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A Different Approach on Availability Modeling of Redundant Structure with Monitoring System

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Abstract. In this paper, we consider a standby redundant structure with a function of switchover processing which may not be not perfect. The switchover processing is governed by a control module whose failure may cause the failure of the whole system. The parameters measuring such an effect of failure of the control module is included in our reliability model. We compute several reliability measures such as reliability function, failure rate, MTBF, mean residual life function, and the steady state availability. We also compare a single unit structure and the redundant structure with regard to those reliability measures. An example is given to illustrate our results.

Key Words : *Redundant structure, Reliability function, Failure rate, MTBF, Mean residual life, Steady state avalability .*

1. INTRODUCTION

The redundant structure is one of the most widely used technique in the reliability design in order to improve the reliability of the system. In a two-unit repairable standby redundant structure, the standby unit starts operating immediately once the active unit is detected to fail and when the failed active unit is repaired, it assumes the position of the standby unit. Thus, these two units alternate their positions as either active or standby whenever the failure or repair occurs. Depending on the readiness(or consequently, the failure rate) of standby unit, it is classified as hot, cold or warm standby unit. While the active unit is operating,

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the cold standby unit does not operate and the hot standby unit operates, while the warm standby does not operate but the preliminary electronic source is laid on during the operation of the active one. More details are given in Elsayed(1996). Kumar and Agarwal(1980) also present excellent summaries for the cold redundant structure. Various techniques for modeling the reliability of a system are discussed in Endrenyi(1978).

The redundant systems having imperfect switchover device have been extensively studied by many authors [Das(1978), Singh(1980), Singh and Goel(1995)]. Recently, Lim(1996) and Lim and Koh(1997) consider a redundant system with a function of switchover processing and suggest a new method of modeling the reliability consideration in which the switchover processing causes an increase of the failure rate of the system. In these papers, they compare a single component system and a redundant system with a function of switchover processing in terms of four reliability measures such as reliability function, failure rate and mean residual life function.

In this paper, we extend the results of Lim and Koh's(1997) to the case of a two -unit warm standby redundant structure(hereafter WSRS). We also obtain the steady state availability of a two-unit WSRS. Finally, in order to investigate the effect of additional components on redundancy, we compare the two-unit WSRS and a single unit structure(hereafter SUS) with respect to several reliability measures and availability measure. In Section 2, we describe the two-unit WSRS with a function of switchover processing. In Section 3, we compute four reliability measures and availability of two-unit WSRS by considering the effect of additional components on redundancy. Section 4 is devoted to compare two-unit WSRS and SUS. An example is given in Section 5.

2. REFERENCE MODEL OF A STANDBY REDUNDANT STRUCTURE

Figure 2.1 shows a reference model of a redundant system with a function of switchover processing which consists of three units: an active unit, a standby unit, and a switchover device. This model is also considered by Lim and Koh(1997). The control module charges the switchover processing in such a way that it monitors the state of the active unit and let the switchover device, which is not 100% perfect, exchange the active unit for the standby unit as soon as the active unit fails.

In a standby redundant structure in Figure 2.1, the failure of control module does affect the operation of system as far as the active unit is working. However, the control module affects the switchover processing if the active unit fails while the control module is in failure state. Hence, it is natural to assume that the switchover processing causes an increase of the failure rate of the system. (That is, the failure of the control module is considered to be a factor in our availability model.) We assume that the increment of the failure rate due to the switchover processing is distributed to each string of the system in such a way that the failure rate of each

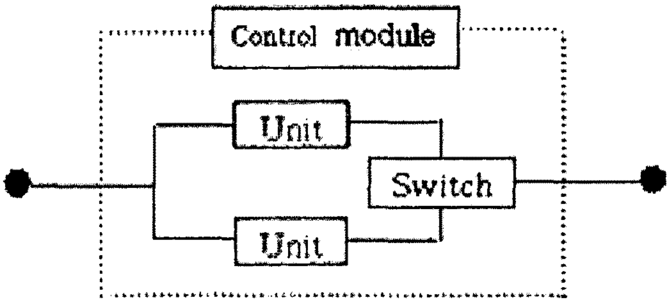


Figure 2.1 A Reference Model of Redundant System with a Function of Switchover Processing

unit increases by $\lambda_\alpha = \alpha\lambda$, where λ_α is relatively smaller than the failure rate of a unit, λ, i.e. $0 \le \alpha \le 1$.

