논문번호 07-01-01

An Integral-Augmented Nonlinear Optimal Variable Structure System for Uncertain MIMO Plants

Jung-Hoon Lee*★

Abstract

In this paper, a design of an integral augmented nonlinear optimal variable structure system(INOVSS) is presented for the prescribed output control of uncertain MIMO systems under persistent disturbances. This algorithm basically concerns removing the problems of the reaching phase and combining with the nonlinear optimal control theory. By means of an integral nonlinear sliding surface, the reaching phase is completely removed. The ideal sliding dynamics of the integral nonlinear sliding surface is obtained in the form of the nonlinear state equation and is designed by using the nonlinear optimal control theory, which means the design of the integral nonlinear sliding surface and equivalent control input. The homogeneous $2v(k)$ form is defined in order to easily select the $2v$ or even k-form higher order nonlinear terms in the suggested sliding surface. The corresponding nonlinear control input is designed in order to generate the sliding mode on the predetermined transformed new surface by means of diagonalization method. As a result, the whole sliding output from a given initial state to origin is completely guaranteed against persistent disturbances. The prediction/predetermination of output is enable. Moreover, the better performance by the nonlinear sliding surface than that of the linear sliding surface can be obtained. Through an illustrative example, the usefulness of the algorithm is shown.

Key words: sliding mode control, variable structure system, nonlinear optimal control

I. Introduction

The most distinct feature of the variable structure system(VSS) is the presence of the sliding mode on the predetermined sliding surface[1]-[3]. Once in the sliding mode, the system is forced to constrain its evolution on the predetermined sliding surface and it results in a dynamic behavior that is largely determined by the design parameters and equation defining the original system can not be obtained for the controlled motion. Therefore, the system in the sliding mode is theoretically robust and insensitive to parameter uncertainties and disturbance[4][5]. The proper design of the sliding surface can results in the almost desired output dynamics. Most of the design methods reported so far yield the linear dynamics in the sliding surface, Some of them are optimal control[6], eigenstructure assignment[7], geometric approach[8], and differential geometric approach[9].

Usually there is a conflict between the static and dynamic accuracy in the VSS having the linear sliding surface. It is required for a high performance system to have the fast response, low overshoot, and high static accuracy[10], The linear sliding surface, however, give the relatively slow speed of response and unsatisfactory performance in certain specific application[11]. Thus, in some practical

*ERI, Dept. of Control and Instrumentation Engineering, Gyeongsang National University

★ Corresponding author

Manuscript received Feb. 2, 2005 ; reviced Dec. 25, 2005

aspects, the linear dynamics can be not sufficient but the additional nonlinear dynamics are more desirable to the sliding dynamics[12]. For the control of a class of the nonlinear system, the desired behavior in the output naturally requires the use of the nonlinear sliding surface. Moreover, even for the control of the linear system, additional nonlinear dynamics can be assigned in order to obtain fast transient dynamics in [11] and [12]. Unfortunately, the reaching phase problems exist. During this reaching phase, the controlled systems may be sensitive to the parameter variations and external disturbances because the sliding mode is not realized[13]. And it is difficult to find the performance designed in the sliding surface from the real output.

Compared to the established works on the VSS, few researches deal with the problems of the reaching phase. One alleviation method is the use of the high-gain feedback[14]. This has the drawbacks related to the high-gain feedback, for example sensitivity to the unmodelled dynamics and actuator saturation[13]. The adaptive rotating or shifting of the sliding surface is suggested to reduce the reaching phase problems in [2][15], and the sliding surface segmentally connected from a given initial condition to the origin is also suggested[16]. But these changing techniques and segmented sliding surface are applicable to only second order systems and those outputs are not predictable. In [17], the exponential term is added to the conventional linear sliding surface is order to make $s(t)=0$ at $t=0$ for removing the reaching phase.

In this paper, an integral-augmented nonlinear optimal variable structure system(INOVSS) for the control of multi-input multi-output(MIMO) systems. The design of such system involves the determination of the integral-augmented nonlinear sliding surface and the corresponding nonlinear control input. The suggested sliding surface is augmented by the integral with the integrand of the state itself and higher order nonlinear terms of the state so that the resulting nonlinear sliding surface can offer significant advantages over the linear one in a variety of circumstances such as state or control constraint. For the design of the nonlinear sliding surface, the nonlinear optimal technique is introduced with minimizing the non-quadratic performance index[18][19]. The homogeneous $2v(k)$ form is defined in order to easily select the $2v$ or even k-form higher order nonlinear terms in the suggested sliding surface. Using the diagonalization method[1][4], the stabilizing control is designed for guaranteeing the chosen nonlinear optimal performance involved in the integral nonlinear sliding surface, whereas the previous nonlinear optimal controller[18]-[21] does not consider the performance robustness against the uncertainties and external disturbances. The performance robustness of the algorithm is shown by means of the stability analysis on the sliding mode on the transformed surface. The effectiveness of the INOVSS such as no reaching phase, the full robustness for the whole trajectory, the improved steady state performance without overshoot, satisfactory performance under state constraint as well as exact predictable output. Finally an example is presented to show the effectiveness of the algorithm compared with a VSS having the integral linear sliding surface.

