조달기간이 확률적인 경우의 예비품 주문정책

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A Spare Ordering Policy with Random Lead Times

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Abstract

A generalized spare ordering policy is treated in this paper. If the operating unit fails before a scheduled ordering time an expedited order is placed at the failure time instant, otherwise a regular order for a spare is placed at the scheduled time. The original unit is replaced when the ordered spare is delivered. The lifetime, regular and expedited lead times have general distributions. The problem is to find the optimum ordering time which minimizes the expected cost rate including the observation, ordering, uptime and down-time costs. Some properties regarding the optimal policy are derived. To explain the spare ordering policy a numerical example is also included.

1. Introduction

Maintenance policies for systems subject to stochastic failures have been treated for several decades [7]. But most of them have assumed that whenever a unit is to be replaced, a new unit is immediately available. If, however as is often the case, the procurement lead time is not negligible, we should consider an ordering policy to determine the ordering time for a spare.

Consider a 1-unit system, where each failed unit is scrapped without repair and each spare is provided only by an order. The original unit begins operating at time 0. The operating unit is continuously kept under observation to detect failure till a specified time t_o or till the instant of failure, whichever occurs earlier. If the original unit fails before t_o, we place an expedited order immediately at the failure time instant and replace the failed unit with the new one as soon as it is delivered. On the other hand, if the operating unit does not fail up to t_o, we place a regular order for a spare at t_o and replace the unit when the ordered spare is delivered. The lifetime and two kinds of lead times have general distributions.

The time between successive replacements is a cycle and the behavior in each cycle repeats. The problem is to find the optimal scheduled ordering time t_o minimizing the expected cost rate.

Symbols

$f(t)$, $F(t)$, m pdf, cdf, and mean value of the lifetime of a unit

$\bar{F}(t)$ $1-F(t)$

$h(t)$ $f(t)/\bar{F}(t)$, instantaneous failure rate of a unit

$h_x(t)$ $[F(t+x)-F(t)]/\bar{F}(t)$, interval failure rate of

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a unit

$g(x)$, $G(x)$, m_r pdf, cdf, and mean value of regular lead time

m_e mean value of expedited lead time

t_o scheduled time for regular order

c_e, c_r ordering costs of expedited and regular orders respectively

c_o observation cost per unit time

c_u uptime cost per unit time operation

c_d downtime cost per unit time due to spare shortage

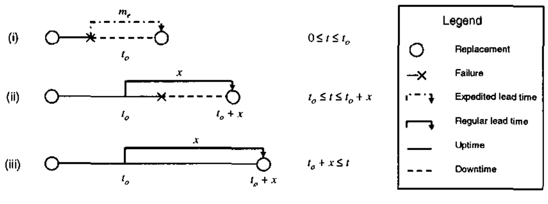
Other symbols are defined when needed.

2. Cost Model

From the renewal reward theorem, the expected cost rate for an infinite time span is the expected cost per cycle divided by the expected cycle length. Since the time between successive replacements is a cycle, the following three mutually exclusive and exhaustive possibilities exist in every cycle (see <Figure 1>) :

(i) the operating unit fails before t_o
(ii) the operating unit fails between t_o and the arrival of the ordered spare
(iii) the operating unit does not fail before the arrival of the ordered spare.

The expected cost per cycle is the sum of the observation, ordering, uptime and downtime costs.



<Figure1> Possible realizations of one cycle

The expected observation and ordering costs per cycle is

$$c_0\left[\int_0^{t_0} t f(t)dt + t_0\int_{t_0}^{\infty} f(t)dt\right] + c_e\int_0^{t_0} f(t)dt + c_r\int_{t_0}^{\infty} f(t)dt$$
$$= c_0\int_0^{t_0}\overline{F}(t)dt + c_e F(t_0) + c_r\overline{F}(t_0) \qquad (1)$$

The expected uptime cost per cycle is

$$c_u\left[\int_0^{t_0} t f(t)dt + \int_0^{\infty}\int_{t_0}^{t_0+x} t f(t)g(x)dtdx + \int_0^{\infty}\int_{t_0+x}^{\infty}(t_0+x)f(t)g(x)dtdx\right]$$
$$= c_u\left[m - \int_0^{\infty}\overline{F}(t_0+x)G(x)dx\right] \qquad (2)$$

