand and Dynamic inventory tracking with multiple plant capacity are modeled to cover the uncertain demand of customer. The constraint for minimum hour limitation of plant running per one start up event is introduced to minimize plants idle time. Furthermore; the Optimal Weight Moving Average Factor for customer demand forecasting is introduced by monthly factors to reduce forecasting error.

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#### 2. LITERATURE REVIEW

#### 2.1 Production Planning

Many production planning methods have been suggested to reduce production costs such as linear cost models, quadratic cost models, fixed cost models, and nonlinear cost models (Hax, 1984). Whist Hoffman and Jacobe (1985) initiate a linear cost model for a single product. Multi stage linear programming models included multiple routings, multiple sources, product mix decision models, and multiple production and decision. In addition, Candea (1977) introduces a linear cost model for a single product. A mathematical programming model with aggregate production planning and machine requirement planning was introduced, the performance of model can be compared with those of the individual aggregate production and machine planning models (Behnezhad and Khoshnevis, 1996). An aggregate production planning model with demand under uncertainty, the demand is presented by a compound Poisson process and a Gaussian approximation is proposed to the compound Poisson process (Silva, 1999). The goal programming is modified to allow a simultaneous solution of a system of complex objectives rather than a single objective (Lee, 1972). Leung et al., (2003) expressed multi-site aggregate production planning with multiple objectives presented by a goal programming approach. The multiple linear programming with the set of efficient extreme point in the decision space obtained by parametric programming models (Bryson, 1991), (Steuer, 1985), and (Yu, 1985). The integrated model for scheduling and planning in a multi-site in order to determine the feasible plan was described by Scswartz (1984). The goal programming model can provide an excellent approximation presented a robust optimization model to solve the aggregate production planning problem in an environment of uncertainty. The production cost, labor cost, and hiring and layoff cost are minimized (Leung and Wu, 2004).

Wang and Fang (2001b) developed a linear programming model with fuzzy parameters such as fuzzy demand, fuzzy machine time, fuzzy machine capacity and fuzzy relevant cost. Wang and Fang (2001a) proposed a model which included fuzzy objective coefficient and fuzzy variable in the aggregate production planning, the solutions presented by fuzzy number with more flexibility to decision-maker with an uncertain environment. Hop and Tabucanon (2000) considered the grouping problem of electronics component families based on their multiple attributes by introducing the concept of fuzziness to express the degree of concordance on the decision.

#### 2.2 Electric Cost

Price of electric power demand depends on two sections of cost i.e. fixed cost and running cost. Firstly,

fixed cost is the capital cost of power station, substation, and the connection between them with the end of utilization. Secondly, running cost depends on cost of the power plant running including Fuel Tariff (FT) (Hamed, 2001). The price of electricity based on non-cooperative game theory by supposing that all of Independent Power Providers (IPPs) can jointly negotiate to supply electricity to a utility. It is reasonable for the utilities that they will purchase the electricity depending on price (Geer *et al.*, 2001).

The electric pricing of TOD is variable cost relating to different seasons in a year. It depends on maximum of monthly demand, and the energy consumption. A customer is able to improve his electric bill by managing electric usage time according to TOD rate charging (Harry, 1989). According to analysis of TOD electric pricing during daily peak demand in summer can provide information on customer response to time-differentiated electric prices. The customer proposed to reduce electric demand during this peak period (Aigner, 1985), (Aigner and Gliali, 1989), (Caves *et al.*, 1989), (Henley and Perison, 1994), (Train and Mehrez, 1994), (Ham *et al.*, 1997), and (Tishler and Lipovestsky, 1997).

The electric pricing of TOU is based on the estimated cost during a particular time block. TOU rate usually divided into three or four time blocks per 24 hours, i.e. on-peek, mid-peek, off-peak, and super offpeak. Most electric TOU studies have found little response, as measured by price elasticity, by business customer to TOU rate (Aigner and Hirschber, 1985), (Park and Action, 1984), (Aigner et al., 1994), and (Woo, 1985). Action and Park (1987) studied estimating individual customer response which mostly shows no change or very modest change in response to TOU prices, the survey found that some of hypotheses have been advanced to explain the pattern of response in large industries. A mixed-integer programming and nonlinear programming models is introduced to analyze customer incentive to install co-generation system in the peak period of TOU (Tsay and Lin, 2000). This customer behavior can be implied that they try to reduce as well as avoid consuming electricity during peak period. The significance of mathematical model is switching the own co-generator during peak period.

#### 2.3 Spreadsheet Solver

The spreadsheet solver is an add-in feature found in recent versions of spreadsheet software such as Lotus 1-2-3, Borland's Quattro Pro, and Microsoft Excel (Winston, 1994). This feature capable solving linear and nonlinear programming problems. Demonstration of spreadsheet solver to solve small aggregate production planning problems described by Albright *et al.* (1999). Demonstrating the usage of a spreadsheet-solver technique in solving single product type expressed by Techawiboonwong and Yenradee (2002). Techawiboonwong and Yenradee (2003) formulated a mathematical model in

spreadsheet format and then used the spreadsheetsolving technique to solve the aggregate production planning for multiple product types where the worker resource can be transferred among the production lines.

There are several interesting references provide other potential approaches and other viewpoints to save cost. The production planing is emphasized to reduce production cost, labor cost, and hiring and firing cost. Load shifting and co-generation was present for electric cost saving. Spreadsheet Solver is used to solve single and multiple product types.

#### 3. LECTRIC SUPPLY

The electric supply based on non-cooperative game theory by supposing that all of IPPs can jointly negotiate to supply electricity both TOD and TOU. It is reasonable for the utilities that they will purchase the electricity depending on price and other situation or factors.

The electric user will have the contract with IPPs either TOD or TOU rates. Single line diagrams of TOD and TOU are the same that can be described as Figure 1, the difference of TOD and TOU meters based on TOD or TOU contract.

There are three different types of electric charge for industry based on electric demand. Those are small size, medium size, and large size industry. This research focuses on investigation of large size industry with electric supply  $\geq 69 \text{ Kilo-Volts (KV)}$ , i.e. 115 KV.