II. A New Vatiable Structure Controller

2.1 Description of Plants and Actuators

The problem of designing the INOVSS controller is considered for an uncertain linear multivariable system:

$$\dot{Y}(t)=(A+\Delta A(Y,t))\cdot Y(t) + (B+\Delta B(Y,t))\cdot U(Y,t)+D(Y,t) \qquad Y^0=Y(0) \tag{1}$$

where $Y(t)\in R^n$ is the state, $A\in R^{n\times n}$ and $B\in R^{n\times m}$ are the system matrices, $\Delta A(Y,t)\in R^{n\times n}$ and $\Delta B\in R^{n\times m}$ represents for the uncertainties as the modeling error, $D(Y,t)$ is the external disturbance, and $U\in R^m$ is the control to be determined. The goal of the INOVSS controller design is to asymptotically stabilize this uncertain MIMO system with quality of the nonlinear optimal performance.

Basically, the assumptions are made as follows:

Assumption 1:

The pair $S(A,B)$ is completely controllable

Assumption 2:(Matching condition)

The uncertainties $\Delta A(Y,t)$ and $\Delta B(Y,t)$ and disturbance $D(Y,t)$ satisfy the matching condition, *i..e.,*

$R(\Delta A(Y,t)) \subset R(B)$

$R(\Delta B(Y,t)) \subset R(B)$

$R(\Delta D(Y,t)) \subset R(B)$

Above all, for effective formulations, $X(t)$ as a new state is transformed from the original $Y(t)$ using the nonsingular coordinate transformation[5][22] T

$$X(t) = \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = T(Y,t) \cdot Y(t) \tag{2a}$$

such that

$$T(X,t) \cdot B(Y,t) = \begin{bmatrix} 0 \\ B_2(X,t) \end{bmatrix} \tag{2b}$$

where $X_1 \in R^{n-m}$ and $X_2 \in R^m$ are the partition of $X \in R^n$. Using (2a), the system (1) can be represented in regular form[23] in X space as

$$\begin{aligned} \dot{X}_1(t) &= A_{11} \cdot X_1(t) + A_{12} \cdot X_2(t) \\ \dot{X}_2(t) &= (A_{21} + \Delta A_{21}'(t)) \cdot X_1(t) + (A_{22} + \Delta A_{22}'(t)) \cdot X_2(t) \\ &\quad + (B_2 + \Delta B_2'(t)) \cdot U(t) + D_2'(X,t) \end{aligned} \tag{3}$$

where $rank(B_2) = m$, $A_{11} \in R^{(n-m)\times(n-m)}$, $A_{12} \in R^{(n-m)\times m}$, $A_{21} \in R^{m\times(n-m)}$, and $A_{22} \in R^{m\times m}$ are known constant matrices transformed from the original A in (1), and X_1^0 and X_2^0 are the initial conditions also transformed from Y^0. The assumption on the boundedness of the uncertainties and disturbance in (3b) is made as

Assumption 3:

The uncertainties and disturbance in (3) can be represented and bounded as the following

$$\Delta A_{21}' = B_2 \cdot \Delta A_{21}, \qquad |\Delta A_{21_{ik}}| < \alpha_{21_{ik}} \tag{4a}$$

$$\Delta A_{22}' = B_2 \cdot \Delta A_{22}, \qquad |\Delta A_{22_{ij}}| < \alpha_{22_{ij}} \tag{4b}$$

$$\Delta B_2' = B_2 \cdot \Delta B_2, \qquad |\Delta B_{2_{ii}}| < \beta_{2_i} < 1 (diagonal) \tag{4c}$$

$$\Delta D_2' = B_2 \cdot \Delta D_2, \qquad |\Delta D_{2_{ij}}| < \gamma_{2_i} \tag{4d}$$

for $i, j = 1,2,...,m$ and $k = 1,2,...,(n-m)$.

Now to the system (3), the integral terms, $X_0 \in R^r$, $r \leq n$ are augmented in general form

$$\begin{aligned} \dot{X}_0(t) &= A_0 \cdot X(t) \\ &= A_{01} \cdot X_1(t) + A_{02} \cdot X_2(t) \qquad X_0^0 \end{aligned} \tag{5}$$

where $A_0 \equiv [A_{01} : A_{02}] \in R^{r\times n}$ is the coefficient matrix for dimensional matching and X_0^0 is the initial condition for X_0. Then, the nominal system of (3) with (5) is described as

$$\dot{X}_0(t) = A_{01} \cdot X_1(t) + A_{02} \cdot X_2(t) \tag{6a}$$

$$\dot{X}_1(t) = A_{11} \cdot X_1(t) + A_{12} \cdot X_2(t) \qquad X_1^0 \tag{6b}$$

$$\dot{X}_2(t) = A_{21} \cdot X_1(t) + A_{22} \cdot X_2(t) + B_2 \cdot U(t) \quad X_2^0. \tag{6c}$$

The choice of A_{01} and A_{02} is included in next design of the integral-augmented nonlinear sliding surface

