한국소음진동공학회논문집 제 17 권 제 7 호, pp. 596~604, 2007.

Effects of Geometric Configuration on the Vibro-acoustic Characteristics of Radial Vibration of an Annular Disc
환형 디스크 형상이 래디얼 진동에 의한 음향방사 특성에 미치는 영향

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(Received April 10, 2007 ; Accepted June 4, 2007)

Key Words : Annular Disc(환형 디스크), Radial Mode(래디얼 모드), Sound Radiation(음향방사)

ABSTRACT

This article investigates the effects of geometric configuration on the vibro-acoustic characteristics of in-plane vibration of a thick annular disc. Disc thickness and outer radius for a given inner radius are selected as independent variables having reasonable ranges. Variations in structural eigensolutions for radial modes are investigated using pre-developed analytical method. Based on these data, far-field sound pressure distributions due to the modal vibrations for a given geometry are also calculated using an analytical solution. Modal sound powers and radiation efficiencies are calculated from the far-field sound pressure distributions and vibratory velocity distributions on the radial surfaces. Based on the results explained above, the geometric configuration that minimizes modal sound radiations in a given frequency range is determined. Finally sound power and radiation efficiency spectra for a unit harmonic force from the selected geometric configuration are obtained from structural and acoustic modal data using the modal expansion technique. Multi-modal sound radiations of the optimized disc that are obtained using proposed analytical methods are confirmed with numerical results. Using the procedure introduced in this article, sound radiation due to in-plane modes within a specific frequency range can be minimized by the disc geometry modifications in a comprehensive and convenient manner.

요 약

이 논문은 후판 환형 디스크의 기하학적인 형상이 래디얼 방향 진동에 의해 방사되는 소음에 미치는 영향을 연구하였다. 디스크의 내경을 고정한 상태에서 두께와 외경을 주어진 범위내에서 변경하면서 이론적인 해를 이용하여 래디얼 모드의 고유진동수와 진동모드의 변화를 검토하였다. 이 결과를 이용하여, 해당 형상을 가진 디스크의 래디얼 방향 고유진동에 의해 발생되는 원격음장, 음향파워 및 방사효율을 계산하였다. 이 결과로부터 고유진동에 의한 음향파워와 방사효율을 최소화할 수 있는 기하학적인 형상을 선택하였다. 마지막으로, 최적화된 디스크의 임의 위치에 단위 하모닉 가진을 가했을 경우에 발생되는 음향파워 및 방사효율 스펙트럼을 구하였으며, 전산해석을 통해 그 정확성을 검증하였다. 이 논문에 소개된 방법을 적용하면 주어진 제한 조건을 만족하면서 목표 주파수 범위내에서 래디얼 진동에 의해 발생되는 소음을 최소화할 수 있는 기하학적인 형상을 간편하고 논리적으로 구할 수 있다.

1. Introduction

Thick annular bodies that can be used to model mechanical or structural components such

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as brake rotors, clutches, flywheels, railway wheels, circular saws, and electrical machinery stators often generate tonal sounds corresponding to in-plane structural vibration modes[1~5]. So, it is necessary to adjust vibro-acoustic characteristics for in-plane modes to efficiently control sound radiation from these components. Even though geometrical alteration is a well-known, traditional way of reducing structural vibration and sound radiation from the structure, this procedure has been applied in a 'trial and error' manner using either experimental or numerical approaches. This is due to the complexity of the problems and the lack of appropriate analytical solutions for vibro-acoustic characteristics of the corresponding structure. But, in the thick annular disc case, structural eigensolutions for in-plane modes are well defined[5,6] and we introduced appropriate analytical solutions for modal sound radiations [6,7].

In this study, as a following step of the previous works introduced in Ref. (6), the effects of geometric configuration on the vibro-acoustic characteristics of a thick annular disc are investigated utilizing pre-developed analytical solutions that is briefly introduced in the first part of each section in this article. Figure 1 illustrates the example disc that is stationary with free-free boundaries and made of an undamped, isotropic material. Table 1 provides nominal values of disc geometry along with material properties that are fixed in this study.

