International Journal of Reliability and Applications
Vol. 8, No. 1, pp. 41-51, 2007

Goodness-of-fit Test for Rayleigh Distribution

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Abstract. In this paper, we use the moments of order statistics derived by Lieblein (1955) to develop the correlation goodness-of-fit test for the Rayleigh distribution. In such we simulate the percentage points of the test statistics for the one-parameter and two-parameter cases. In addition, we calculate the power of the proposed tests based on some alterative distributions. Finally, we apply the procedures developed in the paper to some real data.

Key Words : *Moments of order statistics, correlation coefficient, goodness-of-fit test; power of the test and Monte Carlo simulation.*

1. INTRODUCTION

The two-parameter Rayleigh; $Ray(\theta, \sigma)$ distribution has its pdf as

$$f(x) = \frac{x-\theta}{\sigma^2} \exp\left(-\frac{(x-\theta)^2}{2\sigma^2}\right), x \geq \theta, \sigma > 0, \quad (1.1)$$

where θ and σ are the location and scale parameters, respectively.

The one-parameter pdf of the Rayleigh; $Ray(0, \sigma)$ distribution is given by

$$f(x) = \frac{x}{\sigma^2} \exp\left(-\frac{x^2}{2\sigma^2}\right), x > 0, \ \sigma > 0, \quad (1.2)$$

while the standard Rayleigh; $Ray(0, 1)$ is given by

$$f(x) = x \exp\left(-\frac{x^2}{2}\right), \ x > 0. \quad (1.3)$$

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Rayleigh distributions given in (1.1) - (1.3) are special cases of the Weibull distributions with shape parameter equal to 2, and consequently with failure rate function increasing with time at constant rate.

Dyer and Whisenand (1973) have obtained the best linear unbiased estimates of the scale parameter of the Rayleigh distribution given in (1.2) based on small sample size, while Adatia (1995) has repeated the same technique based on fairly large censored samples. Raqab and Madi (2002) have predicated the total time on test using doubly censored Rayleigh data.

Rayleigh distribution has a wide range of applications including life testing; Polovko (1968), communication engineering; Dyer and Whisenand (1973) and Gross and Clark (1975); clinical trials. Recently, many authors have discussed various applications of the Rayleigh distribution, for example, Yamane (1998) has discussed the applications of the Rayleigh to size selectivity of small prawn pots for the oriental river prawn. Celik (2003 and 2004) has analyzed the wind speed data of Iskenderun, Turkey based on the Rayleigh distribution while Akpinar and Akpimiar (2004) have analyzed the the wing speed data from Agin-Elazig, Turkey by using the Rayleigh distribution. Kuruoglu (2004) has shown that the amplitude distribution of the complex wave fits a generalization of the Raleigh distribution. Also, he has demonstrated that the amplitude distribution is a mixture of Rayleigh distributions.

Goodness-of-fit tests are very important techniques for data analysis in the sense of check whether the given data fits the distributional assumptions of the statistical model. A variety of goodness-of-tests are available in the literature and recently there seems to be significant research on this topic, for more details, see, D'Agostino and Stephens (1986) and Huber-Carol et al. (2002). Correlation coefficient test is considered one of the easiest of such tests, that is because it is only needs special tables introduce from Monte Carlo simulations. The correlation coefficient test was introduced by Filliben (1975) for testing goodness-of-fit to the normal distribution and tables where updated later by Looney and Gulledge (1985). Among others Kinnison (1985, 1989) used the correlation coefficient method to present tables for testing goodness-of-fit to the extreme-value Type-I (Gumbel) and the extreme-value distribution, respectively. Recently, Sultan (2001) has devolved the correlation goodness-of-fit to the logarithmically-decreasing survival distribution. Baklizi (2005) has suggested weighted Kolmogrove-Smirnov type test for grouped Rayleigh data.