2.2 Integral-Augmented Nonlinear Sliding Surface

The conventional sliding surface $S: R^n \to R^m$ for multi-input systems is composed of the set of the m linear sub surface as

$$S(X) \equiv C^T \cdot X = C_1 \cdot X_1 + C_2 X_2 (=0) \tag{7}$$

where $rank(C_2) = m$ and using $S(\cdot) = 0$ and (6b), its ideal sliding dynamics as the reduced subsystem with $(n-m)-th$ order can expressed as

$$\begin{aligned} \dot{X}_1(t) &= (A_{11} - A_{12} C_2^{-1} C_1) \cdot X_1, \\ & X_1(t_s) \qquad t \geq t_s \geq 0 \end{aligned} \tag{8}$$

Since the system (1) is controllable, the pair (A_{11}, A_{12}) is alos controllable[6]. Using this fact, the linear pole assignment are applicable to the system (8) for the design of the sliding surface. However, the dynamics of (8) is not defined from a given initial condition X_1^0 but the state when reaching instant $X_1(t_s)$, hence there can be reaching phase for the initial $X_1^0 \not\subseteq S(\cdot) = 0$ and the output can not be exactly predictable. Moreover, since the surface of (7) only uses the linear elements, it is difficult to resolve the conflict between the opposing requirements of the static and dynamic accuracy which are encountered when designing the linear sliding surface.

In this paper, an integral-augmented nonlinear sliding surface $S(\cdot): R^{n+r} \to R^m$ is proposed by composing of three terms as

$$S(X) \equiv S_L(X) + S_I(X) + S_N(X) (=0) \tag{9}$$

where $S_L(\cdot): R^n \to R^m$, $S_I(\cdot): R^r \to R^m$, and $S_N(\cdot): R^n \to R^m$ are the conventional linear, integral-augmented linear, and intentionally nonlinear integral-augmented terms, respectively as

$$S_L(X) = C_L^T \cdot X = \left[\sum_{j=1}^{n} c_{Lij} \cdot x_j\right] = C_{L1} \cdot X_1 + C_{L2} \cdot X_2 \tag{9a}$$

$$S_I(X) = C_I^T \cdot X_0 = \left[\sum_{j=1}^{n} c_{Iij} \cdot \int_0^t x_j(\tau)d\tau\right] = C_{I1} \cdot X_{01} + C_{I2} \cdot X_{02} \tag{9b}$$

$$S_N(X) = C_N^T \cdot \int_0^t G(X(\tau))d\tau \tag{9c}$$

where $G(\cdot)$ is a nonlinear homogeneous function determined later and C_L, C_I, and C_N are constants as the design parameters. Since it is difficult to split the initial value into the linear and nonlinear integral-augmented terms, the total integral action is considered as

$$X_{IN} = \int_0^t C_I^T \cdot X(\tau) + C_N^T \cdot G(X(\tau))d\tau \tag{10}$$

and let X_{IN}^0 denote its initial condition. For $S(X^0) = 0$ and zeros of the integral terms S_I and S_N in steady state, X_{IN}^0 should satisfy

$$X_{IN}^0 = -\left[\sum_{j=1}^{n} c_{Lij} \cdot x_j^0\right] = -C_{L1} \cdot X_1^0 - C_{L2} \cdot X_2^0 \tag{11}$$

The total integral state converges to zero from this finite value and its rate convergence depends upon the relationship between C_L, C_I, and C_N. Because of $S(X^0) = 0$ for any initial $X^0 \in R^n$, the sliding dynamics of (9) can be defined from any initial $X^0 \in R^n$. Thus, there can be no reaching phase, the controlled system can slide from any given initial condition $X^0 \in R^n$ to the origin, and the full robustness for the whole trajectory can be obtained, furthermore the output of the controlled system can be predictable.

The coefficient of (9), C_L, C_I, and C_N and the structure of $G(X)$ will be designed using nonlinear optimal technique with guaranteed stability in the surface (9). In the sliding mode, (12) is satisfied

$$S(X) = 0 \quad \text{and} \quad \dot{S}(X) = 0 \tag{12}$$

From this fact, the ideal sliding dynamics of the sliding surface with $n-th$ order can be obtained as

$$\dot{X}_1(t) = A_{11} \cdot X_1(t) + A_{12} \cdot X_2(t) \tag{13a}$$

$$\dot{X}_2(t) = -C_{L2}^{-1}\left\{\begin{matrix}(C_{L1}A_{11} + C_{I1}) \cdot X_1(t) \\ +(C_{L1}A_{12} + C_{I2}) \cdot X_2(t) + C_N \cdot G(X(t))\end{matrix}\right\} \tag{13b}$$

or after manipulating

$$\dot{X}(t) = \Lambda \cdot X(t) + \Gamma \cdot \nu(t) \qquad X^0 \tag{14}$$

where $\Lambda \in R^{n\times n}$, $\Gamma \in R^{n\times m}$, and $\nu \in R^m$.

$$\Lambda = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22}\end{bmatrix}, \qquad \Gamma = \begin{bmatrix} 0 \\ B_2 \end{bmatrix} \tag{15}$$

$$\nu(t) = -K_1 \cdot X_1(t) - K_2 \cdot X_2(t) - K_3 \cdot G(X(t)) \tag{16}$$

where

$$K_1 = B_2^{-1}\{A_{21} + C_{L2}^{-1}(C_{L1}A_{11} + C_{I1})\} \tag{17a}$$

$$K_{12} = B_2^{-1}\{A_{22} + C_{L2}^{-1}(C_{L1}A_{12} + C_{I2})\} \tag{17b}$$

$$K_3 = B_2^{-1}C_{L2}^{-1}C_N \tag{17c}$$

The solution of (13) or (14) for a given X^0 gives the integral nonlinear surface coinciding with (9). And using solution of (13) or (14), it is possible to predict/predetermine the real output. Since, as can be seen in (14), it equals to the dynamics of the nominal system (6b) and (6c), the design of the sliding surface is the performance design to the nominal systems of (3), and the reverse argument also holds.