Since downtime occurs in the cases (i) and (ii), the expected downtime cost per cycle is

$$c_d\left[m_e\int_0^{t_0} f(t)dt + \int_0^{\infty}\int_{t_0}^{t_0+x}(t_0+x-t)f(t)g(x)dtdx\right]$$
$$= c_d\left[\int_0^{\infty}\int_{t_0}^{t_0+x} F(t)g(x)dtdx - (m_r - m_e)F(t_0)\right] \qquad (3)$$

Thus the expected cost per cycle is

$$K(t_0) = c_o\int_0^{t_0}\overline{F}(t)dt + c_e F(t_0) + c_r\overline{F}(t_0) + c_u\left[m - \int_0^{\infty}\overline{F}(t_0+x)\,G(x)dx\right]$$
$$+ c_d\left[\int_0^{\infty}\int_{t_0}^{t_0+x} F(t)g(x)dtdx - (m_r - m_e)F(t_0)\right] \qquad (4)$$

The expected cycle length is

$$T(t_0) = \int_0^{t_0}(t+m_e)f(t)dt + \int_0^{\infty}\int_{t_0}^{\infty}(t_0+x)f(t)g(x)dtdx$$
$$= m_e F(t_0) + m_r\overline{F}(t_0) + \int_0^{t_0}\overline{F}(t)dt \qquad (5)$$

Hence the expected cost rate for an infinite time span is

$$C(t_0) = K(t_0)\,/\,T(t_0) \qquad (6)$$

where, $K(t_0)$ and $T(t_0)$ are given by (4) and (5) respectively.

3. Analysis

To analyze the cost model the followings are assumed.

(a) $h(t)$ is strictly increasing (that is, the failure rate of a unit increases as it gets old).
(b) The average lead time of expedited order is smaller than that of regular order but it costs more (i.e., $m_e \le m_r$ and $c_e \ge c_r$).
(c) The downtime cost is larger than uptime cost (i.e., $c_d \ge c_u$) to justify system operation.

Define the numerator of the derivative of $C(t_0)$ in (6)divided by $\bar{F}(t_0)$ as

$$\begin{aligned} p(t_0) = [&(c_e - c_r) - c_d(m_r - m_e)h(t_0) \\ &+ (c_d - c_u)\int_0^\infty h_x(t_0)g(x)dx + (c_0 + c_u)] \cdot T(t_0) \\ &- K(t_0) \cdot [1 - (m_r - m_e)h(t_0)] \end{aligned} \tag{7}$$

Then we have the following theorem regarding the optimum ordering time t_0^* which minimizes the total expected cost rate.

Theorem 1. Suppose that

$$(c_e - c_r) \ge c_d(m_r - m_e).$$

(i) If $p(0) \ge 0$, than the optimum ordering time $t_0^* = 0$, i.e., place a regular order at the same instant when a unit is put in service and never place an expedited order.
(ii) If $p(0) < 0$ and $p(\infty) > 0$, then there exists a finite and unique optimum ordering time t_0^* $(0 < t_0^* < \infty)$ satisfying $p(t_0^*) = 0$.
(iii) If $p(\infty) \le 0$, then the optimum ordering time $t_0^* = \infty$, i.e., place an expedited order at the instant of failure and never place a regular order.

Proof. Differentiating $C(t_0)$ with respect to t_0 and setting it equal to zero implies $p(t_0) = 0$. Further,

$$\begin{aligned} p'(t_0) = [&(c_e - c_r) - c_d(m_r - m_e)h'(t_0) \\ &+ (c_d - c_u)\int_0^\infty h_x'(t_0)\, g(x)] \cdot T(t_0) \\ &+ K(t_0) \cdot (m_r - m_e)h'(t_0) \end{aligned} \tag{8}$$

Since the interval failure rate $h_x(t_0)$ and the instantaneous failure rate $h(t)$ have the same monotone properties (see Barlow and Proschan [2, p.23]), $p(t)$ is strictly increasing. Thus, the existence of t_0^* in the theorem follows trivially.