Throughout this paper, we assume the followings:

(i) All units are independent and have exponential life distributions. The redundant structure is a warm standby redundant structure in such a way that and the standby unit has a failure rate of $\lambda\beta$ while the active unithas a failure rate of λ, where $0 \le \beta \le 1$.

(ii) Repairs occur one at a time (sequential repair) and the repair time is exponentially distributed with a mean of $1/\mu$.

(iii) The probability of successful switchover operation is given by p, $0 \le p \le 1$.

(iv) The type of standby unit in the redundant structure is a warm standby unit. That is, the failure rate of standby unit is between 0 and the failure rate of active unit.

Notations

$R_S(t), R_W(t)$	Reliability function of SUS and WSRS, respectively.
$\Theta_S(t), \Theta_W(t)$	MTBF of SUS and WSRS, respectively.
$r_S(t), r_W(t)$	Failure rate of SUS and WSRS, respectively.
$m_S(t), m_W(t)$	Mean residual life of SUS and WSRS, respectively.
A_S, A_W	Availability of SUS and WSRS, respectively.

3. EVALUATION OF RELIABILITY

3.1 Reliability measures for nonrepairable system

Let F be a distribution function of a life time random variable and f be the probability density function of F. Then the reliability measures considered in this paper are defined as follows.

(i) Reliability function

$$R(t) = 1 - F(t).$$

(i) Failure rate

$$r(t) = f(t)/R(t).$$

(iii) MTBF

$$MTBF = \int_0^\infty R(t)dt.$$

(iv) MRL

$$m(t) = \int_t^\infty R(u)du/R(t).$$

For SUS, it is straightforward to compute those reliability measures since the life distribution of the unit is assumed to be exponential. For two-unit WSRS, we apply the state space method to calculate such reliability measures. The results are summarized as follows.

(i) Reliability function

$$R_S(t) = e^{-\lambda t}. \tag{3.1}$$

$$\begin{aligned} R_W(t) &= P(\text{active unit operates exceeding t}) + \text{P(successful} \\ &\quad \text{switchover)} \times \text{P(active unit} \geq \text{t|successful switchover)} \\ &= \frac{1}{\beta}[(p+\beta)e^{-(1+\alpha)\lambda t} - pe^{-(1+\beta)(1+\alpha)\lambda t}]. \end{aligned} \tag{3.2}$$

(ii) MTBF

$$\Theta_S = 1/\lambda. \tag{3.3}$$

$$\Theta_W = (\beta + 1 + p)/(1+\beta)(1+\alpha)\lambda. \tag{3.4}$$

(iii) Failure Rate

$$r_S(t) = \frac{-dR_S(t)/dt}{R_S(t)} = \lambda. \tag{3.5}$$

$$\begin{aligned} r_W(t) &= \frac{-dR_W(t)/dt}{R_W(t)} \\ &= (1+\alpha)\lambda - \frac{p\beta(1+\alpha)\lambda}{(p+\beta)e^{\beta(1+\alpha)\lambda t} - p}. \end{aligned} \tag{3.6}$$

(iv) MRL

$$m_S(t) = \frac{\int_t^\infty R_S(u)du}{R_S(t)} = \frac{1}{\lambda}. \tag{3.7}$$

$$\begin{aligned} m_W(t) &= \frac{\int_t^\infty R_W(u)du}{R_W(t)} \\ &= \frac{1}{(p+\beta)(1+\alpha)\lambda} + \frac{\beta(p+\beta)}{(1+\beta)(1+\alpha)(p+\beta-pe^{-\beta(1+\alpha)\lambda t})}. \end{aligned} \tag{3.8}$$

We note that the WSRS has an increasing failure rate (IFR), while each component has a constant failure rate(CFR). Hence, it is well known that the WSRS has a decreasing mean residual life (DMRL).

