

Singularity-Circumvented Computation of Green's Functions for 2D Periodic Structures in Homogeneous Medium

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Abstract

This paper suggests a novel method to efficiently calculate the spatial-domain Green's functions of 2D electromagnetic problems. Briefly speaking, this method combines spectral and spatial domain calculation schemes and prevents the Green's functions from poor convergence due to the singularities that complicate the process of the Method of Moment(MoM) applications. For the validation of this proposed method, fields will be evaluated along the spatial distance including zero distance for 2D free-space and periodic homogeneous geometry. The numerical results show the validity of the proposed method and corresponding physics.

Key words : Integro-Differential Equation, Green's Function, 2D Periodicity, Singularity, Radiation and Scattering.

I. Introduction

Much of researchers' attention has been paid to two-dimensional(2D) electromagnetic radiation and scattering problems. Employing integro-differential or integral equations to model the physical phenomena, which are to be numerically solved by the frame work of the MoM, the impulse response Green's functions are the focal point of deciding the efficiency as well as validity of the solutions^[1].

Green's functions are calculated in the spatial domain with the spatial basis and weighting functions during the matrix fill-up process, and they will diverge at singularities where the source and observation points coincide or one of the two is in the other's proximity. Coping with the divergence, Rao *et al.* provided a technique that splits the original Green's function calculation into the numerically stable and analytically integrable parts^{[2]-[4]}. Though this has still been used in much of concerned literature, the efficient treatment of the numerical part totally depends on truncation-related conditions.

This paper presents a novel method to efficiently figure out 2D free-space periodic problems^{[5],[6]} expressed on the spatial domain basis. In accordance with this proposed method, the spatial domain calculation is carried out conventionally for nonsingular spatial points, and is replaced by its counterpart of the spectral domain only for the singularities to avoid the Green functions' divergence at the singularities which complicates the application of the Method of Moment(MoM) to the electromagnetic characterization. In order to validate this

proposed method, the fields of 2D free-space and periodic structure are calculated. Also, the Green's functions of both the cases are handled with the adoption of the proposed method to treat the point where the distance becomes zero. The comparison shows the results obtained by the proposed method are in good agreement with those of the conventional method, and observe what will happen as physics.

II. Theory

Two dimensional(2D) electromagnetic scattering problems have drawn much of researchers' concern and interest in such as radar cross-section(RCS), Nondes-structive Inspection, Waves' Propagation Constant solutions, and so on. The problems are modeled as integro-differential or integral equations, and these equations are discretized and converted to matrix equations which are appropriate for computer work by way of electromagnetic numerical analysis methods like the Method of Moment(MoM)^[1]. In the MoM application, the radiation integral possesses the Green's function corresponding to impulse source-and-field relations, essential to and influencing the overall time of the MoM solution process^[1]. The Green's function for the 2D free-space geometry is the 0th order Hankel function of the second kind as

$$G(\bar{\rho} - \bar{\rho}') = H_0^{(2)}(k_i |\bar{\rho} - \bar{\rho}'|) \quad (1)$$

$\bar{\rho}$, $\bar{\rho}'$, ω and μ mean the field point vector, the source point vector, the angular frequency and permeability, respectively. $k_i = \omega \sqrt{\epsilon \mu}$. The position vectors are

$$\bar{\rho} = x\hat{x} + y\hat{y}, \quad \bar{\rho}' = x'\hat{x} + y'\hat{y}, \quad \bar{r} = \bar{\rho} - \bar{\rho}' \quad (2)$$

While the impedance matrix is filled via basis-expansion and weighting in a spatial domain problem, Eqn. (1) is numerically well-behaved except the singularity occurring when its argument goes to zero or the field point coincides with the source point. In Eqn. (1), the singularity point corresponds to $|\bar{\rho} - \bar{\rho}'| = 0$, where the denominator of Neumann function becomes 0. This entails cumbersome steps of mitigating singularity or reducing the order of singularity with the help of numerical extraction of singularity or approximation^[2].

The new method is proposed to circumvent the singularity of Eqn. (1) using both the spectral form as well as spatial form regarding a spatial-domain based Green's function calculation. The detailed explanation starts as follows. Eqn. (1) of the spatial domain can be expressed in the spectral domain as

$$G(\bar{\rho} - \bar{\rho}') = \int_{-\infty}^{+\infty} \tilde{G}(k_x) dk_x = \int_{-\infty}^{+\infty} \frac{e^{-jk_y|y-y'| - jk_x(x-x')}}{j2k_{iy}} dk_x \quad (3)$$

where j , $\mathcal{T}(k_x)$, k_x and k_{iy} are $\sqrt{-1}$, the spectral domain Green's function, wave numbers in x and y directions, respectively^{[3],[4]}. The corresponding dispersion relation is

$$k_x^2 + k_{iy}^2 = k_i^2 \quad (4)$$

The Sommerfeld type integral like Eqn. (3) customarily takes the integration path in the spectral domain as in Fig. 1.