Now, the nonlinear performance index function is defined as

$$J = \int_0^t X^T QX + \nu^T R\nu + M(X(\tau))d\tau \tag{18}$$

where $Q = \overline{Q^T}\,\overline{Q} \geqq 0$ such that the pair $(\Lambda\,\overline{Q})$ and the output $y = M(X(t))$ are observable, $R > 0$, and $M(X(t))$ is a homogeneous symmetric positive-definite function. A C^1 function $V: R^n \rightarrow R$ is also defined as

$$V(X) = X^T PX + Z(X) \tag{19}$$

such that

$$V(0) = 0 \tag{20a}$$

$$V(X) > 0 \qquad X \in R^n \qquad X \neq 0 \tag{20b}$$

$$\frac{\partial V(X)}{\partial t} = 0 \tag{20c}$$

where $Z(X)$ is a homogeneous positive-definite function. Then, the optimal coefficient K_i and the structure of $G(X)$ can be designed in Theorem 1. Let ∇ imply the gradient with respect to X.

Theorem 1: The optimal coefficient $\nu = \phi(\cdot)$, i.e., K_i and the structure of $G(X)$ for (16) with respect to (18) and (19) are given by

$$K = [K_1 \quad K_2] = R^{-1}\Gamma^T P \tag{21a}$$

$$K_3 G(X) = \frac{1}{2} R^{-1}\Gamma^T \cdot \nabla Z(X) \tag{21b}$$

where $P>0$ is the solution of the Reccati equation and $\nabla Z(X)$ the gradient of $Z(X)$ such that

$$A^T P + PA + Q - PWP = 0 \tag{22a}$$

$$\nabla Z(X) \cdot \Phi \cdot X + M(x) - \frac{1}{4}\nabla Z(X) \cdot W \cdot \nabla Z(X) = 0 \tag{22b}$$

where

$$\Phi \equiv [A - \wp]$$

$$W \equiv \Gamma R^{-1}\Gamma > 0$$

and final cost becomes

$$J + V(X^0) = X^{0^T} P X^0 + Z(X^0). \tag{23}$$

Proof: The derivative of $V(X)$ with respect to time presents

$$\dot{V}(X) = [2X^T P + \nabla Z(X)](AX + \Gamma\nu) \tag{24}$$

Using (21a) and (21b), (24) becomes

$$\begin{aligned}\dot{V}(X) &= [2X^T P + \nabla Z(X)]\left[\Phi \cdot X - \frac{1}{2} W \cdot \nabla Z(X)\right] \\ &= X^T[P\Phi + \Phi^T P] \cdot X + \nabla Z(X)\Phi \cdot X \\ &\quad + X^T PW\nabla Z(X) - \frac{1}{2}\nabla Z(X) \cdot W \cdot \nabla Z(X).\end{aligned} \tag{25}$$

From (22c) and (22d), it follows

$$\begin{aligned}\dot{V}(X) &= -X^T QX - \nu^T R\nu - M(X) \\ &= -L(X(t), \phi(t), t)\end{aligned} \tag{26}$$

which implies that $\dot{V}(\cdot)$ is negative for all time, i.e. the closed loop system (14) with (16) is asymptotically stable. Moreover, the optimal cost becomes

$$\begin{aligned}J &= \int_0^\infty -\dot{V}(\tau) d\tau \\ &= X^{0^T} P X^0 + Z(X^0)\end{aligned} \tag{27}$$

Now, a Hamiltonian is defined as

$$\begin{aligned}H(X, \nabla V^T(X), \nu) &= L(X, \phi, t) + \nabla V^T(X) \cdot [AX + \Gamma\nu] \\ &= X^T QX + \nu^T R\nu + M(X) + [2X^T P + \nabla Z(X)](AX + \Gamma\nu)\end{aligned} \tag{28}$$

and by letting $\frac{\partial}{\partial\nu} H(\cdot, \cdot, \cdot) = 0$, (21a) and (21b) can be obtained. Furthermore with (22a) and (22b), it can be shown that

$$H(X, \nabla V^T(X), \nu) = [\nu - \phi(\cdot)]^T R[\nu - \phi(\cdot)] \tag{29}$$

which completes the proof of Theorem 1.

At this point, the special function for the candidate of $G(X)$ are defined in the following.

Definition 1: homogeneous k(respectively, $2v$)-form[18]

Let a function $\psi: R^n \to R^p$ a homogeneous k (respectively, $2v$)-form if k is positive integer and $\psi(a \cdot X) = a^{k(2v)}\psi(X)$ for all $a \in R$ and $X \in R^n$. Particularly, when $p=1$, a homogeneous $k(2v)$-form $\psi: R^n \to R$ is non negative definite if $\psi(X) \ge 0$ for all $X \in R^n$.