3.2 Reliability measure for repairable system

Availability is one of the most important reliability measures for a repairable system since it explains both reliability and maintainability. The unavailability of a system is the probability that the system is in failure state when it is needed to operate. The unavailability is defined as 1 - availability. It is common practice that the annual down-time can be used to as a measure representing the reliability of telecommunication system, which is computed by the following formula.

$$\text{Annual down} - \text{time(min/year)} = \text{unavailability} \times 525600.$$

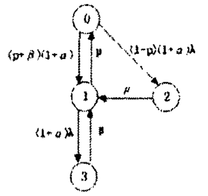
We obtain the steady state availability of the SUS and two-unit WSRS by using the state space method. More details on the state space method are discussed in Bellcore[1]. For the SUS, it is well known that the availability is given by

$$A_S = \mu/(\lambda + \mu). \tag{3.9}$$

For the two-unit WSRS, we define four states of the system and draw the state transition diagram(STD) as shown in Figure 3.1. The states 2 and 3 represent the failure of the system. The state 2, which represents uncoverage outage, is caused by the malfunction of the switchover device and the state 3 is due to the failure of both units.

Define P_i as the probability of the system being in state i, $i = 0, 1, 2, 3$. Then the P_i's must satisfy $P_i \sum_{j\neq i} \lambda_{ij} = \sum_{j\neq i} \lambda_{ji} P_j$, subject to the restraint $\sum_{i=0}^{3} P_i = 1$, where λ_{ij} is the transition rate from state i to state j. It implies that the flow rate out of the state equals the flow rate into the state for any state.

On the basis of the state transition diagram in Figure 3.1, the flow rate equations can be established as follows:



State	Description
0	Duplex
1	Simplex
2	Uncoverage Outage
3	Failure

Figure 3.1 State Transition Diagram(STD) of WSRS

$$\begin{aligned}
(1+\beta)(1+\alpha)\lambda P_0 &= \mu P_1 \\
[\mu+(1+\alpha)\lambda]P_1 &= (p+\beta)(1+\alpha)\lambda P_0 + P_2/R + \mu P_3 \\
P_2/R &= (1-p)(1+\alpha)\lambda P_0 \\
\mu P_3 &= (1+\alpha)\lambda P_1 \\
P_0+P_1+P_2+P_3 &= 1.
\end{aligned} \tag{3.10}$$

Solving these equations of (3.10), we obtain the availability of the WSRS as

$$A_W = \frac{\mu^2+\mu(1+\beta)(1+\alpha)\lambda}{\mu^2+\mu(1+\beta)(1+\alpha)\lambda+\mu^2R(1-p)(1+\alpha)\lambda+(1+\beta)(1+\alpha)^2\lambda^2}. \tag{3.11}$$

4. COMPARISON OF SUS AND TWO-UNIT WSRS

In this section, we compare SUS and two-unit WSRS in terms of four reliability measures and the steady state availability.

When the control device does not cause the increase of failure rate of the system, i.e. $\alpha = 0$, it is clear that the two-unit WSRS outperforms the SUS with respect to reliability measures considered. However we have somewhat different results when the control device has an effect on the performance of the system. The following theorems summarize such results.

Theorem 4.1. There exits a $p^* \in [0,1]$ such that $\Theta_S \geq \Theta_W$ for $0 \leq p \leq p^*$ and $\Theta_S \leq \Theta_W$ for $p^* \leq p \leq 1$, where $p^* = (1+\beta)\alpha$.