The simple numerical experiment reveals the the phase of the integrand of Eqn. (3) grows and tends to prevent convergence with the increasing distance between the source and field points. Conversely, it is noteworthy that Eqn. (3) converges very fast and makes calculation most efficient when $\bar{\rho}$ approaches $\bar{\rho}'$. Thus, it is proposed that Eqn. (3) is adopted for only the spatial singularity point and Eqn. (1) for all the other spatial

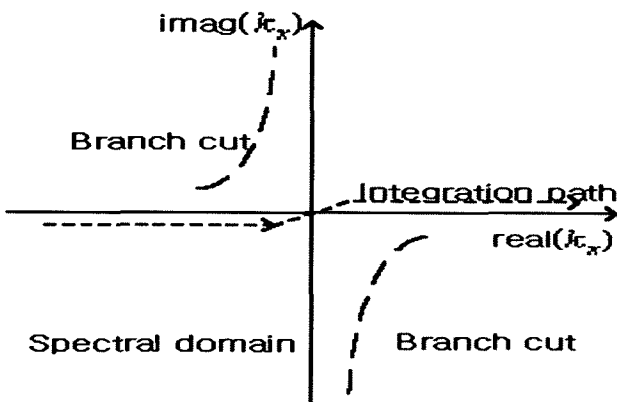


Fig. 1. Sommerfeld integration path.

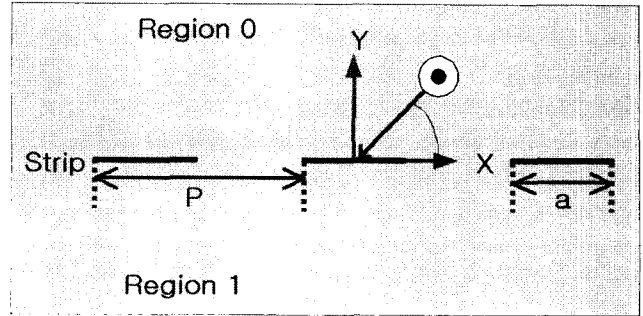


Fig. 2. TM incidence on periodic strips interfacing regions 0 and 1 that have the same permittivity ϵ_1 : Incidence angle is ϕ_0 .

points, taking only the advantages of Eqn.'s (1) and (3).

Now strips are periodically positioned in one medium as in Fig. 2, in common with [5], [6].

The Green's function $G(\bar{\rho} - \bar{\rho}')$ here can be expressed as

$$G(\bar{\rho} - \bar{\rho}') = \sum_{n=-\infty}^{\infty} H_0^{(2)}(k_0 \sqrt{(\Delta_x - nP)^2 + \Delta_y^2}) e^{-n\beta P} \quad (5)$$

where

$$\beta = -k_0 \cos \phi_0, \quad \Delta_x = x - x' \quad \text{and} \quad \Delta_y = y - y'$$

P and a means periodicity and width of the strips. The computation time of Eq. (2) grows due to the oscillatory phase- and singularity-oriented slow convergence of the Green's function.

III. Numerical Validation

A couple of examples are presented for proving the validity of the proposed method. As the first example, before going to the periodic case, the 2D Free-space Green's function is used to represent the field behavior. Firstly, we assume that one 2D transverse lineal strip with one end as the source point (where electric current line source is placed), and the observation is made at an arbitrary point along the line. Here we consider the 2D Hankel function the Z-component of the electric field and the argument of the Green's function is the wave-number times the distance between the source and observation points. Varying the observation point, the Green's function is evaluated at frequency of 300 MHz by the proposed and conventional methods.

Conventionally, a number of works have attempted to use the approximate expressions and carefully approach the singularity. However, they can not exceed the limitations of the approximation. As seen in Fig. 3, the conventional method produces discontinuity due to the avoidance of the spatial domain logarithmic singularity,

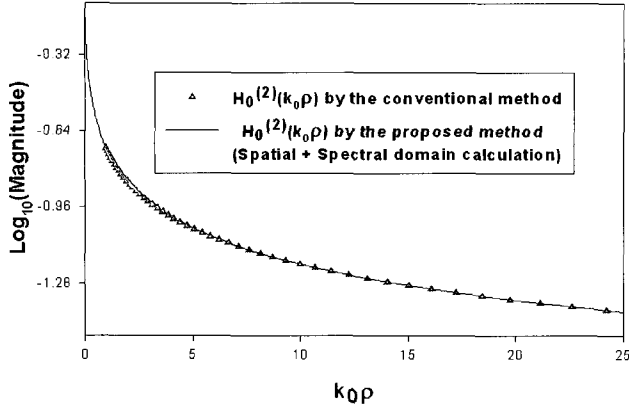


Fig. 3. Green's function of the 2D free-space.

but the proposed method returns the favor and presents the finite field value in a stable manner. As a sophisticated application of the proposed method from the simple case above, it can be exploited to see the electromagnetics on the following structure.