To investigate the existence of the solution such that (22b), (22b) can be represented by supposing that for $v = 2, 3, \ldots, q (k = 4, 5, \ldots, 2q)$, $M_{k(2v)}$ is given non-negative definite homogeneous $k(2v)$-form as

$$\nabla Z_{2v(k)}(X)\Phi \cdot X + M_{2v(k)}(X) = 0 \tag{30}$$

$$v = 2, 3, \ldots, q (k = 4, 5, \ldots 2q)$$

where

$$M(X) = \sum_{v=2}^{q} M_{2v}(X) + \frac{1}{4}\sum_{v=2}^{q} \nabla Z_{2v}(X) \cdot W \cdot \nabla Z_{2v}(X) \quad \text{for } 2v\text{-form} \tag{31a}$$

$$= \sum_{k=4}^{2q} M_k(X) + \frac{1}{4}\sum_{k=4}^{2q} \nabla Z_k(X) \cdot W \cdot \nabla Z_k(X) \quad \text{for } k\text{-form} \tag{31b}$$

$$Z(X) = \sum_{v=2}^{q} Z_{2v}(X) \quad \text{for } 2v\text{-form} \tag{32a}$$

$$= \sum_{k=4}^{2q} Z_k(X) \quad \text{for } k\text{-form} \tag{32b}$$

The existence of the solution for (30) is reviewed in Lemma 1

Lemma 1: Let $\Phi \in R^{n \times n}$ be Hurwitz matrix and $M: R^m \to R$ be a homogeneous $k(2v)$-form. Then, there exists a unique homogeneous $k(2v)$-form $Z: R^n \to R$ such that (30). Furthermore, if $M(\cdot)$ is a non-negative(resp. positive) definite, then, $Z(\cdot)$ is a non-negative(resp. positive) definite.

Proof: For convenience, define the Kronecker sum[24] as

$$\overset{k}{\oplus} A \equiv A \oplus A \oplus \ldots \oplus A \tag{33}$$

where A appearing k times and so forth, and define $M(X) \equiv \psi X^{[k]}$ where $\Phi = -\psi \left(\overset{k}{\oplus A} \right)^{-1}$ where $\overset{k}{\oplus A}$ is invertable since A (cad hence $\overset{k}{\oplus A}$) is asymptotically stable. Now, note that

$$\begin{aligned} \nabla Z(X) \Phi \cdot X &= \Phi \frac{d}{dX} (X^{[k]} A \cdot X) \\ &= \Phi (A \oplus A \oplus ... \oplus A) X^{[k]} \\ &= \Phi \left(\overset{k}{\oplus A} \right) X^{[k]} \\ &= -\psi \cdot X^{[k]} \\ &= -M(X). \end{aligned} \tag{34}$$

Thus, the feature of $M(X)$ is the same that of $G(X)$.

In Lemma 1, $M(X)$ is a positive definite homogeneous k-form, where k is necessary even. Futhermore, let $Z(X)$ be the positive-definite, then, since $\nabla Z(X) \Phi \cdot X < 0$ for $X \in R^n$, $X \neq 0$, it follows that $Z(X)$ is a Lyapunov function for (19). Finally, one can find a stable $G(X)$ using $2v$- or even k-form $M(\cdot)$ and $Z(\cdot)$. In [12], a homogeneous odd k-form is selected for the nonlinear term in the sliding surface without integration.

Now, the coefficient matrices of the sliding surface (9) can be obtained from (17a)-(17c), (21a), and (21b) as

$$C_{L2}^{-1}(C_{L1}A_{11} + C_{l1}) = B_2 K_1 - A_{21} \tag{35a}$$

$$C_{L2}^{-1}(C_{L1}A_{12} + C_{l2}) = B_2 K_2 - A_{22} \tag{35b}$$

$$C_{L2}^{-1}(C_N \cdot G(X)) = B_2 K_3 G(X) = \frac{1}{2} R^{-1} \Gamma^T \nabla Z(X) \tag{35c}$$

Particularly, if $C_{L2} = I_m$ without loss of generality, then simply

$$C_{L1}A_{11} + C_{l1} = B_2 K_1 - A_{21} \tag{36a}$$

$$C_{L1}A_{12} + C_{l2} = B_2 K_2 - A_{22} \tag{36b}$$

$$C_N = B_2 K_3 = \frac{1}{2} R^{-1} \Gamma^T \qquad \nabla Z(X) = G(X). \tag{36c}$$

Therefore, one can determine the optimal coefficient matrices of the integral nonlinear sliding surface (9). One example on the nonlinear dynamics for the integral nonlinear sliding surface will be given.