Proof: From the equations (3.3) and (3.4), it is easy to see that the results hold. The value of p^* can be obtained by solving the following equation with respect to p.

$$(\beta+1+p)/(1+\beta)(1+\alpha)\lambda = 1/\lambda$$

■.

We also compare the SUS and the two-unit WSRS in terms of failure rate and mean residual life function. The results are formally stated in the following theorem.

Theorem 4.2.

(i) Suppose that $p > \alpha/(1+\alpha)$. Then there exist a point $t^* \in R^+$ such that $r_W(t) \le r_S(t)$ for $0 \le t \le t^*$ and $r_W(t) \ge r_S(t)$ for $t^* \le t \le \infty$, where $t^* = \frac{1}{\beta(1+\alpha)\lambda}\ln(\frac{p(\beta(1+\alpha)+\alpha)}{\alpha(p+\beta)})$.

(ii) Suppose that $p > \alpha(1+\beta)$. Then there exist a point $t^* \in R^+$ such that $m_S(t) \le m_W(t)$ for $0 \le t \le t^*$ and $m_S(t) \ge m_W(t)$ for $t^* \le t \le \infty$, where $t^* = \frac{1}{\beta(1+\alpha)\lambda}\ln(\frac{p(\beta(1+\alpha)+\alpha)}{\alpha(1+\beta)(p+\beta)})$.

Proof:

(i) First, we note that the SUS has a monotone increasing failure rate. It is easy to see that

$$r_W(t) \to \lambda(1+\alpha)(1-p) \;\; as \;\; t \to 0 \tag{4.1}$$

and

$$r_W(t) \to (1+\alpha)\lambda \;\; as \;\; t \to \infty. \tag{4.2}$$

Since $p > \alpha/(1+\alpha)$, we obtain the following inequality

$$r_W(0) = \lambda(1+\alpha)(1-p) < \lambda = r_S(0). \tag{4.3}$$

The result follows immediately from the monotonicity of the failure rate function of the WSRS and (4.1), (4.2), and (4.3).

(ii) The proof can be done in the similar manner ■.

We note that the condition for existence of a turning point in the MRL is that the value of p is greater than the turning point of the MTBF in Theorem 1. Since the actual probability of the successful switchover, p, is close to 1.0, all such a conditions are satisfied in most of real situations.

Using the formulas given in (3.9) and (3.11), we can compare the SUS and two-unit WSRS in terms of steady state availability.

Theorem 4.3. Let $\gamma = (1+\beta)\lambda + \mu^2 R$. Given that $\frac{-\gamma+\sqrt{\gamma^2-4(1+\beta)\lambda\mu(\mu R-1)}}{2(1+\beta)\lambda} < \alpha < \frac{-(1+\beta)\lambda+\sqrt{(1+\beta)^2\lambda^2+4(1+\beta)\lambda\mu}}{2(1+\beta)\lambda}$, there exists a $p^* \in [0,1]$ such that $A_S \ge A_W$ for

$0 \leq p \leq p^*$ and $A_S \leq A_W$ for $p^* \leq p \leq 1$, where $p^* = 1 - \frac{\mu-(1+\beta)\alpha(1+\alpha)\lambda}{\mu^2 R(1+\alpha)}$.

Proof: We note that A_W is non-decreasing in p and A_S is a constant. Hence, it is sufficient to show that when $p = 0$, $A_S \geq A_W$ and when $p = 1$, $A_S \leq A_W$. It is somewhat tedious but straightforward to show that when $p = 0$, $A_S \geq A_W$ if $\alpha > \frac{-\gamma+\sqrt{\gamma^2-4(1+\beta)\lambda\mu(\mu R-1)}}{2(1+\beta)\lambda}$. Thus, the existence and uniqueness of p^* is established. The value of p^* can be obtained by solving the following equation with respect to p

$$\frac{\mu^2 + \mu(1-\beta)(1+\alpha)\lambda}{\mu^2 + \mu(1+\beta)(1+\alpha)\lambda + \mu^2 R(1-p)(1+\alpha)\lambda + (1+\beta)(1+\alpha)^2\lambda^2} = \frac{\mu}{\lambda+\mu}. \blacksquare$$