In this test geometry, there is a dielectric substrate whose bottom and top are interfacing with the PEC backing and the thick square PEC cylinder, respectively. Besides, we can see another PEC scatterer above the PEC-backed slab and distant from the conductor 1. The input parameters w_1 (conductor 1's width), h_1 (conductor 1's length) and h_2 (conductor 2's length) are set identically as $1 \lambda_0$ for the sake of convenience, with w_2 (conductor 2's width) assumed much smaller than h_2 . h_{strip} (conductor 2's height), spacing between the two conductors, d_r (thickness of the slab) are given $15 \lambda_0$, $10 \lambda_0$ and $2.5 \cdot 10^{-3} \lambda_0$ in order. The substrate permittivity is provided with 2.2. As for the illumination, the Transverse Magnetic(TM) wave is applied with the angle

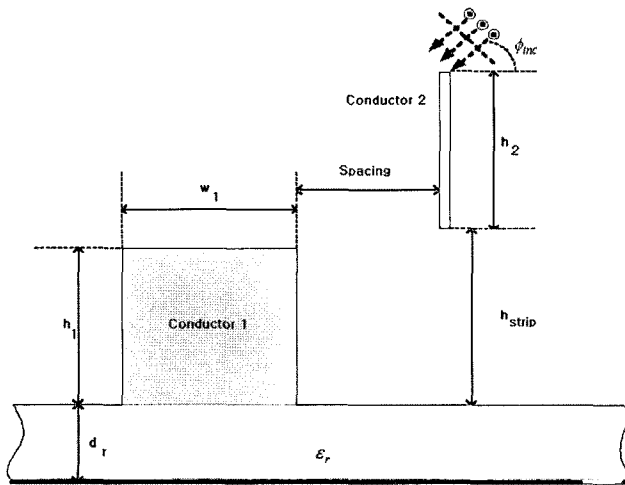
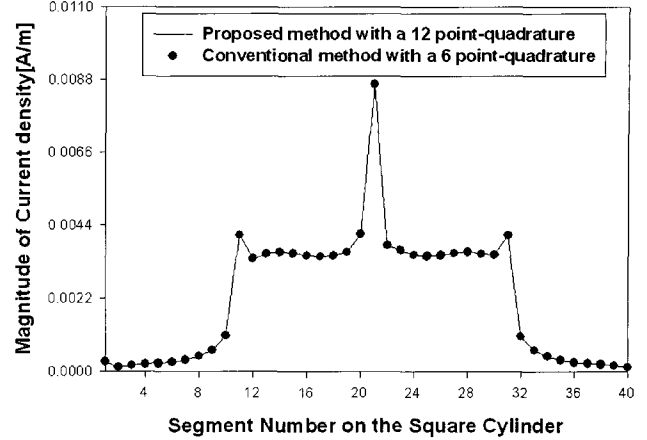
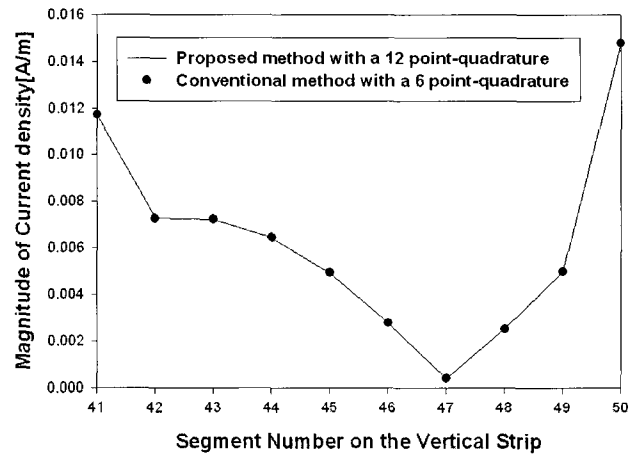


Fig. 4. TM illumination onto conductor 2(strip) apart from conductor 1(cylinder) on the PEC-backed slab.