Example 1: Consider that $V(X)$ be of the form

$$V(X) = X^T P X + \frac{1}{2} (X^T \overline{Z} X)^2 \qquad \overline{Z} > 0 \tag{37}$$

with $v = q = 2$, which corresponds to (19) with $Z(X) = 1/2 (X^T \overline{Z} X)^2$. From (22b), one can obtain

$$\begin{aligned} M(X) + (X^T \overline{Z} X) \cdot X^T (\Phi^T \overline{Z} + \overline{\Phi}) X& \\ - (X^T \overline{Z} X)^2 \cdot X^T \overline{Z} W \overline{Z} X = 0& \end{aligned} \tag{38}$$

Hence, if one select $\overline{Z}$ such that

$$(\Phi^T \overline{Z} + \overline{Z} \Phi) + \overline{R} = 0 \tag{39}$$

for $R > 0$, then $M(X)$ becomes

$$\begin{aligned} M(X) = (X^T \overline{Z} X) \cdot X^T (\overline{R}) X& \\ + (X^T \overline{Z} X)^2 \cdot X^T \overline{Z} W \overline{Z} X& \end{aligned} \tag{40}$$

Finally

$$G(X) = (X^T \overline{Z} X) \cdot \overline{Z} X^T \tag{41}$$

which results in the stable design of the nonlinear dynamics (9c) for the sliding surface of (9). Thus, the nonlinear term $G(X)$ in (9c) can be selected by the choice of $\overline{R}$ in (39) through determination of $M(X)$.

2.3 Stabilizing Input and Stability Anaysis

The control input should be designed to satisfy the existence condition of the sliding mode, but it is not simple for multi input plants. The vector version of the existence condition of the sliding mode, as well known, is

$$\lim_{s \to 0} s_i \cdot \dot{s}_i < 0 \qquad i = 1, 2, ..., m \tag{42}$$

or

$$\lim_{s \to 0} s_i \cdot \dot{s}_i \leftarrow \eta |s_i| \qquad i = 1, 2, ..., m \tag{43}$$

so called direct switching approach in [5]. It is, however, difficult to use these condition directly for uncertain general multi input plants because it is dependent of the structure of $C_{L2} B_2(X,t)$. The looser condition than (42) and (43) so called Lyapunov function approach including the norm-bounded approach for multi-input nonlinear systems is as follows[5]:

$$S^T \dot{S} = \sum_{i=1}^{m} s_i \cdot \dot{s}_i = s_1 \cdot \dot{s}_1 + s_2 \cdot \dot{s}_2, ..., + s_m \cdot \dot{s}_m < 0. \tag{44}$$

If (42) and (43) are satisfied, then (44) is, but its reciprocal argument generally does not hold[5].

In this paper, based on Assumption 1, 2, 3, diagonalization method will be utilized with Theorem 2

Theorem 2: *The equation of the sliding mode is invariant with respect to the nonlinear transformations*

$$S^*(X) = H_s(X,t) \cdot S(X), \qquad \det H_s(X,t) \neq 0 \tag{45a}$$

$$U^*(X) = H_u(X,t) \cdot U(X), \qquad \det H_u(X,t) \neq 0 \tag{45b}$$

where det denote determinate of a matrix.

Proof: See [1] or [4]

This theorem means that the sliding mode is governed by the same (9) if the components of the controlled vector undergo discontinuity on the new transformed surface $S^*(X)=0$ or the components of the new control vector $U^*(X)$ undergo discontinuity on the already chosen surface $S(X)=0$.

With transforming $S(X)=0$ into $S^*(X)=0$, consider a corresponding control function having the form of

$$U(X_1,X_2) = U_{eq}(X_1,X_2) + \Delta U(X_1,X_2) \tag{46}$$

where $U_{eq}(X_1,X_2)$ is the equivalent control for the nominal system of (3) directly determined according to the choice of the integral nonlinear sliding surface (9) as

$$\begin{aligned} U_{eq}(X_1,X_2) = -\left[C_{L2}B_2\right]^{-1}\{ & (C_{L1}A_{11} + C_{L2}A_{21} + C_{I1}) \cdot X_1 \\ & + (C_{L1}A_{12} + C_{L2}A_{22} + C_{I2}) \cdot X_2 + C_N G(X)\} \end{aligned} \tag{47}$$

which governs the desired main nonlinear dynamics to be optimal for (18), and ΔU cancels out the uncertainties and external disturbances in order to maintain the sliding mode on pre-specified surface form X^0 to origin

$$\Delta U = (X_1,X_2) = -\left\{ \begin{aligned} & \Psi_1 \cdot X_1 + \Psi_2 \cdot X_2 + \Psi_3 \cdot G(X) \\ & + \delta \cdot sgn(S^*) + \kappa \cdot S^* \end{aligned} \right\} \tag{48}$$

where S^* implies the transformed a new sliding surface by $H_s(X,t) = \left[C_{L2}B_2\right]^{-1}$ as

$$S^* = \left[C_{L2}B_2\right]^{-1} \cdot S \tag{49}$$

and the switched gain matrices $\Psi_0 \in R^{m\times r}$, $\Psi_1 \in R^{m\times(n-m)}$, $\Psi_2 \in R^{m\times m}$, and δ and $\kappa \in dia\left[R^{m\times m}\right]$ can be selected by the inequalities as follows:

$$\Psi_{0ij} = \begin{cases} >0 & \text{for} \quad s_i^* \cdot x_{0j} > 0 \\ <0 & \text{for} \quad s_i^* \cdot x_{0j} < 0 \end{cases} \tag{50a}$$

$$\Psi_{1ik} = \begin{cases} > (\alpha_{21ik} + \beta_{2i} \cdot K_{1ik})/(1-\beta_{2i}) & \text{for} \quad s_i^* \cdot x_{1k} > 0 \\ < (\alpha_{21ik} + \beta_{2i} \cdot K_{1ik})/(1-\beta_{2i}) & \text{for} \quad s_i^* \cdot x_{1k} < 0 \end{cases} \tag{50b}$$