5. EXAMPLE

For the purpose of illustration of our results, we modify the redundant structure considered by Lim and Koh(1997) in such a way that a switchover device is added and the standby units are assumed to be warm standby. Figure 5.1 shows the modified structure in which three units are arranged in series and they are forming two rows in parallel. The controller monitors the active units and make the switchover device do switchover processing as soon as any of active units is detected to fail. We refer this structure as Two-unit WSRS for the switchover device.

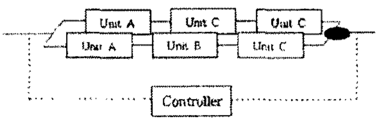


Figure 5.1 The Modified Structure of the Optical Transportation System

In Lim and Koh(1997), all units are independently operating and have exponential life distributions with failure rates shown in Table 5.1. Since Unit A, B, and C are connected in series, it can be easily shown that both active units and standby units are exponentially distributed with the failure rate being equal to the sum of failure rates of three units, which results in 26,000 FITs. Finally, since the increment of the failure rate would not be greater than the failure rate of controller, we assume that the proportion of increment of the failure rate, α, is given by 0.223.

Table 5.1 Failure Rate of PBAs (Unit:FIT).

PBA	Unit A	Unit B	Unit C	Controller
Failure Rate	9,000	7,500	9,500	5,800

We also consider an altenative structure consisting of Unit A, Unit B and Unit C which are connected in series. This structure is refered to as structure S.

We evaluate the reliability measures of two structures in terms of reliability function, MTBF, failure rate and mean residual life and the results are summarized in Table 5.2.

Table 5.2 Reliability Measures of Simple Structure and Redundant Structure.

Reliability Measures	SUS	Two-unit WSRS
$R(t)$	$e^{-2.6t}$	$\frac{1}{\beta}[(p+\beta)e^{-3.18t} - pe^{-3.18(1-\beta)t}]$
MTBF	0.385	$\frac{1+p+\beta}{3.18(1+\beta)}$
$r(t)$	2.600	$3.18 - \frac{3.18p\beta}{(p+\beta)e^{-3.18(1+\beta)t}}$
$m(t)$	0.385	$\frac{1}{3.18(p+\beta)} + \frac{\beta(p+\beta)}{1.223(1+\beta)(p+\beta-pe^{-3.18\beta t})}$

(Unit of time:10^5 hours)

Table 5.3 represents the values of MTBF of two-unit WSRS for various choices of p and β. As expected from Theorem 4.1, the MTBF of two-unit WSRS is less than the MTBF of SUS when the probability of successful switchover, p, is very small.

Table 5.3 The MTBF of two-unit WSRS for Various Values of p and β.

(MTBF of the SUS = 0.385)

p	$\beta=0.0$	$\beta=0.3$	$\beta=0.6$	$\beta=1.0$
0.0	0.314485	0.314485	0.314485	0.314485
0.2	0.377382	0.362868	0.353796	0.345934
0.4	0.440279	0.411250	0.393106	0.377382
0.6	0.503176	0.459632	0.432417	0.408831
0.8	0.566073	0.508015	0.471728	0.440279
1.0	0.628970	0.556397	0.511038	0.471728

Figure 5.2 shows the failure rates of SUS and two-unit WSRS for $p = 0.99$ and $\beta = 0.6$. It is noted that the failure rate of two-unit WSRS is smaller than that of SUS, but it is reversed after the point $t^* = 0.224$. The mean residual life functions of two structures are shown in Figure 5.3. The mean residual life of two-unit WSRS is greater than that of SUS in the early stage and then is smaller after the point $t^* = 0.1167$. These outcomes are corresponding to the results of Theorem 4.2.