# Electric Power Supply From IPPs

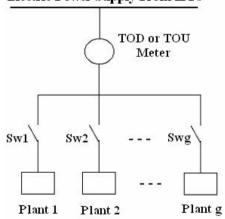


Figure 1. TOD or TOU Electric power supply

There are three different types of electric charge for industry based on electric demand. Those are small size, medium size, and large size industry. This research focuses on investigation of large size industry with electric supply  $\geq$  69 Kilo-Volts (KV), i.e. 115 KV.

Large size industry consumes the average electricity  $\geq 1,000$  KW.Hr within 15 minutes by measuring through a single Watt-meter while average consumption

in three months is more than 250,000 KW.Hr. Electric cost can be separated into two rates i.e. TOD and TOU as described in Table 1 and Table 2.

**Table 1.** TOD rate for large size industry

Voltage	Deman	d Charge (E	Energy Charge		
(KV)	Peak	Partial	Off-Peak	(Baht/KW.Hr)	
< 22	332.71	68.22	0	1.7314	
22-23	285.05	58.88	0	1.7034	
≥ 69	224.30	29.91	0	1.6660	

Note: - Peak : 6:30 p.m. - 9:30 p.m. (every day) - Patial : 8:00 a.m. - 6:30 p.m. (every day) - Off-Peak : 9:30 p.m. - 8:00 a.m. (every day)

Time blocks of TOD defined by certain time of a day and every day are the same. Time blocks of TOU defined by electric demand in a day by separate working days and holidays.

Normally, the electric demand of TOD in Off-Peak period is low. Therefore; IPPs stimulated their customer to use electricity in these periods by omitting the demand charge as shows in Table 1.

**Table 2.** TOU rate for large size industry

Voltage	Demand Charge	Energy Charge (Baht/KW.Hr) Peak Off-Peak		Service Charge
(KV)	(Baht/KW)			(Baht/month)
< 22	210.00	2.8408	1.2246	228.17
22-23	132.93	2.6950	1.1914	228.17
≥69	74.14	2.6136	1.1726	228.17

Note

Timing diagram over 24 hours per day of TOU and TOD can be described in Figure 2. TOD identifies period of 24.00~08.00 and 21.30~24.as Off-Peak hours, but period of 08.00~18.30 is Partial Peak hours, and period of 18.30~21.00 is Peak. On the other hand for TOU, period time 24.00~09.00 and 22.00~24.00 are Off-Peak hours, and 09.00~22.00 is Peak. The different perception on time of electric consuming between TOD and TOU induced the author to find out the electric cost minimization method.

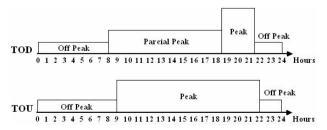


Figure 2. TOD and TOU time period

## 4. MODEL DEVELOPMENT FOR TOS

This article presents a new optimization model that considered the electric source switching, Dynamic inventory, and minimum limit hours of plant operation per one start up. The major objective is to minimize the electric cost while satisfying customer demands and system requirements. The optimization models compose TOU and TOD electric types to be the TOS electric type with hourly switching consideration while satisfying daily customer demands. The formulation of mathematical model is based on Mixed Integer Linear Programming (MILP). Triangular fuzzy demand is applied to cover the uncertainty of customer demand with several fuzzy factor adjustment. Daily demands have been combined to be a monthly demand. Weighted Moving Average is applied for demand forecasting while weighted factors optimization is presented to minimize forecasting error. To adjust inventory level based on Off-Peak hours, the Dynamic inventory constraint is presented. The continuous plant operation is introduced to avoid frequently start-stop during production.

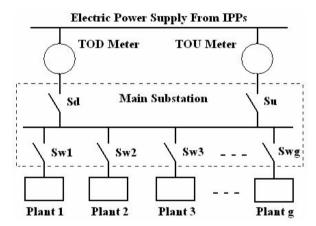


Figure 3. TOS Single line diagram

The Off-Peak periods of both electric types in Figure 2 are cheapest electric cost, but they are in the different time. If we can supply both electric power lines to electric user and switching them to select the cheapest cost for each hour, the electric bill can be decreased properly. This research created this switching method and was named as "Time of Switching: TOS". To support the TOS, single line diagram can be described in Figure 3. Where Sd, Su, and Sw are TOD switch, TOU switch and plant switches respectively.

#### 4.1 Model Formulation

To support the switching of TOS, the deterministic production model will be formulated. Decision variables and parameters for mathematical formulation will be assigned as below.

: Product type, i = 1, ..., 3 types.

: Electric Type, j = 1, 2 (TOD, TOU).

: Period hours, h = 1, 2, ..., 24 (hr).

: Planning horizon, t = 1 day.

: Number of plants running in period hour h, g = 1, 2, ..., 7 plants.

 $E_{gi}$ : Electricity used by plant g to produce one ton of product i (Kw/hr).

 $M_{ig}$ : Capacity of plant g to produce product i over 24 hours. (ton).

 $r_{ig}$ : Minimum running hours of plant g to produce product *i* per one starting event (*hr*).

: Initial start hour of plant g to produce product i

 $Ii_{max}$ : Maximum inventory stock of product i (ton).

 $Ii_{min}$ : Minimum inventory stock of product i (ton).

 $c_{jh}$ : Electric cost of electric type j in period hour h(**B**/KW.hr).

 $d_{it}$ : Demand of product i on daily t (ton).