Let $X_{1:n} \leq X_{2:n} \leq \ldots \leq X_{n:n}$ be the order statistics from the Rayleigh distribution given in (1.3). Then, the pdf of the r-th order statistic is given by

$$f_{r:n}(x) = C_{r:n}\{F(x)\}^{r-1}\{1 - F(x)\}^{n-r}f(x), \qquad 0 < x < \infty, \qquad (1.4)$$

where $C_{r:n} = \frac{n!}{(r-1)!(n-r)!}$. For more details, see David (1981 and 2003) and Arnold, Balakrishnan and Nagaraja (1992). Lieblein (1955) has shown that the first single moments of the r-th order statistic from the Rayleigh distribution in (1.3) is given by

$$\mu_{r:n} = n(\pi/2)^{1/2}\binom{n-1}{r-1}\sum_{i=0}^{r-1}\frac{(-1)^{r-1-i}}{(n-i)^{3/2}}\binom{r-1}{i}. \tag{1.5}$$

In this paper, we use the single moments of the $r-$th order statistic given in (1.5) to develop the goodness of fit tests for one- and two-parameter Rayleigh distributions. In Section 2, we develop test for the one-parameter case, while in Section 3, we develop test for the two-parameter case. In Section 4, we calculate the power of the tests based on some different alternative distributions. In addition, we present some simulated examples. In Section 5, we apply the proposed test for some real data. Finally in Section 6, we draw a conclusion.

2. TEST FOR THE ONE-PARAMETER CASE

Let $X_{1:n}, X_{2:n}, \ldots, X_{n-r:n}$ denote a Type-II right-censored sample from the distribution in (1.2), and let $Z_{i:n} = X_{i:n}/\sigma, i = 1, 2, \ldots, n-r$, be the corresponding order statistics from the standard distribution in (1.3). Let us denote $E(Z_{i:n})$ by $\mu_{i:n}$, then $E(X_{i:n}) = \sigma\mu_{i:n}, i = 1, 2, \ldots, n-r$. The correlation-type goodness-of-fit test in this case may be formed as

H_0 : F is correct, that is $X_1, X_2, \ldots, X_n$ have $Ray(0, \sigma)$ given in (1.2) versus,
H_1 : F is not correct, that is $X_1, X_2, \ldots, X_n$ have another pdf,

and the statistic used to run the test is given by

$$T_1 = \sum_{i=1}^{n-r} X_{i:n}\mu_{i:n} \Bigg/ \sqrt{\sum_{i=1}^{n-r} X_{i:n}^2 \sum_{i=1}^{n-r} \mu_{i:n}^2}, \tag{2.1}$$

this statistic represents the correlation between $X_{i:n}$ and $\mu_{i:n}, i = 1, 2, \ldots, n-r$. By using the moments $\mu_{i:n}, i = 1, 2, \ldots, n-r$ given in (1.5), the statistic T_1 is simulated through Monte Carlo method based on 10,001 runs. Table 2.1 represents the percentage points of T_1 for sample sizes $n = 5, 10, 15, 20, 25, 30$ and different censoring ratios $p = \frac{n-r}{n} = 1.0, 0.8, 0.6$.

As we can see from Table 2.1, the percentage points of T_1 increases as the sample size increases as well as the significance level increases for censoring ratios $p = 1.0, 0.8, 0.6$.

3. TEST FOR THE TWO-PARAMETER CASE

Let $X_{1:n}, X_{2:n}, \ldots, X_{n-r:n}$ denote a Type-II right-censored sample from the distribution in (1.1), and let $Z_i = X_{i+1:n} - X_{1:n}$ and $\nu_i = \mu_{i+1:n} - \mu_{1:n}, i = 1, 2, \ldots, n-r-1$,