We also calculate the unavailability of each of structures S and R for various values of p and β. For both structures, the mean repair time is assumed to be equal to 2 hours. For the purpose of calculation, the values of β are taken as 0, 0.3, 0.6, 0.9 and 1.0 and the values of p are assumed to vary from 0.0 to 1.0(0.2). For such values of p and β, we also compute annual down-time in minutes.

Table 5.4 represents the values of unavailability and annual down-time. Availability for each value of p and β is directly obtained by subtracting the corresponding unavailability from 1.0. In all cases, the annual down time decreases fast as the successful switchover probability increases. Table 5.4 shows that the SUS outperforms the other when the probability of successful switchover is small. Such results agree with the results obtained in Theorem 4.3. It is also noted that the turning point(p^*)

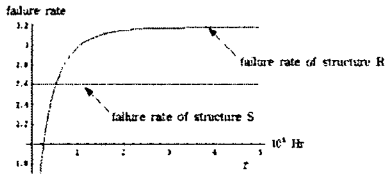


Figure 5.2 Comparison of the Failure Rates

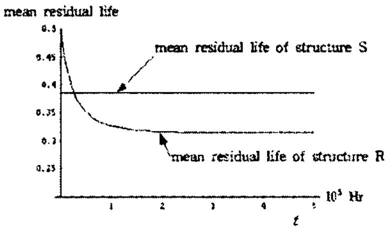


Figure 5.3 Comparison of Mean Residual Life

increases as the value of β is increasing. The values of p^* for various choices of β are listed in Table 5.5.

Table 5.4 Unavailability(U.A) and Annual Down-time(A.D) of the Redundant Structure.

(Unavailability of the SUS=2.56×10^{-5}, Annual Down-time of the SUS=13.6653).

p	$\beta = 0.0$		$\beta = 0.3$	
	UA	AD	UA	AD
0.0	3.179699E-05	16.7124974	3.179699E-05	16.7124974
0.2	2.543796E-05	13.3701892	2.543802E-05	13.3702211
0.4	1.907884E-05	10.0278385	1.907896E-05	10.0279023
0.6	1.271964E-05	6.6854453	1.271983E-05	6.6855410
0.8	6.360368E-06	3.3430096	6.360611E-06	3.3431372
1.0	1.011081E-09	0.0005314	1.314392E-09	0.0006908

p	$\beta = 0.6$		$\beta = 1.0$	
	UA	AD	UA	AD
0.0	3.179699E-05	16.7124974	3.179699E-05	16.7124974
0.2	2.543808E-05	13.3702530	2.543816E-05	13.3702955
0.4	1.907908E-05	10.0279661	1.907924E-05	10.0280511
0.6	1.272001E-05	6.6856366	1.272025E-05	6.6857642
0.8	6.360854E-06	3.3432647	6.361177E-06	3.3434347
1.0	1.617698E-09	0.0008503	2.022097E-09	0.0010628

Table 5.5 Turning Point (p^*) for Given β.

β	0.0	0.3	0.6	1.0
p^*	0.18234431	0.18234605	0.18234779	0.18235011

6. CONCLUSIONS

In this paper we consider the single unit structure (SUS) and two-unit warm standby redundant structure with the function of the switchover processing (two-unit WSRS). Assuming the constant failure rates of all units comprising the structures, we evaluate several reliability measures such as reliability function, MTBF, failure rate, mean residual function and steady state availability of each structure. We make analytic comparisons between the SUS and the two-unit WSRS in terms of those measures and obtain the various relations. Our observations are summarized as follows:

For MTBF and availability, the SUS is better than the two-unit WSRS when the probability of successful switchover, p, is small, but the two-unit WSRS is better than the SUS for other values of p. That is, even though many engineers adopt the redundant structure in order to improve the relaibility of a system, they would not achieve their goal if the successful switchover probability is low. For failure rate and mean residual life function, there exists a turning point before which the two-unit WSRS outperforms the SUS if certain considerations are satisfied.

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