## **Decision Variable**

Decision variable will consist of three parts as following

> : Quantity of product i produced in plant g  $X_{igh}$ in hour *h* (ton)

: Quantity of product i produced in a day t (ton)

: Inventory of product *i* in a day *t* (ton)

 $R_{jh} = \begin{cases} 1 & \text{;} & \text{If electric type } j \text{ selected in period } h \\ 0 & \text{;} & \text{Otherwise} \end{cases}$ 

 $P_{igh} = \begin{cases} 1 & \text{;} & \text{If plant } g \text{ selected to produce product } i \text{ in period } h \\ 0 & \text{;} & \text{Otherwise} \end{cases}$ 

The mathematical formulation to generate the switching of TOS with minimizing electric cost consideration can be described as following:

#### **Objective Function**

$$\operatorname{Min} \sum_{ijgh} c_{jh} R_{jh} X_{igh} E_{gi} P_{gih} \tag{1}$$

#### subject to

$$\sum_{gh} X_{igh} = X^{it}$$
  $\forall i, t$  (2)

$$X^{it} + I_{i,t-1} - I_{it} = d_{it}$$
  $\forall i, t$  (3)

$$I_{it} \ge Ii_{\min}$$
  $\forall i, t$  (4)

$$I_{it} \le Ii_{\text{max}}$$
  $\forall i, t$  (5)

$$\sum_{h} X_{igh} P_{igh} \le M_{ig} \qquad \forall i, g \quad (6)$$

$$\sum_{h=s_{ie}}^{h=r_{ig}} P_{igh} = r_{ig} \qquad \forall i, g \quad (7)$$

$$X^{it}, X_{igh}, d_{it}, I_{it} \ge 0$$
  $\forall i, g, h, t$  (8)  $R_{jh}, P_{igh} = 0, 1 (Binary)$   $\forall i, g, h, j$  (9)

Objective function (1) is proposed to minimize the electric cost. Inventory holding and labor cost are neglected to consider since the labor cost is least varying in production level and inventory holding cost is very low in cement industry. The electric cost occurs up to electric type selected in period hour h through decision variable  $R_{ih}$ . Constraint (2) is summation of product i produced in plant g in hours h, implied the number of product i produced in a day t. Constraint (3) is balance inventory of product i in planning horizon day t. Equation (4), (5) are minimum and maximum limit of inventory stock of product i. Equation (6) shows maximum quantity of product i can be produce in plant g over 24 hours. During starting up the plant; motors consumed high electricity. Therefore, start up event should least as possible and continuous plant running should long as possible. Equation (7) shows the minimum running hours  $r_{ig}$  of plant g to produce product i per one starting event. Plant g started on hour  $h = s_{ig}$  and continuous running to hour  $h = r_{ig}$ . Equation (8), (9) are nonnegative and binary constraints.

The planning period is hourly and planning horizon is one day. Equation (2) imply the summation over hourly production  $X_{igh}$  is the daily production  $X^{it}$ . By substituting equation (2) into production-inventory balance equation (3), we can have daily production-inventory balance in form of hourly production as following:

$$\sum_{g} \sum_{h=1}^{t} X_{igh} = \left[ d_{it} + I_{it} - I_{i,t-1} \right]$$
  $\forall i, t$  (10)

#### 4.2 Dynamic Inventory

Objective function (1) is proposed to minimize electric cost. Therefore, production level  $X^{it}$  in production-inventory balance equation (3) must be as small as possible. It mean that the production  $X^{it}$  will produce only to make production-inventory balance equation (3) in order to cover customer demand  $d_{it}$  and keep ending inventory  $I_{it}$  at lower bound of minimum inventory limit while the customer demand of next planning horizon t is out of consideration. If the customer demand in next planning horizon is large number, mathematical equation will force manufacturers to produce during peak period to cover their customer demand.

To increase the ending inventory  $I_{it}$  up from lower bound of minimum inventory limit  $I_{min}$ , the Dynamic inventory  $I_{it}^d$  is introduced. Level of Dynamic inven-

tory is related to plant capacity during lowest electricity cost hours  $X_{igh_L}$  on the planning day t. If customer demand  $d_{it}$  is not large, Dynamic inventory level will be large. On the other hand, if customer demand  $d_{it}$  is large, Dynamic inventory will lower. Relationship between plant capacity during hours of lowest electric cost and Dynamic inventory can be described in term of production-inventory balance as equation (11).

$$I_{it}^{d} = X_{igh_L} + I_{i,t-1} - d_{it}$$
  $\forall i, g, h, t$  (11)

Therefore, two possibilities of ending inventory level can occur. Firstly, if level of Dynamic inventory  $I_{ii}^d$  is less than minimum inventory limit  $I_{i_{\min}}$ , level of ending inventory limited by minimum inventory limit  $I_{i_t} \ge I_{i_{\min}}$  as equation (4). Secondly, if Dynamic inventory  $I_{ii}^d$  is more than minimum inventory limit  $I_{i_{\min}}$ , level of ending inventory limited by Dynamic inventory  $I_{ii} \ge I_{ii}^d$ . Thus, the Dynamic inventory is always less than ending inventory as described in equation (12).

$$I_{it} \ge I_{it}^d \qquad \forall i, t \quad (12)$$

where

 $X_{igh_L}$ : Capacity of plant g to produce product i in lowest electric cost hours  $h_L$ 

 $I_{it}^d$ : Dynamic inventory of product i in period t

#### 4.3 Fuzzy Demand

In real world, most customer demands are not deterministic, they are uncertainty. This mathematical model represents stochastic customer demand. A multiproduct, multiperiod production planning provided in stochastic optimization model with constraints of production level, electric cost, and inventory variable is formulated to describe the problem. This research assumes that the customer demand is random variable representted by triangular fuzzy number.

The random demand is direct effect to electric cost and inventory level. Hence, this system is seen as a sto-chastic process. For these reasons, in the following parts, the basic model will be further developed in such a manner that it can deal with the uncertain demand.

Let  $d_{it}$  be demand for each type of product i in period t. In general, this demand is uncertain, but an estimation in terms of triangular fuzzy numbers denoted by  $\tilde{d}_{it} = (d_{it}, \alpha_{it}, \beta_{it})$  where  $d_{it}$ ,  $d_{it} - \alpha_{it}$ ,  $d_{it} + \beta_{it}$  are the most probable, the minimum and maximum values of the tri-angular fuzzy demand respectively.

Because of the fuzziness of customer demand, production-inventory balance equation is not an equation in terms of crisp mathematics any longer, but in term of a fuzzy equation. The equation is satisfied in terms of a degree of truth.