Table 2.1. Percentage Points of T_1

p	n	0.5%	1%	2%	2.5%	5%	10%	20%	30%	40%	50%
1.0	5	.9189	.9325	.9454	.9490	.9603	.9695	.9778	.9825	.9856	.9881
	10	.9525	.9590	.9662	.9683	.9741	.9800	.9853	.9881	.9901	.9918
	15	.9640	.9697	.9750	.9764	.9809	.9852	.9890	.9911	.9925	.9937
	20	.9722	.9761	.9795	.9807	.9847	.9880	.9911	.9928	.9940	.9949
	25	.9752	.9792	.9835	.9844	.9871	.9901	.9926	.9939	.9948	.9956
	30	.9804	.9826	.9856	.9866	.9891	.9914	.9935	.9947	.9956	.9962
0.8	5	.9080	.9242	.9411	.9446	.9578	.9682	.9776	.9825	.9861	.9888
	10	.9516	.9598	.9671	.9692	.9756	.9814	.9863	.9890	.9910	.9925
	15	.9666	.9723	.9768	.9784	.9826	.9862	.9900	.9919	.9933	.9944
	20	.9748	.9785	.9825	.9836	.9867	.9894	.9921	.9936	.9947	.9956
	25	.9801	.9831	.9856	.9865	.9890	.9915	.9936	.9948	.9957	.9963
	30	.9835	.9859	.9881	.9889	.9908	.9928	.9946	.9957	.9964	.9969
0.6	5	.8916	.9111	.9279	.9362	.9512	.9644	.9752	.9817	.9861	.9892
	10	.9355	.9494	.9581	.9614	.9694	.9770	.9838	.9872	.9895	.9914
	15	.9604	.9664	.9718	.9738	.9794	.9839	.9882	.9907	.9923	.9936
	20	.9708	.9745	.9786	.9799	.9840	.9875	.9908	.9925	.9939	.9949
	25	.9750	.9789	.9826	.9836	.9864	.9895	.9924	.9939	.9950	.9958
	30	.9791	.9819	.9850	.9859	.9885	.9912	.9936	.9949	.9958	.9964

where $\mu_{i:n}$ be the corresponding moments of order statistics from the standard distribution given in (1.3). The correlation-type goodness-of-fit test in this case may be formed as

H_0 : F is correct, that is $X_1, X_2, \ldots, X_n$ have $Ray(\theta, \sigma)$ given in (1.1) versus,

H_1 : F is not correct, that is $X_1, X_2, \ldots, X_n$ have another pdf,

and the statistic used to run the test is given by

$$T_2 = \sum_{i=1}^{n-r-1} (Z_i)(\nu_i) \Big/ \sqrt{\sum_{i=1}^{n-r-1} Z_i^2 \sum_{i=1}^{n-r-1} \nu_i^2}, \tag{3.1}$$

this statistic represents the correlation between Z_i and $\nu_i, i = 1, 2, \ldots, n-r$.

Once again by using the moments $\mu_{i:n}, i = 1, 2, \ldots, n-r$ given in (1.5), the statistic T_2 is simulated through Monte Carlo method based on 10,001 runs. Table 3.1 represents the percentage points of T_2 for sample sizes $n = 5, 10, 15, 20, 25, 30$ and different censoring ratios p.

From Table 2, we see that, the percentage points of T_2 increases as the sample size increases as well as the significance level increases for censoring ratios $p = 1.0, 0.8, 0.6$.

Table 3.1 Percentage Points of T_2

p	n	0.5%	1%	2%	2.5%	5%	10%	20%	30%	40%	50%
1.0	5	.8764	.8927	.9134	.9186	.9350	.9500	.9642	.9720	.9773	.9816
	10	.9158	.9317	.9447	.9485	.9598	.9697	.9780	.9827	.9857	.9880
	15	.9422	.9525	.9605	.9629	.9714	.9781	.9839	.9871	.9893	.9910
	20	.9571	.9642	.9705	.9721	.9779	.9832	.9877	.9901	.9918	.9930
	25	.9630	.9692	.9749	.9767	.9818	.9860	.9898	.9917	.9931	.9942
	30	.9701	.9757	.9798	.9810	.9849	.9882	.9913	.9930	.9941	.9950
0.8	5	.8609	.8757	.8944	.9047	.9285	.9470	.9619	.9701	.9765	.9812
	10	.9094	.9256	.9404	.9444	.9563	.9680	.9773	.9823	.9856	.9882
	15	.9442	.9534	.9615	.9643	.9718	.9787	.9847	.9880	.9901	.9918
	20	.9595	.9653	.9716	.9736	.9788	.9836	.9882	.9907	.9924	.9937
	25	.9694	.9744	.9786	.9804	.9838	.9877	.9910	.9928	.9940	.9950
	30	.9717	.9769	.9814	.9826	.9863	.9895	.9924	.9939	.9950	.9958
0.6	5	.8908	.8938	.8991	.9017	.9156	.9365	.9584	.9674	.9760	.9828
	10	.8745	.8986	.9197	.9265	.9458	.9601	.9722	.9790	.9830	.9861
	15	.9223	.9368	.9487	.9522	.9636	.9727	.9806	.9849	.9878	.9899
	20	.9459	.9546	.9631	.9659	.9733	.9798	.9855	.9887	.9907	.9923
	25	.9578	.9654	.9718	.9740	.9790	.9840	.9886	.9909	.9926	.9938
	30	.9683	.9729	.9767	.9781	.9827	.9868	.9907	.9926	.9940	.9949