A sufficient condition for the optimality in the theorem, $(c_e - c_r) > c_d(m_r - m_e)$, has been widely used in spare ordering policies [3, 4, 6]. However it should be noted that the assumption dose not economically justify placing an expedited order since the additional cost for the expedition $(c_e - c_r)$ is large than the savings obtained from the expedition $c_d(m_r - m_e)$. Hence the theorem is meaningful only when there exist such intangibles as loss of goodwill, reputation and credit which are difficult to be quantified and included in downtime cost.

Numerical example

For the purpose of illustration let us consider the following case : Both the lifetime and regular lead time are gamma distributed with integer modulus.

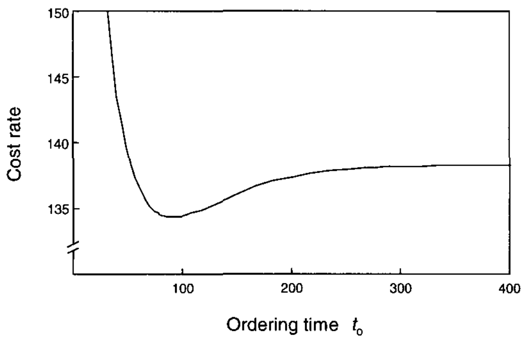
Lifetime cdf

$$F(t) = 1 - [1 + 0.03t + (0.03t)^2/2]\exp(-0.03t)$$

Regular lead time cdf

$$G(x) = 1 - (1 + 0.05x)\exp(-0.05x).$$

Expedited lead time follows any general distribution with mean $m_e = 20$.
The cost parameters are c_o = \$10, c_e = \$12,000, c_r = \$8,000, c_u=\$20, c_d = \$80.
In this case example, mean time to failure of a unit m = 100 and mean value of regular lead time m_r = 40.



<Figure 2> Cost rate as function of ordering time t_0

<Figure 2> shows how the expected cost rate $C(t_0)$ changes with respect to the scheduled ordering time t_0. The optimum ordering time $t_0* = 94$ and the corresponding cost rate is $C(t_o*)=134$.

4. Concluding Remarks

This study includes some previous works as special cases. Let us denote the Dirac-delta function as $\delta(x)$, and the pdf of expedited lead time as $g_e(x)$. (Notice that we need only mean value m_e for expedited order to derive the cost model.) If $g_e(x) = g(x) = \delta(x)$ and $c_o = c_u = 0$, it reduces to the age replacement policy of Barlow and Hunter [1]. If $g_e(x) = g(x) = \delta(x-L)$, and $c_o = c_u = 0$ it reduces to Osaki [5]. If $g_e(x) = \delta(x-L_e)$, $g(x) = \delta(x-L_r)$, and $c_o = c_u = 0$ it reduces to Kaio and Osaki [3]. If $g_e(x) = \delta(x-L_e)$, $g(x) = \delta(x-L_r)$ and $c_u = 0$ it reduces to Sridharan [6].

References

[1] Barlow, R. E. and Hunter, L.(1960), "Optimum Preventive Maintenance Policies", *Operations Research*, Vol. 8, pp. 90-100.

[2] Barlow, R. E. and Proschan, F.(1965), *Mathematical Theory of Reliability*, Wiley, New York.

[3] Kaio, N. and Osaki, S.(1977), "Ordering Policies with Two Types of Lead Times", *Microelectronics and Reliability*, Vol. 16, pp. 225-229.

[4] Kalpakam, S. and Hameed, A. S.(1981), "Ordering Policies with Random Lead Times", *Microelectronics and Reliability*, Vol. 21, No. 5, pp. 737-741.

[5] Osaki, S.(1977), "An Ordering Policy with Lead Time", *International Journal of Systems Science*, Vol. 8, pp. 1091-1095.

[6] Sridharan, V.(1993), "A Note on Ordering Policies with Cost of Observation and Lead Times", *International Journal of Production Economics*, Vol. 32, pp. 91-97.

[7] Wang, H.(2002), "A Survey of Maintenance Policies of Deteriorating Systems", *European Journal of Operational Research*, Vol. 139, pp. 469-489.