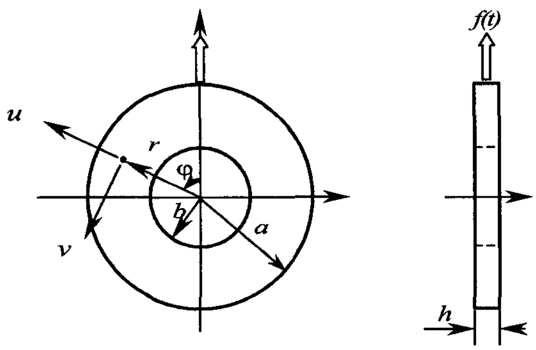

Fig. 1 A thick annular disc

Table 1 Disc dimensions and material properties

Outer diameter (a)	139.0 mm
Inner diameter (b)	82.5 mm
Thickness (h)	31.5 mm
Mass density (ρ_d)	7905.9 kg/m^3
Young's ratio (E)	218 GPa
Poisson's ratio (ν)	0.305

Through the parametric studies on disc thickness and outer radius with the inner radius fixed to its nominal value, optimal geometric configuration that minimize modal sound radiation within a given frequency range is obtained. In addition, multi-modal sound radiations from a disc having this optimal geometric configuration are investigated and compared with those from the original disc. The results are confirmed using numerical results.

Primary assumptions are as follows: (1) Structural and acoustic systems are linear time-invariant. (2) The complicating effects such as fluid loading and acoustic scattering from the disc edges are negligible. (3) Free and far field sound pressure at an observation point (r_p) is generated only by the vibratory motions at the inner or outer radial surfaces and normal surfaces do not contribute to total far-field sound pressure.

Chief objectives of this article are as follows. (1) Examine effects of geometric configuration on the structural modal dataset. (2) Investigate variations in modal sound radiation introduced by the alterations in the disc geometry. (3) Suggest the optimal outer radius and thickness within given ranges for a nominal inner radius. (4) Study effects of geometric alteration on the multi-modal sound radiation from a thick annular disc.

2. Effects of Disc Geometry on the Structural Eigensolution

As one can imagine, structural eigensolutions of a thick annular disc are affected by the geometric configuration as well as by its material properties. To examine the effects of outer diameter a and thickness h on the structural eigensolutions for a given inner radius b, two non-dimensional parameters, radii ratio ($\beta=b/a$) and thickness ratio ($\alpha=h/a$) are defined. Then non- dimensional eigenvalue (λ_q) and mode shape (ψ_q) for the selected modes are evaluated varying above two parameters within the appropriate ranges.

For a given geometric configuration, the output variables are obtained using the transfer matrix method that has been introduced by Irie et al.[5] and Lee[6]. For the better understandings, overall procedure of this method is briefly outlined here. In this method, the displacements and forces at a radial position are related to those at the inner radial edge by transfer matrix $[T(\xi)]$ as

$$\{z(\xi)\}=[T(\xi)]\{z(\beta)\} \tag{1}$$

where, $\xi=r/a$ and $\beta=b/a$ represent normalized radial coordinates at an arbitrary radial location (r) and at the inner edge ($r=b$) respectively. Refer to Fig. 1 and Table 1 for more information. Furthermore, $\{z(\xi)\}$ is the state vector that expresses normalized displacements and forces at x and satisfying following relation

$$\frac{d}{d\xi}\{z(\xi)\}=[U(\xi)]\{z(\xi)\} \tag{2}$$

From Eq. (1) and (2), the transfer matrix at frequency ω was obtained from the following relation

$$[T(\xi)]=\exp\left(\int_\beta^\xi [U(\xi')]d\xi'\right)=[I]+\frac{1}{1!}\left[\int_\beta^\xi [U(\xi')]d\xi'\right]+\frac{1}{2!}\left[\int_\beta^\xi [U(\xi')]d\xi'\right]^2+\cdots \tag{3}$$

By applying the boundary conditions at $\xi=\beta$ and $\xi=1$, the modal data set (λ_q, ψ_q) for the q^{th} radial mode was determined from the transfer matrix $[T(\xi)]$. The accuracy of this approach has been confirmed in the previous study by comparing the results with numerical and experimental results[6].