4. THE POWER OF THE TESTS

In this section, we calculate the power of the proposed tests by replacing the $Ray(\theta, \sigma)$ random variates generator in the simulation programs with the generators from the alternative distributions including; normal, lognormal, cauchy, Weibull, gamma and mixture of two exponential distributions. Based on different sample sizes, different censoring ratios and 10,001 runs, the power is calculated to be

$$\text{Power} = \frac{\#\text{ of rejection of } H_0}{10,001},$$

where H_0 is rejected if $T_1(T_2) \geq$ the corresponding percentage points given in Table 2.1 (Table 2), and $T_1(T_2)$ is evaluated from the alternative distributions.

Tables 4.1 and 4.2 represent the power of the tests for the one-parameter and two-parameter cases, respectively. The different alternative distributions considered are:

1. Normal distribution $N(\mu, \sigma)$
2. Lognormal $LN(\mu, \sigma)$
3. Weibull distribution with shape a, scale parameter σ and location parameter μ, $W(\mu, \sigma, a)$,

4. Exponential distribution with scale parameter σ and location parameter μ, $EXP(\mu, \sigma)$,

5. Gamma distribution with shape parameter k, scale parameter σ and location parameter μ, $G(\mu, \sigma, k)$

6. Cauchy distribution with scale parameter σ and location parameter μ, $C(\mu, \sigma)$.

7. Mixture of two exponential distributions, $MTE(\theta_1, \theta_2, w) = wf_1(\theta_1) + (1 - w)f_2(\theta_2)$.

Table 4.1 and 4.2 indicate that the correlation test has good power to reject sample from the chosen alternative distributions. Also, the power increases as the sample sizes increase for all given censoring ratios $p = 1.0, 0.8$ and 0.6 as well as the significance level increases.

Examples:

In order to show the performance of the test for $Ray(\theta, \sigma)$ distribution in both cases (one-parameter and two-parameter), we simulate four sets of order statistics each of size 20; they are

Table 4.1. Power of The Test Based on One-parameter Case ($\sigma = 1$).

		$N(0,1)$		$W(0,1,5)$		$W(0,1,10)$		$EXP(0,1)$		$LN(0,1)$	
p	n	5%	10%	5%	10%	5%	10%	5%	10%	5%	10%
1.0	5	.9748	.9802	.1743	.3806	.7338	.9282	.5137	.6038	.4751	.5590
	10	.9987	.9993	.6585	.8407	.9996	.9999	.7462	.8128	.7376	.7963
	15	1.0000	1.0000	.9264	.9791	1.0000	1.0000	.8726	.9171	.8762	.9130
	20	1.0000	1.0000	.9891	.9980	1.0000	1.0000	.9383	.9651	.9404	.9594
	25	1.0000	1.0000	.9987	.9997	1.0000	1.0000	.9704	.9833	.9704	.9839
	30	1.0000	1.0000	.9999	1.0000	1.0000	1.0000	.9880	.9941	.9892	.9943
0.8	5	.9758	.9806	.0342	.1683	.1782	.5930	.4204	.5242	.2859	.3719
	10	.9989	.9991	.3941	.6234	.9704	.9959	.6453	.7355	.4712	.5646
	15	1.0000	1.0000	.7216	.8590	.9999	1.0000	.7884	.8494	.5993	.6804
	20	1.0000	1.0000	.9102	.9614	1.0000	1.0000	.8774	.9179	.7131	.7778
	25	1.0000	1.0000	.9711	.9924	1.0000	1.0000	.9218	.9530	.7725	.8363
	30	1.0000	1.0000	.9917	.9980	1.0000	1.0000	.9568	.9746	.8304	.8843
0.6	5	.9776	.9801	.0000	.0260	.0000	.0888	.3505	.4512	.1510	.2360
	10	.9993	.9994	.1167	.3086	.5913	.8726	.5217	.6201	.2338	.3256
	15	1.0000	1.0000	.4207	.6306	.9762	.9951	.6653	.7442	.3248	.4163
	20	1.0000	1.0000	.6541	.8195	.9993	.9998	.7632	.8300	.3776	.4737
	25	1.0000	1.0000	.8027	.9106	1.0000	1.0000	.8187	.8762	.4245	.5258
	30	1.0000	1.0000	.9028	.9639	1.0000	1.0000	.8736	.9183	.4761	.5815