$$\Psi_{2il} = \begin{cases} > (\alpha_{22il} + \beta_{2i} \cdot K_{1il})/(1-\beta_{2i}) & \text{for} \quad s_i^* \cdot x_{2l} > 0 \\ \leftarrow (\alpha_{22il} + \beta_{2i} \cdot K_{1il})/(1-\beta_{2i}) & \text{for} \quad s_i^* \cdot x_{2l} < 0 \end{cases} \tag{50c}$$

$$\Psi_{3iz} = \begin{cases} > (\alpha_{22iz} + \beta_{2i} \cdot K_{1iz})/(1-\beta_{2i}) & \text{for} \quad s_i^* \cdot g(x)_z > 0 \\ \leftarrow (\alpha_{22iz} + \beta_{2i} \cdot K_{1iz})/(1-\beta_{2i}) & \text{for} \quad s_i^* \cdot g(x)_z < 0 \end{cases} \tag{50d}$$

$$\delta_i = \begin{cases} > \gamma_{2i}/(1-\beta_{2i}) & \text{for} \quad s_i^* > 0 \\ \leftarrow \gamma_{2i}/(1-\beta_{2i}) & \text{for} \quad s_i^* < 0 \end{cases} \tag{50e}$$

$$\kappa_i > 0 \tag{50f}$$

for $i,l=1,2,\ldots,m$, $j=1,2,\ldots,r$, $k=1,2,\ldots,(n-m)$, and $z=1,2,\ldots,n$. From (2b) and (36), the original control for (1) can be found as

$$U_Y(Y) = U_X(X)|_{X=T\cdot Y} = U_X(T\cdot Y) \tag{51}$$

The stability of the closed loop system and the existence condition of the sliding mode will be investigated.

Theorem 3: The closed loop system (1) with (51) is totally asymptotically stable with respect to $S^*(X)=0$, eventually to origin of $(n+r)$ order state space provided that (9) is asymptotically stable.

Proof: Take Lyapunov candidate function as

$$V(X) = 1/2 S^{*T} \cdot S^*. \tag{52}$$

From (3) and (46), the derivative of $S^*(t)$ becomes

$$\begin{aligned} \dot{S}^*(t) = & (C_{L1}A_{11} + C_{L2}A_{21} + C_{I1}) \cdot X_1 \\ & + (C_{L1}A_{12} + C_{L2}A_{22} + C_{I2}) \cdot X_2 + C_N G(X) \\ & + U_{eq}(X) \\ & + \left\{ \begin{aligned} & \Delta A_{21} \cdot X_1 + \Delta A_{22} \cdot X_2 + D_2(X,t) \\ & - \Delta B_2 U_{eq}(X) + \Delta U(X) \end{aligned} \right\} \end{aligned} \tag{53}$$

Rearranging (53), it follows

$$\begin{aligned} \dot{S}^*(t) = & -\Psi_0 \cdot X_0 \\ & - \left[\Delta B_2 (C_{L2}B_2)^{-1}(C_{L1}A_{11} + C_{L2}A_{21} + C_{I1}) - \Delta A_{21} + (I_m - \Delta B_2) \cdot \Psi_1\right] \cdot X_1 \\ & - \left[\Delta B_2 (C_{L2}B_2)^{-1}(C_{L1}A_{12} + C_{L2}A_{22} + C_{I2}) - \Delta A_{22} + (I_m - \Delta B_2) \cdot \Psi_2\right] \cdot X_2 \\ & - \left[\Delta B_2 (C_{L2}B_2)^{-1} C_N + (I_m - \Delta B_2) \cdot \Psi_3\right] \cdot G(X) \\ & - \left[\delta sgn(S^*) - D_2(X,t)\right] \\ & - \kappa \cdot S^*. \end{aligned} \tag{54}$$

Finally, one can easily show that

$$s_i^* \cdot \dot{s}_i^* < \kappa_i \cdot s_i^{*2} \qquad i=1,2,\ldots,m. \tag{55}$$

Then from (55), the algorithm guarantees the sliding mode at every point on the new sliding surface. And therefore, based on theorem 2, the motion equation in the sliding mode on the sliding surface (9) is invariant, and the controlled system is asymptotically stable to the new sliding surface and eventually to the origin in $n+r$ state space.

By Theorem 3, the optimal performance designed by Theorem 1 for the nominal system of (1) is guaranteed for all uncertainties and external disturbance. As a result, the choice of the sliding surface implies the nonlinear optimal performance design, and control input selection does the robust stabilizing design for the pre-designed performance against the system uncertainties and disturbance, i.e. design separation. In addition, it does not need to consider the reachability to the sliding surface[25] during the reaching phase.