First, the effect of β on the non-dimensional eigenvalues is examined for selected free-free radial modes in the range of $0.5\le\beta\le0.7$. In our investigation, β is controlled by adjusting a for the b and h fixed to their nominal values.

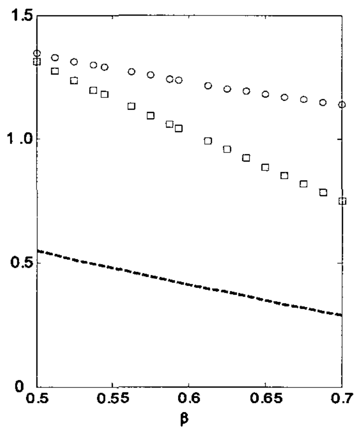

Fig. 2 Effect of the radii ratio ($\beta=b/a$) on the non-dimensional eigenvalues. Key: - - -, q=2 mode; ○○○, q=0 mode; □□□, q=3 mode

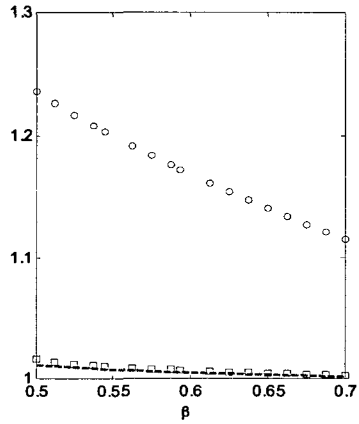

Fig. 3 Effect of the radii ratio ($\beta=b/a$) on the mode shapes. Key: - - -, q=2 mode; ○○○, q=0 mode; □□□, q=3 mode

Results are shown in Fig. 2 where normalized eigenvalues for selected in-plane modes are clearly dependent on the radii ratio in the given range. Note that the non-dimensional eigenvalues of 3 radial modes are inversely proportional to β in the given range. The effects of β on the mode shapes are also studied. Since, the displacements of radial modes are assumed to be sinusoidal, mode shape for a given β is characterized by the modal displacement ratio ($U_b=U_{Iq}/U_{Oq}$). Here, U_{Iq} and U_{Oq} are displacement amplitudes at the outer and inner edges for radial mode q respectively.

The results are given in Fig. 3 where U_b for 3 selected modes approach 1 as β increases. But, the rate of change is mode dependent.

The U_b for q=0 mode is most sensitive to β and those for the q=2 and q=3 mode are almost constant in the given range.

Finally, the effects of thickness ratio ($\alpha=h/a$) on the non-dimensional eigenvalues are examined for first three radial modes of the sample disc. As in the case of the radii ratio, non-dimensional parameter α is controlled by adjusting h for the fixed a and b. But, as one can figure out from the details of Eqs. (1~3) where all the components of the transfer matrix are independent of disc thickness, the structural eigenvalues of radial modes for given inner and outer radii are independent of disc thickness. Consequently, ϕ_q and U_β for the 3 modes are also constant regardless of α. So, the results are not explained in this article.

3. Effects of Disc Geometry on the Modal Sound Radiation

Previous studies illustrate that sound radiation from a vibrating structure depends on its geometry, vibrating mode and frequency(9~12). Generally, geometric configuration of a thick annular disc affects its modal sound radiation in two different ways. First, as one can see from Section 2, natural frequencies and modes shapes of a thick annular disc are affected by its geometry and these alterations affect the modal sound radiations from the disc. On the other hand, since the sizes and locations of sound generating surfaces are determined by geometric configuration of the disc, its modal sound radiations are directly affected by the disc geometry. In this section, the effects of α and β on modal sound radiation are studied by investigating variations of modal acoustic power Π_q and the modal radiation efficiency σ_q accompanied by alterations in these parameters. As in the case of the structural analysis, Π_q and σ_q are analytically evaluated varying α and β within the given ranges.