The production-inventory balance equation as described in equations (3), (10), and Dynamic inventory equation (11) will change. By substituting the demand

 $d_{it}$  with fuzzy demand  $\tilde{d}_{it}$ , the new fuzzy equation can be described as equation (13), (14) and (15) respectively. Where  $\cong$  is a fuzzy equation, called soft equation. It implies that the production-inventory balance equation (14) is met in terms of a degree of truth. In other words, it indicates that the sum of production level  $X^{it}$  at this period and the inventory level at the end of the previous period  $I_{i,t-1}$  minus the inventory level at the end of this period  $I_{it}$  should not much larger or less than the demand  $\tilde{d}_{it}$ . The more difference the less degree of truth or satisfaction of customer demand.

$$I_{it} \cong \sum_{g} \sum_{h=1}^{t} X_{igh} + I_{i,t-1} - \tilde{d}_{it} \qquad \forall i, t \quad (13)$$

$$I_{it} \cong X^{it} + I_{i,t-1} - \tilde{d}_{it} \qquad \forall i, t \qquad (14)$$

$$I_{it}^{d} \cong X_{igh_{t}} + I_{i,t-1} - \tilde{d}_{it} \qquad \forall i, g, h, t$$
 (15)

Given level of customer satisfaction is  $\theta$  with production and inventory plan to cover the demand  $\tilde{d}_{it}$  for product i in period t. Where  $\mu_{\tilde{d}_{it}}$  is the fuzzy demand of product i of the planning period t, then soft equations (13), (14) and (15) are equivalent to the following formulas:

$$\mu_{\tilde{d}_{it}}(\sum_{g}\sum_{h=1}^{t}X_{igh} + I_{i,t-1} - I_{it}) \ge \theta \qquad \forall i, t \quad (16)$$

$$\mu_{\tilde{d}_{it}}(X^{it} + I_{i,t-1} - I_{it}) \ge \theta \qquad \forall i, t \quad (17)$$

$$\mu_{\tilde{d}_{i,t}}(X_{igh_t} + I_{i,t-1} - I_{it}^d) \ge \theta$$
  $\forall i, g, h, t$  (18)

Formula (14) can also be expressed in the same types of fuzzy demands  $\tilde{d}_{it}$  of the production-inventory balance equation in single stage. It is equivalent to the dynamic balance equation for product i from the beginning of the planning horizon to a certain period t.

In another way, equation (14) can be rewritten as:

$$I_{i,t-1} + X^{it} - I_{it} \cong \tilde{d}_{it}$$
  $\forall i, t$  (19)

This mean that;

$$t = 1, (19) \Rightarrow I_{i,0} + X^{i1} - I_{i1} \cong \tilde{d}_{i1} \forall i$$

$$t = 2, (19) \Rightarrow I_{i,1} + X^{i2} - I_{i2} \cong \tilde{d}_{i2} \forall i$$

$$\vdots$$

$$t = \text{T-1}, (19) \Rightarrow I_{i,T-2} + X^{i,T-1} - I_{i,T-1} \cong \tilde{d}_{iT-1} \forall i$$

$$t = T$$
,  $(19) \Rightarrow I_{i,T-1} + X^{i,T} - I_{i,T} \cong \tilde{d}_{i,T} \quad \forall i$ 

The accumulation of daily production  $X^{it}$  from t = 1 day to t = T or t = 1, 2, ..., T can be obtained by following formula:

$$I_{i0} + \sum_{t=1}^{T} X^{it} - I_{iT} \cong \sum_{t=1}^{T} \tilde{d}_{it}$$
  $\forall i$  (20)

Referring to level of customer satisfaction  $\theta$ , the

soft equation (20) is equivalent to the following formula:

$$\mu_{\tilde{r}_{it}}(I_{i0} + \sum_{t=1}^{T} X^{it} - I_{it}) \ge \theta \qquad \forall i \quad (21)$$

where 
$$\tilde{r}_{it} = \sum_{t=1}^{T} \tilde{d}_{it}$$
 is the total fuzzy demand of

product i from the beginning of the planning horizon t to any day T.

Equation (21) is the production-inventory balance equation (14) under a fuzzy environment. Substituting equation (20) into the production-inventory balance equation (14), then can be rewritten as following:

$$\mu_{\tilde{d}_{it}}(X^{it} + I_{i,t-1} - I_{it}) \ge \theta \qquad \forall i, t \quad (22)$$

or 
$$\mu_{\tilde{r}_{it}}(I_{i0} + \sum_{t=1}^{T} X^{it} - I_{it}) \ge \theta$$
  $\forall i, t \quad (22a)$ 

#### 4.4 Triangular Fuzzy Demand

A preferably acceptable level  $\theta$  denotes decision maker's satisfaction with the production and inventory plan to cover customer demands, a soft equation may be defuzzified in the sense that the cumulative demand of product i from period 1 to period t should be met at the degree of truth  $\theta$ .

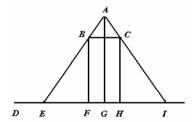


Figure 4. Demand considered as a triangular fuzzy number

Consider a fuzzy number  $\tilde{d} = (d, \alpha, \beta)$  where d, d- $\alpha$ , d+ $\beta$  are the most probable, minimum and maximum values of the fuzzy number (Figure 4). We can meet the followings: DG = d,  $GE = \alpha$ ,  $GI = \beta$ , and  $BF = CH = \theta$ 

In the triangular AEG, we have:

$$\frac{BF}{AG} = \frac{EF}{EG}$$

$$\Rightarrow EF = \left(\frac{BF}{AG}\right)(EG) = \left(\frac{\theta}{1}\right)\alpha = \theta\alpha$$

$$\Rightarrow GF = GE - EF = \alpha - \theta\alpha = \alpha(1 - \theta)$$

Similar with triangular AGI

$$\frac{CH}{AG} = \frac{HI}{GI}$$

$$\Rightarrow HI = \left(\frac{CH}{AG}\right)(GI) = \left(\frac{\theta}{1}\right)\beta = \theta\beta$$

$$\Rightarrow GH = GI - HI = \beta - \theta\beta = \beta(1 - \theta)$$

In order to ensure that the membership of the fuzzy number is larger than or equal to 0, the values of the fuzzy number  $\tilde{d}$  should be in the range of FH only.