Table 4.2. Power of The Test Based on Two-parameter Case $\theta = 0.0, \sigma = 1.0$.

		$LN(1,5)$		$W(0,1,0.3)$		$MTE(4,.2,.5)$		$G(0,1,0.4)$		$C(0,1)$	
p	n	5%	10%	5%	10%	5%	10%	5%	10%	5%	10%
1.0	5	.8614	.9053	.7312	.8083	.4911	.5983	.3906	.5004	.2825	.3757
	10	.9984	.9991	.9824	.9905	.8826	.9263	.7935	.8625	.5899	.6779
	15	1.0000	1.0000	.9997	1.0000	.9842	.9925	.9459	.9705	.7788	.8427
	20	1.0000	1.0000	1.0000	1.0000	.9984	.9993	.9873	.9944	.8905	.9308
	25	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9980	.9995	.9463	.9677
	30	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9997	.9998	.9757	.9865
0.8	5	.6449	.7044	.5304	.6064	.3520	.4333	.2853	.3696	.0644	.2069
	10	.9790	.9868	.9291	.9589	.7878	.8530	.6697	.7604	.3945	.5161
	15	.9989	.9994	.9944	.9968	.9514	.9688	.8905	.9343	.6339	.7195
	20	1.0000	1.0000	.9997	.9997	.9908	.9953	.9654	.9818	.7911	.8417
	25	1.0000	1.0000	1.0000	1.0000	.9979	.9986	.9923	.9957	.8888	.9224
	30	1.0000	1.0000	1.0000	1.0000	.9993	.9994	.9987	.9993	.9371	.9552
0.6	5	.4121	.5202	.3379	.4450	.1644	.2689	.2014	.2995	.0322	.0613
	10	.8568	.9043	.7856	.8570	.4820	.5813	.5041	.6153	.2157	.3504
	15	.9763	.9867	.9487	.9700	.6903	.7626	.7477	.8280	.4803	.5963
	20	.9970	.9985	.9921	.9956	.8150	.8619	.8968	.9359	.6771	.7597
	25	1.0000	1.0000	.9988	.9991	.8932	.9261	.9576	.9730	.8080	.8646
	30	1.0000	1.0000	.9996	.9999	.9323	.9585	.9860	.9928	.8853	.9233

1. Sample from $Ray(0,1)$: one-parameter case of Rayleigh distribution with location parameter is equal to 0 and scale parameter is equal to 1.
2. Sample from $Ray(1,2)$: two-parameter case of Rayleigh distribution with location parameter is equal to 1 and scale parameter is equal to 2.
3. Sample from $W(0,1,10)$: two-parameter case of the Weibull distribution with location parameter is equal to 0, scale parameter is equal to 1 and shape parameter equal to 10.
4. Sample from $LN(1,5)$: two-parameter case of the lognormal distribution with location parameter is equal to 1 and scale parameter is equal to 5.

The order statistics samples with the analogous moments of order statistics from $Ray(0,1)$, Table 2.1 and Table 3.1 are used to run the test. The results of the tests at 5% significance level are given below:

		Decision	
Distribution	P	90%	95%
$W(0,1,10)$	1.0	R	R
	0.8	R	R
	0.6	R	R
$Ray(0,1)$	1.0	A	A
	0.8	A	A
	0.6	A	A

		Decision	
Distribution	P	90%	95%
$LN(1,5)$	1.0	R	R
	0.8	R	R
	0.6	R	R
$Ray(0,1)$	1.0	A	A
	0.8	A	A
	0.6	A	R