First, modal surface velocities of the disc having a specific geometric configuration are defined from the mode shapes and natural frequencies. Then, modal far-field sound pressures are calculated from these modal surface velocities using following Eq. (6). Refer to Fig. 4 for the relevant configuration.

$$P_q(R,\theta,\phi) = P_{qI}(R,\theta,\phi) + P_{qO}(R,\theta,\phi) \tag{4a}$$

$$P_{qO}(R,\theta,\phi) = \frac{\rho_0 e^{ik_q R}}{\pi k_q R\sin\theta}\left|\ddot{u}_{qO}\right| h \times \frac{Sinc(k_q \sin\theta\, h/2)(-i)^{q+1}}{H_q^{1\,\prime}(k_q a\sin\theta)}\cos q\phi \tag{4b}$$

$$P_{qI}(R,\theta,\phi) = \frac{\rho_0 e^{ik_q R}}{\pi k_q R\sin\theta}\left|\ddot{u}_{qI}\right| h \times \frac{Sinc(k_q \sin\theta\, h/2)(-i)^{q+1}}{H_q^{2\,\prime}(k_q b\sin\theta)}\cos q\phi \tag{4c}$$

$$Sinc(x) = \frac{\sin(x)}{x}. \tag{4d}$$

Here, k_q is the acoustic wave number of the q^{th} mode, H_q is the q^{th} order Hankel function, $|\ddot{u}_{qO}|$ and $|\ddot{u}_{qI}|$ are the amplitudes of vibratory acceleration on outer and inner radial edges respectively. The modal sound radiation for the q^{th} radial mode(Γ_q) is defined from the calculated sound pressures on a large sphere surrounding the disc.

Finally, Π_q and σ_q are obtained from the sound pressure data. Details and validation of this procedure can be found in (6). In this particular study, the amplitudes of modal vibrations are intentionally adjusted to get the same modal velocity amplitudes regardless of variations in the natural frequencies for a given geometric configuration.

3.1 Effect of the Radii Ratio

First, the effect of b is investigated in the range of 0.5~0.7 which is used in the structural analysis. The results of this investigation are summarized in Figs. 5 and 6 where σ_q are clearly affected by β. Note that the characteristics of variations are mode dependent. Π_q and σ_q for q=2 mode fluctuate slowly and have their maximum at β=0.58. Also, Π_q and σ_q for q=3 mode fluctuate much more rapidly than those of q=2 mode having 2 valleys in the given range. On the other hand, the factors for q=0 mode monotonically decrease with β without any significant fluctuation within the given range.

For a limiting case of $\beta \to 0$ when the annular disc turns into a circular disc, that is out of the scope if this study. And, for the other limiting case when $\beta \to 1$, the annular disc can be considered as a thin cylinder that can be solved using the method explained in references (9, 10).

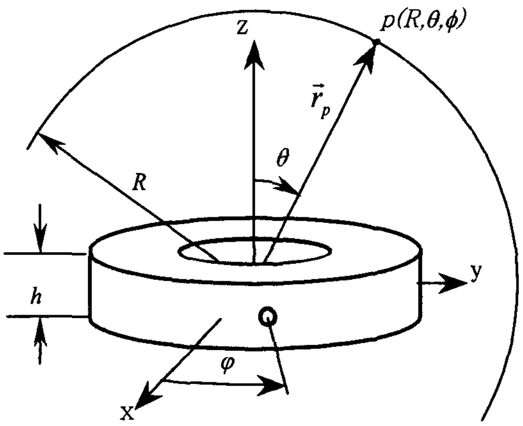

Fig. 4 Sound Radiation from the radial vibration of a thick annular disc in spherical coordinate system