In other words, the values of the fuzzy number should be in the range of  $[d\text{-}(1\text{-}\theta)\alpha,\ d\text{+}(1\text{-}\theta)\beta]$ . More generally, the fuzzy demand  $\bar{d}_{it}$  should be in the range of  $[d_{it}-(1-\theta)\alpha_{it},d_{it}+(1-\theta)\beta_{it}]$ .

## 4.5 Fuzzy Model

Forecasting demand is the approximation of future demand while considering the actual past demand. It may either be more than or less than or equal to actual demand in the period. In this research;  $d_{it}$  is forecasting demand and uncertain customer demand based on fuzzy demand  $\tilde{d}_{it}$ . Let  $\tau_{it} = \alpha_{it} = \beta_{it}$ ; the fuzzy demand  $\tilde{d}_{it}$  should be in the range of  $d_{it} \pm (1-\theta)\tau_{it}$ .

When considering Dynamic inventory equation (15), substitute demand  $d_{it}$  by fuzzy demand  $\tilde{d}_{it}$ , the dynamic inventory equation with fuzzy demand can be described as following:

$$I_{it}^{d} = X_{igh_t} + I_{i,t-1} - [d_{it} \pm (1-\theta)\tau_{it}]$$
  $\forall i, g, h, t$  (23)

Substitute the best value of  $\tilde{d}_{it}$  into equation (13), (22), and (15). Then combine adjustable inventory equation (23), and (12) to the mathematical model. The completed fuzzy model can be described as following:

## **Objective Function**

$$\operatorname{Min} \quad \sum_{ijgh} c_{jh} R_{jh} X_{igh} E_{gi} P_{gih} \tag{24}$$

subject to

$$\sum_{gh} X_{igh} = \left[ d_{it} \pm (1 - \theta) \tau_{it} + I_{it} - I_{i,t-1} \right] \qquad \forall i, t \quad (25)$$

$$I_{\min} \le I_{it}$$
  $\forall i, t$  (26)

$$Ii_{\max} \ge I_{it}$$
  $\forall i, t$  (27)

$$I_{it}^d = X_{igh_t} + I_{i,t-1} - [d_{it} \pm (1-\theta)\tau_{it}] \quad \forall i, g, h, t$$
 (28)

$$I_{it} \ge I_{it}^d \qquad \forall i, t \quad (29)$$

$$\sum_{h} X_{igh} P_{igh} \le M_{ig} \qquad \forall i, g \quad (30)$$

$$\sum_{h=s_{in}}^{h=r_{ig}} P_{igh} = r_{ig} \qquad \forall i, g \quad (31)$$

$$X_{igh}, X_{igh_{L}}, I_{it}, I_{it}^{d}, d_{it} \ge 0$$
  $\forall i, g, h, t$  (32)

$$R_{ih}, P_{igh} = 0.1(Binary)$$
  $\forall i, g, h, j$  (33)

The production-inventory balance equation (14) in the original fuzzy optimization model is expressed in term of constraints (25) and (28), by the decision maker's level of customer satisfaction with production and inventory plan to covered customer demand.

#### 5. MODEL APPLICATION

The optimization model is applied to cement industry, seven cement mills are under consideration. The proposal of this application is to, firstly, simulate the incurred cost under the optimization schedule, and secondly, compare those costs with the actual cost in a half test year. To ensure that the analysis is not biased, the actual cement demands are taken from historical data of first half of year 2004.

Since the calculation of electric cost is based only on the production of cement. The electric bill covers not only the plant consumption but also other electric uses such as lighting, welding, maintenance work and other utilities. It is necessary to verify the calculated cost with the actual cost incurred. Using the cement demand to determine the electric cost by applying the electric rates on the energy use as calculated from the cement production. The actual cost is minus 1% approximately.

Cement consists of clinker, gypsum, and limestone. They are treated and stored in different places. The real time proportional weigh-feeder and belt conveyors are used to transmit raw material to the ball mill. Different sizes of cement powder from the outlet of the ball mill are separated by SEPOL separator and are conveyed to store in cement silo containing many types of cement separately. The dispatching will be loaded from cement silo and can also be packed into bulk trucks, bulk train or cement bags.

#### 5.1 Optimal Weight Moving Average Factor

The best forecasting method should have the least difference between forecasting and actual. Generally, at the time of forecasting, we do not know how much will the forecasting error. This research presents the forecasting method that can minimize the forecasting error by Optimal Weight Moving Average Factor: OWMAF.

Three cement products are considered in the experiment. They are type I cement, type mixed cement, and type III cement. The actual cement demand in first six months of year 2004 described in Table 4. Monthly demand had been changed to daily demand to fulfill 24 hour electricity cost optimization. By trial and error, the three days Weighted Moving Average forecasting is found suitable and it can be expressed in equation (34). To maximize the accuracy of forecasting that means minimizing forecasting error. The sum square error of forecasting demand and actual demand are considered to find out the OWMAF for each production type. Objective function (35) is the minimization of sum square error. Equation (36) describes the forecasting value of next period t+1. Constraint (37) and (38) are the summation of weighting and non-negativity constraint.

$$F_{t+1} = w_{f=0}D_t + w_{f=1}D_{t-f} + w_{f=2}D_{t-f}$$
 (34)

#### Objective function

$$Min \sum_{t} e_{t+1}$$
 (35)

# Subject to

$$\sum_{t} F_{t+1} = \sum_{f} w_{f} D_{t-f}$$

$$\sum_{f} w_{f} = 1$$
(36)
(37)

$$\sum_{f} w_f = 1 \tag{37}$$

$$w_f, D_t \ge 0 \qquad \forall f, t \qquad (38)$$

where; t: number of day in considered month, t = 1, 2, ..., 31

f: moving average day, f = 0, 1, 2 day

 $w_f$ : optimal weight value of moving average day f  $F_{t+1}$ : forecasting value of day t+1

 $D_{t-f}$ : actual demand of day t-f

 $e_{t+1}$ : sum square error of forecasting period

$$e_{t+1} = (F_{t+1} - D_{t+1})^2$$

Apply equation (35) to (38) to the actual cement demand in first six months of year 2004 with three days Weighted Moving Average and used the spreadsheetsolver. The optimal result of three days OWMAF (f = 0, f = 1, and f = 2) is described in Table 3. The daily actual cement demand for first six months of year 2004 combined into monthly demand as Table 4.