5. APPLICATION

The set of data on the endurance of deep ball bearings analyzed by Lieblein and Zelen (1956) consists of the number of million revolutions before failure for each of the 23 ball bearings in life test. The first 20 of the data are:
17.88, 42.12, 51.96, 68.84, 93.12, 127.96, 28.92, 45.60, 54.12, 68.64, 98.64, 128.01, 33.0, 48.48, 55.56, 68.88, 105.12, 173.4, 41.52, 51.84
The required moments of order statistics when are calculated from (1.5). Then by using these moments and data life, we calculate the value T_1 given in (2.1) for different values of p as follows:

p	Calculated T_1	Simulated T_1 from Table 2.1	
		5%	10%
1.0	.9933	.9848	.9880
0.8	.9926	.9866	.9895
0.6	.9908	.9838	.9874

As we can see the Rayleigh distribution fits the data at 5% and 10%. So, we recommend the Rayleigh distribution for the given data. This is also clear from the probability plots and the empirical cumulative distribution function (EDF) in Figure 5.1.

6. CONCLUSION

The goodness of fit test of the Rayleigh distribution (one-parameter and two-parameter cases) have been developed. The power of the proposed tests based on some other distributions are also calculated by using Monte Carlo simulation as presented in Tables 4.1 and 4.2. As we can see from Table 4.1 and 4.2, the test has good power to reject the Rayleigh distribution Ray(θ, σ) when data are coming from the considered alternative distributions. Finally, an application is investigated.

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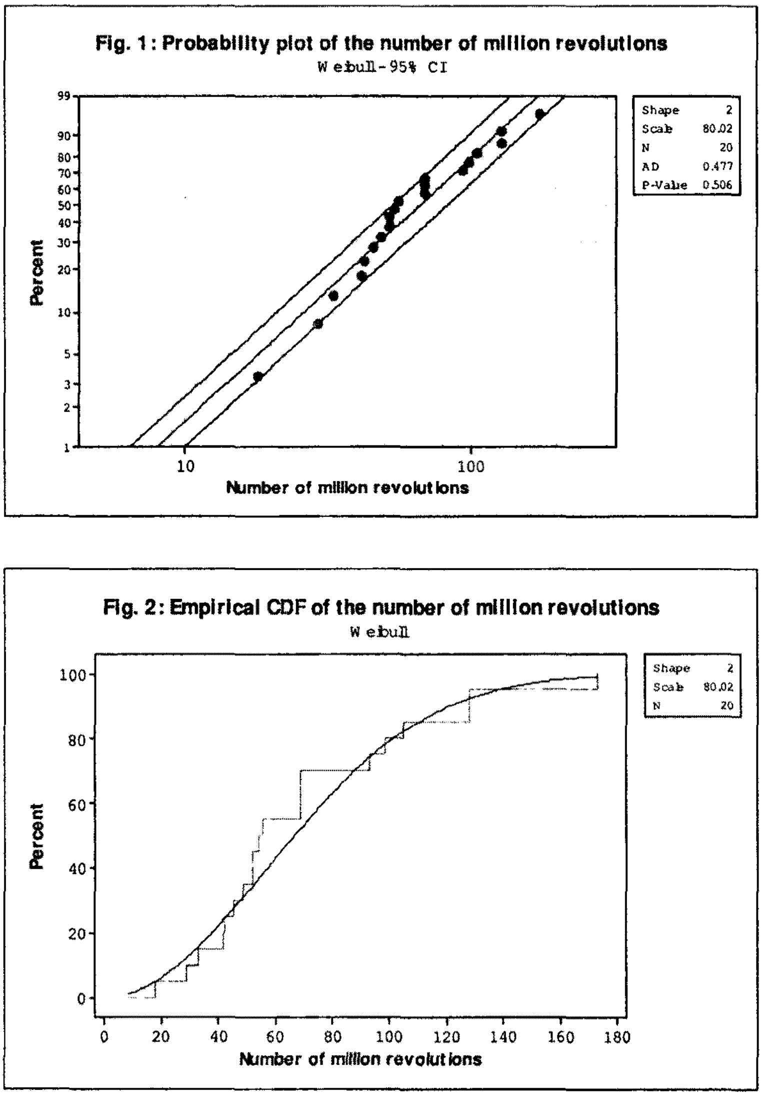

Figure 5.1. Probability plot of the number of million revolutions and empirical CDF of the number of million revolutions