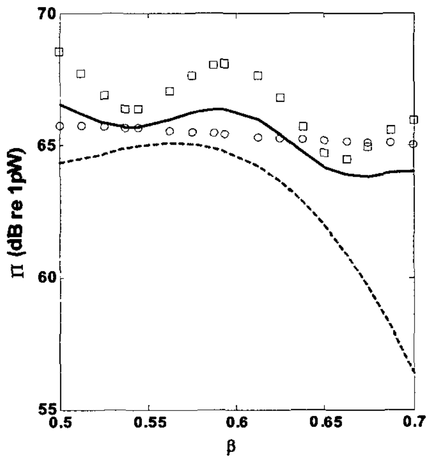

Fig. 5 Effect of the radii ratio ($\beta=b/a$) on the modal sound powers. Key: - - -, q=2 mode; ○○○, q=0 mode; □□□, q=3 mode; ——, averaged

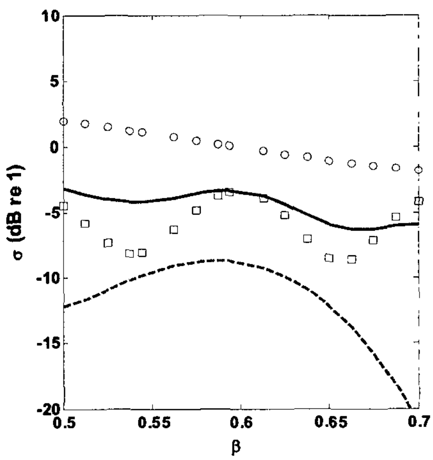

Fig. 6 Effect of the radii ratio ($\beta=b/a$) on the modal sound radiation efficiencies. Key: - - -, q=2 mode; ○○○, q=0 mode; □□□, q=3 mode; ——, averaged

3.2 Effect of the Thickness Ratio

Next, the thickness ratio α is selected as the independent variable and it is varied from 0.025 to 0.35 with a nominal value α=0.227.

Even though the structural eigensolutions of an annular disc are independent of α, sound radiation from the modal vibrations are affected by the a since overall radiating area for a given outer radius is determined by α. Results are summarized in Figs. 7 and 8. As expected, Π_q and σ_q monotonically increase with increase in α. Also, as one can see from Eq. (4), both P_qI and P_qO vanish for all modes in the limiting case of $\alpha \rightarrow 0$. Consequently, Π_q=0 and σ_q cannot be defined for any q.

Finally, as shown in Fig. 7, sound power from radial mode is very small when a is smaller than 0.05. Therefore, sound radiation from the radial modes can be ignored over the range of $\alpha \leq 0.05$.

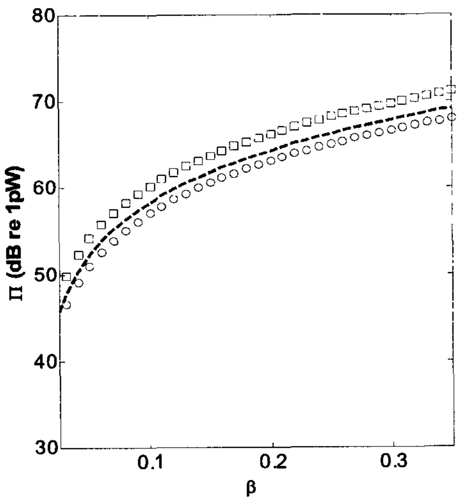

Fig. 7 Effect of the thickness ratio ($\alpha=h/a$) on the modal sound powers. Key: - - -, q=2 mode; ○○○, q=0 mode; □□□, q=3 mode

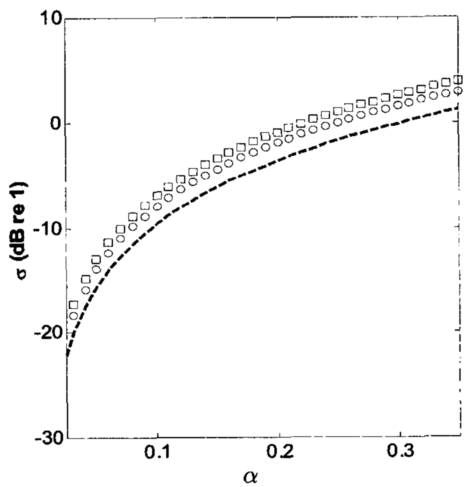

Fig. 8 Effect of the thickness ratio ($\alpha=h/a$) on the modal sound radiation efficiencies. Key: - - -, q=2 mode; ○○○, q=0 mode; □□□, q=3 mode