### 5.2 Demand Forecasting

Based on the actual demand described in Table 4 and OWMAF show in Table 3. The forecasting result of three days Weight Moving Average can be calculated with daily demand consideration and combined to be the monthly demand as shown in Table 4. The OWMAF is used to forecast the demand for each month in next planing year.

### 5.3 Production Planning

According to TOS charge, one day planning horizon is used as the operation period for a model in seven cement mills. Each time step in the optimization procedure is one hour. The day of the month in the experiment can be divided into two cases which are normal working days and holidays including traditional holidays. Each case has different Peak and Off-Peak periods. The operation periods on each day start from 00.00 a.m. which is the beginning of Off-Peak for both cases, and end when satisfied inventory balance equation (25) and Dynamic inventory equation (28) is satisfied.

Number of holidays in each month will be taken into account. There are 11, 9, 9, 12, 12, and 9 number of holidays in January, February, March, April, May, and June in year 2004 respectively. The holiday can be di-

Tabl	le 3.	OW	VΜ	٩F

	Product Type (Tons)									
Month	Type I			Type Mixed				Type III		
	f = 0	f=0 $f=1$ $f=2$ $f=0$ $f=1$ $f=2$				f = 0	f=1	f=2		
January-2004	0.849	0.000	0.151	0.611	0.306	0.084	0.495	0.056	0.448	
February-2004	0.731	0.018	0.252	0.711	0.280	0.009	0.809	0.018	0.173	
March-2004	0.329	0.000	0.671	0.933	0.000	0.067	0.226	0.536	0.238	
April-2004	1.000	0.000	0.000	1.000	0.000	0.000	0.771	0.173	0.056	
May-2004	0.872	0.000	0.128	0.742	0.155	0.102	0.817	0.000	0.183	
June-2004	0.772	0.200	0.028	0.639	0.264	0.097	0.176	0.367	0.457	

Table 4. Actual demand and forecasting demand

	Product Type (Tons)							
Month	Тур	Type I		Mixed	Type III			
	Actual	Forecasting	Actual	Forecasting	Actual	Forecasting		
January-2004	189,854	188,015	109,746	107,681	27,635	27,744		
Febeuary-2004	196,596	196,983	123,550	124,654	27,207	27,372		
March-2004	226,995	227,221	121,741	120,841	27,876	27,545		
April-2004	208,421	209,265	106,384	107,080	25,552	5,961		
May-2004	239,768	236,799	122,461	122,771	28,248	28,086		
June-2004	282,843	283,480	121,295	119,525	26,063	26,282		
Total	1,344,477	1,341,762	705,177	702,553	162,580	162,991		

	Product Type (Tons)							
Types of Planning Day	Type I		Туре	Type Mixed		Type III		
	Min.	Max.	Min.	Max.	Min.	Max.		
Working Day	4,000	19,000	2,000	19,000	800	19,000		
First Holiday	11,500	19,000	3,500	19,000	1,500	19,000		
Second Holiday	14,500	19,000	5,000	19,000	2,000	19,000		
Third Holiday	17,500	19,000	6,500	19,000	2,800	19,000		

**Table 5.** Limit level of inventory ( $Ii_{min}$ ,  $Ii_{max}$ )

**Table 6.** Cumulative demand with different level of  $\theta$  and  $\tau$ 

Product	Cumulative Demand (Tons)									
Types			$\theta$ =0.85, $\tau$ =30	$\theta$ =0.90, $\tau$ =10	$\theta$ =0.90, $\tau$ =20	$\theta$ =0.95, $\tau$ =30				
Type I	1,344,477	1,341,762	1,281,383 ~1,402,141	1,328,345 ~1,355,180	1,314,927 ~1,368,597	1,321,636~1,361,889				
Type Mixed	705,177	702,553	670,938 ~734,167	695,527 ~709,578	688,501 ~716,604	692,014~713,081				
Type III	162,580	162,991	155,656~170,325	161,361~164,621	159,731 ~166251	160,546~165,436				

**Table 7.** Capacity of each cement mill plant

Product	Cement Mill Plants (T/Hr)							
Types	CM # 1	CM # 2	CM # 3	CM # 4	CM # 5	CM # 6	CM # 7	
Type I	218.37	210.96	207.87	210.78	209.83	211.29	213.85	
Type Mixed	278.96	272.81	255.16	261.48	261.05	265.37	264.83	
Type III	122.28	122.87	123.21	129.00	129.02	128.99	128.64	

Table 8. Electric power consumption in kilo-Watt hour per ton of product

Product	Cement Mill Plants (KW.Hr/T)							
Types	CM # 1	CM # 2	CM # 3	CM # 4	CM # 5	CM # 6	CM # 7	
Type I	36.88	36.99	36.34	42.74	42.68	40.14	39.82	
Type Mixed	28.77	28.58	29.93	29.71	30.39	34.79	35.01	
Type III	51.26	50.63	50.33	48.11	48.18	47.92	47.85	

vided into three cases based on the sequence number; they are first holiday, second holiday, and third holiday. Each case has significant effect to inventory level. The limitation of inventory level on working days and on holidays is described in Table 5. Furthermore; on normal working days, level of inventory is not only larger than or equal to minimum limit level. But it must also be larger than or equal to Dynamic inventory as expressed in equation (28) based on which one is bigger.

The cumulative six months actual demand and fuzzy demand is described in Table 6. Forecasting demand is the fuzzy demand at the level of  $\theta$  =1.00 and  $\tau$  =0.00. The calculation result of fuzzy demand has been adjusted by level of  $\theta$  and  $\tau$  to cover different degree of customer satisfaction.

#### 5.4 Plant Basic Data

To describe the performance of optimization model, the actual plant basic data are necessary. Process of cement mill is continuous process. The capacity of each cement mill plant, electric power consumption in Kilo-Watt Hour per Ton of product, and electric power consumption in Kilo-Watt Hour per hour are presented in Table 7, Table 8, and Table 9 respectively.