(a) Electric current density vs. the segments of conductor 1



(b) Electric current density vs. the segments of conductor 2

Fig. 5. Electric current densities on the conductors in Fig. 4.

of 45° . The electric current densities on the conductors' surfaces are obtained as follows. As useful information to see the results, all the segments assigned to the conducting surfaces are numbered sequentially from the left bottom vertex of conductor 1 along the conductor 1's closed line and continually through the lowest point of conductor 2 to its top.

Regarding the current induced on conductor 1, it appears almost the same as J_{sz} in [7] that deals with only the part corresponding to segments 22 through 40 (or segments 18 back through 1). Particularly, segment 20 coincides with the vertex that faces directly the impinging wave and most of the charges gather (edge behavior). The accumulation of the charges at the edges is also seen in conductor 2's case. It is noticeable that during the numerical integration to fill-up the impedance matrix, the conventional calculation technique requires the quadrature points twice more than the points for the proposed method, to have the same accuracy. This represents that the present method saves the overall

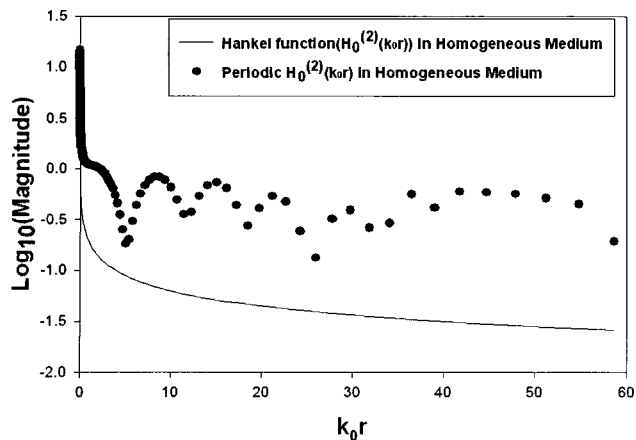


Fig. 6. Comparing the Green's functions of the 2D free-space and a periodic structure in the homogeneous media example with P of 1λ and 1 GHz.

computation time and accuracy, while the other technique is very likely to run across the singularity with having more quadrature points.

Now the work will concern the structure of 2D periodic strips specified with P of 1λ (wavelength) and frequency of 1 GHz. With regard to this, Eq. (5) is used as the Green's function whose computation method can be verified, compared with $H_0^{(2)}(k_0 |\bar{\rho} - \bar{\rho}'|)$ as is usually done. The results of them are given in Fig. 6.

The free space Green's function shows the typical behavior of $r^{-1/2}$ variation with the increasing distance. On the contrary, the result of Eq. (3) is calculable at distance zero and varies up and down, representing the mutual couplings of nearby strips, and will approach that of the free space if the periodicity is assumed

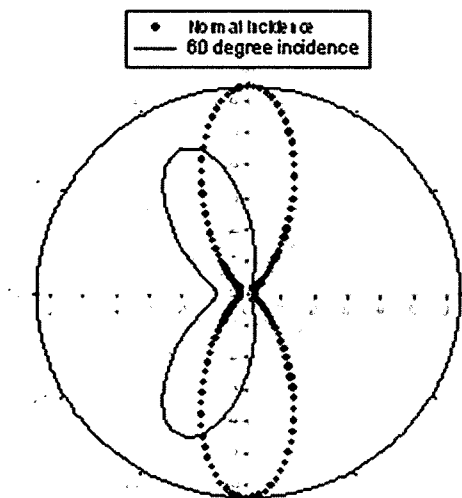


Fig. 7. Echo width results due to the TM wave incidence with normal angle and 60° .

infinite. Based upon the evaluation of the 2D Green's function for the periodic structure, we are going to have the following scattering behaviors with different angles of TM wave illuminations.

The echo width(2D version of Radar Cross Section) is presented through the far-field radiation integral with the current obtained by the proposed method, which agrees well with physics. Last but not least, it is noted that the computation time of the proposed method amounts to 90 seconds less than that of the others, say, about 200 seconds.

IV. Conclusion

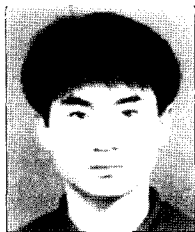
A new and efficient method is proposed to calculate the spatial-domain 2D free-space periodic Green's functions for electromagnetic radiation and scattering problems. This method can simply circumvent the very poor convergence problems resulting from the spatial singularities by combining spectral and spatial domain calculation schemes and extended from simple free-space solution to solving periodic structures in homogeneous media.

This work was supported by grant No.(KRF-2006-311-D 00739) of the Korea Research Foundation.

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