3.3 Optimal Configuration Based on Modal Sound Radiations

Since acoustic system of this study is assumed to be linear time invariant, overall sound radiation from the disc can be expressed as linear combination of the individual modal sound radiations of the disc. So, an optimal configuration that has minimum sound radiation in a given frequency range can be estimated as a configuration that has minimum modal sound radiation within that frequency range. Eqs. (4~5) state that P_q depends on disc geometry as well as vibrating mode and frequency. This can be confirmed with Figs. 5~8 where modal sound powers for individual modes are affected by β and α. Consequently, overall sound radiation from radial vibration of a thick annular disc can be controlled by adjusting these geometric parameters. For example, one can reduce the sound radiation from q=3 radial mode by increasing or decreasing β from 0.594 for a given b=165 mm.

As the first step, target frequency range where overall sound radiation is to be minimized should be defined. Then, the feasible ranges for the two parameters should be selected considering practical limitations on them. In this particular study, the target frequency range is defined as 0~8 kHz since

Table 2 Sound radiation from original and optimized disc

Mode	Π_q (dB re 1 pW)		σ_q (dB re 1)	
	Original	Modified	Original	Modified
q=2	65.40	65.61	0.091	1.088
q=0	64.76	64.95	-8.713	-9.823
q=3	68.08	66.34	-3.451	-8.015
Averaged	66.33	65.67	-3.310	-4.182

most noise problems of the practical application of this study are in this frequency range. Also, considering durability issue and requirements in the geometric configuration, the range of β is set to be $0.5 \le \beta \le 0.6$ while b and h are fixed their nominal values. Based on these criteria the optimal configuration can be easily determined using Figs. 5~6. As one can see from these two figures, averaged modal sound radiations for three modes can be minimized by setting β as 0.545. The effects of this modification are summarized in Table 2 where modal sound powers and radiation efficiencies for the optimized disc are compared with those for the original one. As shown in the table, Π_q and σ_q for q=3 mode decrease significantly by adopting the optimal configuration. On the other hand, those for q=0 and q=2 mode slightly increase. Consequently, averaged modal sound power and radiation efficiency decrease and one can expect reduction in the overall sound radiation with this modification.

4. Effects of Geometry on the Multi-modal Sound Radiation

4.1 Modal Formulation

As explained in Section 3.3, the overall sound radiation from a disc can be expressed as a linear combination of the modal sound radiation of the disc. If a harmonic excitation is applied on the disc, surface velocity $\{v\}$ and far-field sound pressure $\{P\}$ can be expressed as combination of those for individual excitation as follows[7]:

$$\{v\} = \sum_i \eta_i \{\Phi_i\} \qquad\qquad (5)$$
$$\{P\} = \sum_i \eta_i \{\Gamma_i\}$$

where η_i is the modal participation factor, $\{\Phi_i\}$ is the modal surface velocity and $\{\Gamma_i\}$ is the modal sound radiation for mode i respectively.