# 6. OPTIMIZATION RESULTS

The optimization model is applied to the system for reducing electric billing in seven cement mill plants. The research takes one month of running time to obtain the results for this billing period. The plant basic data, number of demand, limitation of inventory, and other constraints are obtained. The optimization of MILP can be implemented by using the Spreadsheet-Solver. The optimization results of MILP can be described as below:

Table 10 shows the optimization result of cement plants that are suitable to produce which cement product. The results "1" implied that this plant is suitable or selected to produce this product.

Table 11 expresses the capacity of optimal plant selection for each type of cement production. The operation manager can match the plants and the product types according to these optimization results.

The optimal TOS electricity is results of switching

T 11 A	T1 / '	4.	. 1.1	337 44 1 1
Table 9.	. Electric energy	consumption	in kild	-Watt hour per hour

Product		Cement Mill Plants (KW.Hr/Hr)							
Types	CM # 1	CM # 2	CM # 3	CM # 4	CM # 5	CM # 6	CM # 7		
Type I	8,053.48	7,803.41	7,553.99	9,008.73	8,955.54	8,481.18	8,515.50		
Type Mixed	8,025.67	7,796.91	7,636.93	7,768.57	7,933,31	9,232,22	9,271.69		
Type III	6,268.07	6,220.90	6,201.15	6,206.19	6,216.18	6,181.20	6,155.42		

Table 10. Optimal cement plant selection

Product		Cement Mill Plants							
Types	CM # 1	CM # 2	CM # 3	CM # 4	CM # 5	CM # 6	CM # 7		
Type I	1	1	1	0	0	0	1		
Type Mixed	0	0	0	1	1	0	0		
Type III	0	0	0	0	0	1	0		

**Table 11.** Capacity of optimal plants selection (Tons)

Product		Cement Mill Plants									
Types	CM # 1	CM # 2	CM # 3	CM # 4	CM # 5	CM # 6	CM # 7	CM # 1			
Type I	218.37	210.96	207.87	0	0	0	213.85	851.05			
Type Mixed	0	0	0	261.48	261.05	0	0	522.53			
Type III	0	0	0	0	0	128.99	0	128.99			

Table 12. TOS optimal electricity charge on workdays (Baht/KW.Hr)

Electric Type		Hour Number										
Electric Type	1	1 2 3 4 5 6 7 8 9 10 11										
TOS	1.1726	1726   1.1726   1.1726   1.1726   1.1726   1.1726   1.1726   1.1726   1.1726   1.1726   1.6660   1.6660										

Table 12. (Continue)

Electric Type		Hour Number										
Electric Type	12	- 17   13   14   15   16   17   18   118 10~18 30171 30~77 001 73   74									24	
TOS	1.6660	.6660   1.6660   1.6660   1.6660   1.6660   1.6660   1.6660   1.6660   1.6660   1.6660   1.1726   1.172									1.1726	

**Table 13.** Optimization results of plant g assigned to produced product i in hour  $h\left(P_{igh}\right)$ 

Plants		Hour Number											
1 lants	1	2	3	4	5	6	7	8	9	10	11	12	
CM # 1	218.37	218.37	218.37	218.37	218.37	218.37	218.37	218.37	218.37	0	0	0	
CM # 2	210.96	210.96	210.96	210.96	0	0	0	0	0	0	0	0	
CM # 3	207.87	207.87	207.87	207.87	207.87	207.87	207.87	207.87	207.87	0	0	0	
CM # 4	261.48	261.48	261.48	261.48	261.48	261.48	0	0	0	0	0	0	
CM # 5	0	0	0	0	0	0	0	0	0	0	0	0	
CM # 6	128.99	128.99	128.99	0	0	0	0	0	0	0	0	0	
CM # 7	0	0	0	0	0	0	0	0	0	0	0	0	

among TOU and TOD can be separated to holiday and workday. On holiday; optimal TOS electricity charge is 1.726 Baht per Kilowatt-Hour for both weekend and traditional holiday. On workdays; optimization results avoid producing at period time 18:30~21:30 because these periods cause too high electricity cost. The result of TOS optimal electricity charge over production hours can be described in Table 12.

Table 13 describes the optimization results of plant g assigned to produced product i in hour h on January 05, 2004 with level of  $\theta$  =0.85 and  $\tau$  =30. Cement mill # 6 expresses the minimum running hour limitation i.e. 3 hours to produce cement type III according to equation (31). From time 23.00~24.00 is only two hours; it should be shift to the next planing day t. Therefore; the planing time is start on time 23.00. The result "0" implied that

<b>Table</b>	13.	(Continue)	١
Table	13.	Continue	)

Plant		Hour Number												
Tant	13	14	15	16	17	18	18.00~18.30	21.30~22.00	23	24				
CM # 1	0	0	0	0	0	0	0	0	218.37	218.37				
CM # 2	0	0	0	0	0	0	0	0	0	0				
CM # 3	0	0	0	0	0	0	0	0	207.87	207.87				
CM # 4	0	0	0	0	0	0	0	0	0	0				
CM # 5	0	0	0	0	0	0	0	0	0	0				
CM # 6	0	0	0	0	0	0	0	0	0	0				
CM # 7	0	0	0	0	0	0	0	0	0	0				

Table 14. Maximum of plants capacity running

Plant		Hour Number												
Tiant	1	2	3	4	5	6	7	8	9	10	11	12		
CM # 1	218.37	218.37	218.37	218.37	218.37	218.37	218.37	218.37	218.37	218.37	218.37	218.37		
CM # 2	210.96	210.96	210.96	210.96	210.96	210.96	210.96	210.96	210.96	210.96	210.96	210.96		
CM # 3	207.87	207.87	207.87	207.87	207.87	207.87	207.87	207.87	207.87	207.87	207.87	207.87		
CM # 4	261.48	261.48	261.48	261.48	261.48	261.48	261.48	261.48	261.48	261.48	261.48	261.48		
CM # 5	261.05	261.05	261.05	261.05	261.05	261.05	261.05	261.05	261.05	261.05	261.05	261.05		
CM # 6	128.99	128.99	128.99	128.99	128.99	128.99	128.99	128.99	128.99	128.99	128.99	128.99		
CM # 7	213.85	213.85	213.85	213.85	213.85	213.85	213.85	213.85	213.85	213.85	213.85	213.85		