The sound power $\Pi(\omega)$ radiated by the vibrating disc and the corresponding radiation efficiency $\sigma(\omega)$ resulting from an arbitrary harmonic excitation $f(t)$ with an angular frequency ω were also calculated from the far-field sound pressure distribution on the surface of a sphere surrounding the disc. Sound power was determined from the product of the surface-average of the sound intensity and the surface area of the sphere. The following expressions describe the procedure.

$$\Pi(\omega) = \left(\frac{1}{2}\right)\int_0^{2\pi}\int_0^{\pi} \frac{P^H(\omega)P(\omega)}{\rho_0 c_0} R^2 \sin\theta \, d\theta \, d\phi \qquad (6a)$$

$$\sigma(\omega) = \Pi(\omega) \Big/ A_s < |\dot{v}|^2 >_{t,s} \qquad (6b)$$

where A_s is the total area of all radiating surface sand subscripts t and s indicate time and spatial averaging. The time-space average of the surface velocity on A_s is given by

$$< |\dot{v}|^2 >_{t,s} = \frac{1}{4\pi h(a+b)} \left\{ \int_{-h/2}^{h/2} \int_0^{2\pi(a+b)} \dot{U}^2 dl \, dz \right\} \qquad (6c)$$

where dimensions a, b and h are as given in Fig. 1 and $\dot{U}$ is the surface velocity in the r direction, respectively.

4.2 Effects of Optimization

In Section 3.3, a geometric configuration that generates minimum modal sound radiations

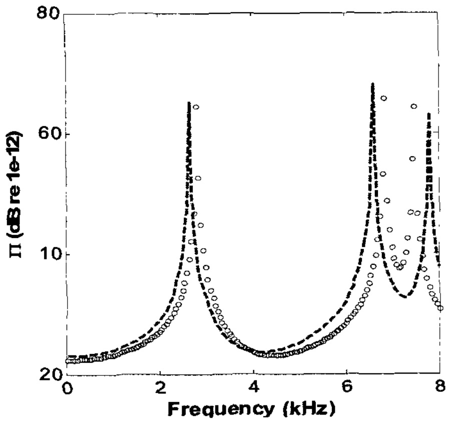

Fig. 9 Sound power spectra $\Pi(\omega)$ for original and optimized discs. Key: - - -, Original; OOO, Modified

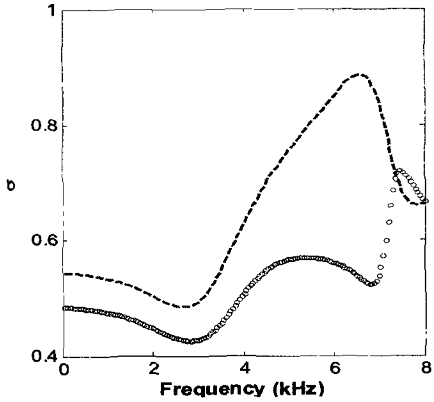

Fig. 10 Radiation efficiency spectra $\sigma(\omega)$ for original and optimized discs. Key: - - -, Original; OOO, Modified

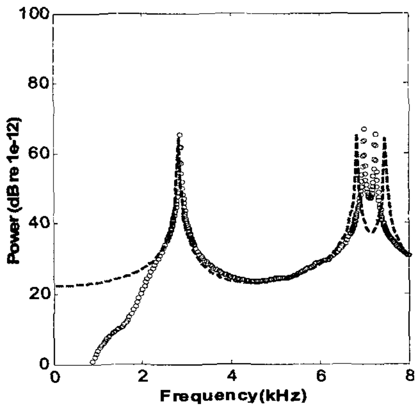

Fig. 11 Comparison of sound power spectra $\Pi(\omega)$ for optimized disc. Key: - - -, Analytical solution; OOO, Numerical analysis

within the feasible ranges of geometric parameters has been selected. In this section, multi-modal sound radiation characteristics of this optimized disc are compared with those of the original disc. The procedure introduced in the previous section is used to obtain the multi-modal radiations for the original and modified discs. In this particular study, the excitation to the disc is assumed to be a unit harmonic force in the radial direction. The results are given in Figs. 9 and 10. As one can see in these figures, $\Pi(\omega)$ and $\sigma(\omega)$ for the disc significantly decrease with optimized geometric configuration showing that this procedure can be effectively used to reduce multi-modal sound radiation. Please note that peaks for individual modes are shifted a little due to the changes in the natural frequencies of the disc.