Table 14. (Continue)

Plant		Hour Number												
1 Idill	13	14	15	16	17	18	18.00~18.30	21.30~22.00	23	24				
CM # 1	218.37	218.37	218.37	218.37	218.37	218.37	218.37	218.37	218.37	218.37				
CM # 2	210.96	210.96	210.96	210.96	210.96	210.96	210.96	210.96	210.96	210.96				
CM # 3	207.87	207.87	207.87	207.87	207.87	207.87	207.87	207.87	207.87	207.87				
CM # 4	261.48	261.48	261.48	261.48	261.48	261.48	261.48	261.48	261.48	261.48				
CM # 5	261.05	261.05	261.05	261.05	261.05	261.05	261.05	261.05	261.05	261.05				
CM # 6	128.99	128.99	128.99	128.99	128.99	128.99	128.99	128.99	128.99	128.99				
CM # 7	213.85	213.85	213.85	213.85	213.85	213.85	213.85	213.85	213.85	213.85				

Table 15. Electric cost

			Level of	$\theta$ and $\tau$		
Month	Actual	Forecasting	$\theta = 0.85$	$\theta = 0.9$	$\theta = 0.9$	$\theta = 0.95$
	Actual	Porecasting	$\tau = 30$	$\tau = 10$	$\tau = 20$	$\tau = 30$
January-2004	43,757,053	14,026,299	14,000,003	13,552,705	13,694,900	13,623,387
Febeuary-2004	49,492,668	14,648,520	15,409,796	14,834,049	14,966,459	14,926,258
March-2004	50,900,377	16,496,499	17,129,280	16,566,972	16,728,846	16,635,308
April-2004	48,131,458	14,627,051	15,325,120	14,736,924	15,003,778	14,858,171
May-2004	47,645,464	17,440,844	18,210,618	17,579,679	17,752,526	17,732,747
June-2004	52,041,351	17,932,898	18,606,398	17,943,342	18,137,632	18,052,350
Total	291,968,370	95,172,110	98,681,215	95,213,671	96,284,142	95,828,221

plants do not select to produce in those hours.

If customer demand rise up to maximum of optimal plant capacity. The optimization of TOS electricity on Off-Peak cannot perform. All plants must run over all period hours (see Table 14. Because of economic downturn during considered period the demand was never maximum capacity.

This research intends to cover any level of customer satisfaction by adjusting level of  $\theta$  and  $\tau$ . The electric cost also increases directly to level of customer satisfaction, it can be between minimum and maximum. Table 15 displays result of maximum electric cost for each level of  $\theta$  and  $\tau$  adjustment compared with actual electric cost.

Cost Types		Level of Customer Sa	atisfaction	
Cost Types	$\theta = 0.85, \ \tau = 30$	$\theta = 0.9, \ \tau = 10$	$\theta = 0.9, \ \tau = 20$	$\theta = 0.95, \ \tau = 30$
Actual Cost (Baht)	289,048,687	289,048,687	289,048,687	289,048,687
Fuzzy Model Cost (Baht)	103,615,275	99,974,355	101,098,349	100,619,632
Demand Charge (Baht)	12,570,000	12,570,000	12,570,000	12,570,000
Saved (Baht)	172,863,411	176,504,332	175,380,338	175,859,055
Percent of Saved	59.80%	61.06%	60.68%	60.84%

**Table 16.** Fuzzy model cost.

The actual cost and fuzzy model cost described in Table 16, the actual cost will minus 1% approximately to verify that the rate is being applied correctly to the electricity used, i.e. (291,968,370)(0.99) = 289,048,687 Baht. Fuzzy model cost must be 5% increased for probability of plant breakdown. Maximum electric demand of seven experimental plants was 70 MW, additional electric demand charge for TOD occurred on the Partial Peak only. Because on the Peak period i.e. 6:30 p.m.-9:30 p.m. all plant operations are avoided. Hence, demand charge of TOD for voltage ≥ 69 KV is (29.91 Baht/KW)(70,000 KW) = 2,093,700 Baht per month, this means 12,562,200 Baht per six months.

Table 16 describes the final result of electric cost saving. Saving cost is comparison of actual cost plus electric demand charge and fuzzy model cost. The amount of saving based on level of customer satisfaction. Maximum saved is 61.06% while level of customer satisfaction 90% ( $\theta = 0.90$  and  $\tau = 10$ ), and minimum saved is 59.80% while level of customer satisfaction 85% ( $\theta = 0.85$ , and  $\tau = 30$ ).

# 7. CONCLUSIONS

A new optimization model has been developed in order to minimizing the electric cost or electric bill while satisfying level of customer demand. The model uses the decomposition in space of TOU and TOD electricity by creating the TOS electricity. TOS can be generated by the combination of TOU and TOD, and then switched over both electric types to minimize the electric cost. To reduce the complication of model, FT and electricity for utility are omitted.

The mathematical model has been formulated with the objective function to minimize the electric cost by applying TOS electricity with binary constraint. Triangular fuzzy demand and dynamic inventory tracking with multiple plant capacity are introduced to cover the uncertain demand of customer. The constraint for minimum hour limitation of plant running per one start up event is presented to minimize plant idle time. Moreover; the OWMAF for demand forecasting is also presented. Application is made to the multiple cement mill plants. To implement the model solution the Spreadsheet-Solver is used. A simulation run on part of the system in

a test for six months over seven cement mill plants was presented.

The optimization result shows the creation of TOS electricity, and suitability of plants and product types selection. Finally, cost saving more than 61% or more than 177 million Baht of the actual electric cost. Dynamic inventory constraint (28) can be adjusted the inventory level to prevent the peak load in next planning day, it also causes high saving. The maximum expectation of plant break down and electricity cost for utilities are taken from historical data that are 5% and 1% respectively. Plant break down can be decreased by scheduling preventive maintenance on Peak period hours when not producing.

The mathematical model was designed for general plant application while multiple plants are available. In case of single plant application; optimization of plant selection cannot perform.

The experiment was done with the past data. Application of this mathematical model is suitable for the countries that supply electricity by IPPs.

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