Finally, the results are confirmed with numerical analyses. Sound power spectra $\Pi(\omega)$ for the optimized disc has been calculated using commercial FEM and BEM codes[14,15] and compared with the results from analytical solutions given above. The results are explained in Fig. 11. As one can see in the figure, the two results match relatively well each other even though there are some discrepancies in the peak locations. These discrepancies were introduced due to the differences between the numerical eigenvalues and corresponding analytical ones.

5. Conclusions

This article has examined the effects of geometric configuration of a thick annular disc on its vibro-acoustic characteristics of radial vibration. The effects of three key parameters (inner radius, outer radius and thickness) on the modal sound radiations within a target frequency range have been examined using pre-developed analytical solution. Based on the results, an optimal configuration that has minimal modal sound radiations in the given frequency range was selected. The modal expansion technique has been used to calculate sound power and radiation efficiency spectra in the given frequency range for this optimal configuration. A comparative evaluation of analytical and numerical results shows that this procedure has sufficient accuracy in selecting a geometry having minimal sound radiation in the target frequency range.

The procedure introduced in this article can be efficiently used to obtain an optimal configuration that has minimal sound radiation within a target frequency range satisfying given geometric constraints. In a future study, this procedure will simultaneously consider both out-of-plane and in-plane components of the disc vibration. Modal interaction effects and sound radiation from coupled modes will also be studied.

References

(1) Bhuta, P. G. and Jones, J. P., 1971, “Symmetric Planar Vibrations of a Rotating Disc”, J. Acoust. Soc. Am. Vol. 35, No. 7, pp. 982~989.

(2) Burdess, S., Wren, T. and Fawcett, J. N., 1987, “Plane Stress Vibration in Rotating Discs”, Proceedings of the Institution of Mechanical Engineers 201, pp. 37~44.

(3) Chen, J. S. and Jhu, J. L., 1996, “On the In-plane Vibration and Stability of a Spinning Annular Disc”, Journal of Sound and Vibration, Vol. 195, No. 4, pp. 585~593.

(4) Tzou, K. I., Wickert, J. A. and Akay, A., 1998, “In-plane Vibration Modes of Arbitrary Thick Discs”, Journal of Vibration and Acoustics, Vol. 120, pp. 384~391.

(5) Irie, T., Yamada, G. and Muramoto, Y., 1984, “Natural Frequencies of In-plane Vibration of Annular Plates”, Journal of Sound and Vibration, Vol 97, No. 1, pp. 1711~175.

(6) Lee, H., 2005, “Acoustic Radiation from Radial Vibration Modes of a Thick Annular Disk”, Transactions of th Korean Society for Noise and Vibration Engineering, Vol. 15, No. 4, pp. 412~420.

(7) Lee, H., Singh, R., 2005, “Comparison of Two Analytical Methods Used to Calculate Sound Radiation from Radial Vibration Modes of a Thick Annular Disc”, Journal of Sound and Vibration, Vol. 285, pp. 1210~1216.

(8) Lee, H., Singh, R., 2005, “Determination of Sound Radiation from a Simplified Disc-brake Rotor by a Semi-analytical Method”, Noise Control Eng. J. Vol. 52, No. 5, pp. 225~239.

(9) Junger, M. C. and Feit, D., 1985, Sound, Structures, and Their Interactions, MIT Press, New York.

(10) Williams, E. G., 1999, Fourier Acoustics, Academic Press, San Diego.

(11) Wang, C. and Lai, J. C. S., 2000, “The Sound Radiation Efficiency of Finite Length Acoustically Thick Circular Cylindrical Shell under Mechanical Excitation I: Theoretical analysis”, Journal of Sound and Vibration, Vol. 232, No. 2, pp. 431~447.

(12) Sandman, B. E., 1976, “Fluid Loading Influence Coefficients for a Finite Cylindrical Shell”, J. Acoust. Soc. Am. Vol. 60, No. 6, pp. 1256~1264.

(13) SDRC, 2000, I-DEAS User’s Manual Version 8.2, USA.

(14) NIT, 1999, SYSNOISE User’s manual V. 5.4, Belgium.