To show the effectiveness of the algorithm, an example will be presented

III. Illustrative Example

The control of an example uncertain MIMO plant is presented for the purpose of the performance comparison between with and without the intentional nonlinear term in the INOVSS algorithm. A $4-th$ order MIMO plant is considered as

$$\dot{X}=\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -a_{11}(t) & -a_{12}(t) & 0 & 0 \\ -a_{21}(t) & -a_{22}(t) & 0 & 0 \end{bmatrix}\cdot X+\begin{bmatrix} 0 & 0 \\ 0 & 0 \\ b_1(t) & 0 \\ 0 & b_2(t) \end{bmatrix}\cdot U+\begin{bmatrix} 0 \\ 0 \\ d_1(t) \\ d_2(t) \end{bmatrix} \tag{56}$$

Table 1 Selected control gains

		Ψ_{01}		Ψ_1		Ψ_2		Ψ_3		δ	κ
		x_{01}	x_{02}	x_1	x_2	x_3	x_4	$g(x)_3$	$g(x)_4$		
ΔU_1	+	1	0	13	4	10	0	6	0	25	0.1
	-	-1	0	-13	-4	-10	0	-6	0	-25	
ΔU_2	+	0	1	4	13	0	10	0	6	25	0.1
	-	0	-1	-4	-13	0	-10	0	-6	-25	

where $X^T=[x_1 \ \ x_2 \ \ x_3 \ \ x_4]$ is the state, system parameters $a_{ij}(t)$ gains $b_i(t)$, and disturbance $d_i(t)$ of the plant are assumed such that

$$a_{ij}(t)=-1+\Delta a_{ij}(t) \qquad -0.5<\Delta a_{ij}(t)<0.5 \tag{57a}$$

$$b_i(t)=1+\Delta b_i(t) \qquad -0.5<\Delta b_i(t)<0.5 \tag{57b}$$

$$|d_i(t)|<4. \tag{57c}$$

The nominal matrices of (56) becomes

$$A_{11}=O \quad A_{12}=I_2 \quad A_{21}=\begin{bmatrix} -1 & -1 \\ -1 & -1 \end{bmatrix}. \tag{58}$$
$$A_{22}=O, \quad \text{and} \quad B_2=I_2$$

To show the potential of the nonlinear dynamics of the algorithm, as an example problem, the constraint on the states x_3 and x_4 are additionally imposed as

$$|x_3|<21 \qquad \text{and} \qquad |x_4|<11. \tag{59}$$

To effectively solve these constraints, the INOVSS algorithm will be designed. Above all, Q and R in (18) is selected as

$$Q=\begin{bmatrix} 10 & 0 & 0 & 0 \\ 0 & 10 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad R=\begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix} \tag{60}$$

which results in the linear coefficient matrices of (16), K_i, $i=1,\ 2,\ 3$ such as

$$K_1=\begin{bmatrix} 11.09902 & 1.09902 \\ 1.09902 & 1.09902 \end{bmatrix} \tag{61a}$$

$$K_2=\begin{bmatrix} 5.67102 & 0.19380 \\ 0.19380 & 5.67102 \end{bmatrix} \tag{61b}$$

$$K_3=\begin{bmatrix} 0 & 0 & 5.0 & 0 \\ 0 & 0 & 0 & 5.0 \end{bmatrix}. \tag{61c}$$

Next, to specify the stable nonlinear element $G(X)$ in (16), the form of $Z(X)$ in (19) is chosen as the same in (37). By analyzing the dynamics (14) resulted from modifying $\overline{R}$ in (39), one can determine the $M(X)$ and $G(X)$ so that (59) is satisfied as

$$M(X)=(X^T\overline{Z}X)(X^T\overline{R}X)+(X^T\overline{Z}X)^2X^T\overline{Z}W\overline{Z}X \tag{62}$$

$$G(X)=(X^T\overline{Z}X)\cdot\overline{Z}X^T \tag{63}$$

where

$$\overline{R}=\begin{bmatrix} 0 & 0 & -0.118 & 0 \\ -0.118 & 0 & 0.15 & 0 \\ 0 & 0.15 & 0.3 & 0 \\ 0 & 0 & 0 & 0.3 \end{bmatrix} \tag{64}$$

$$\overline{Z}=\begin{bmatrix} 0.2517 & -0.00489 & 0.00000 & -0.00012 \\ -0.00489 & 0.26730 & 0.00012 & 0.00000 \\ 0.00000 & 0.00012 & 0.01325 & -0.02647 \\ -0.00012 & 0.00000 & -0.00068 & 0.02647 \end{bmatrix}. \tag{65}$$

Then, from (36), (62), and (63), the coefficient matrices of the nonlinear sliding surface can be designed as

$$C_{l1}=\begin{bmatrix} 10.09902 & 0.09902 \\ 0.09902 & 0.09902 \end{bmatrix}, \qquad C_{l2}=O \tag{66a}$$

$$C_{L1}=\begin{bmatrix} 5.67102 & 0.19380 \\ 0.19380 & 5.6102 \end{bmatrix}, \qquad C_{L2}=\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \tag{66b}$$

$$C_N=\begin{bmatrix} 0 & 0 & 5.0 & 0 \\ 0 & 0 & 0 & 5.0 \end{bmatrix}. \tag{66c}$$

At the stage of the design of the INOVSS, the switching gains Ψ_0, Ψ_1, Ψ_2, δ, and κ are selected as in Table 1. For the three initial conditions,

$$X^{0^T} = [40 - 20 \quad 0 \quad 0], \quad [30 - 15 \quad 0 \quad 0], \quad [20 - 10 \quad 0 \quad 0] \tag{67}$$

the simulations are carried out with 2 $[msec]$ sampling and under the condition of parameter variations and external disturbance in (56) as

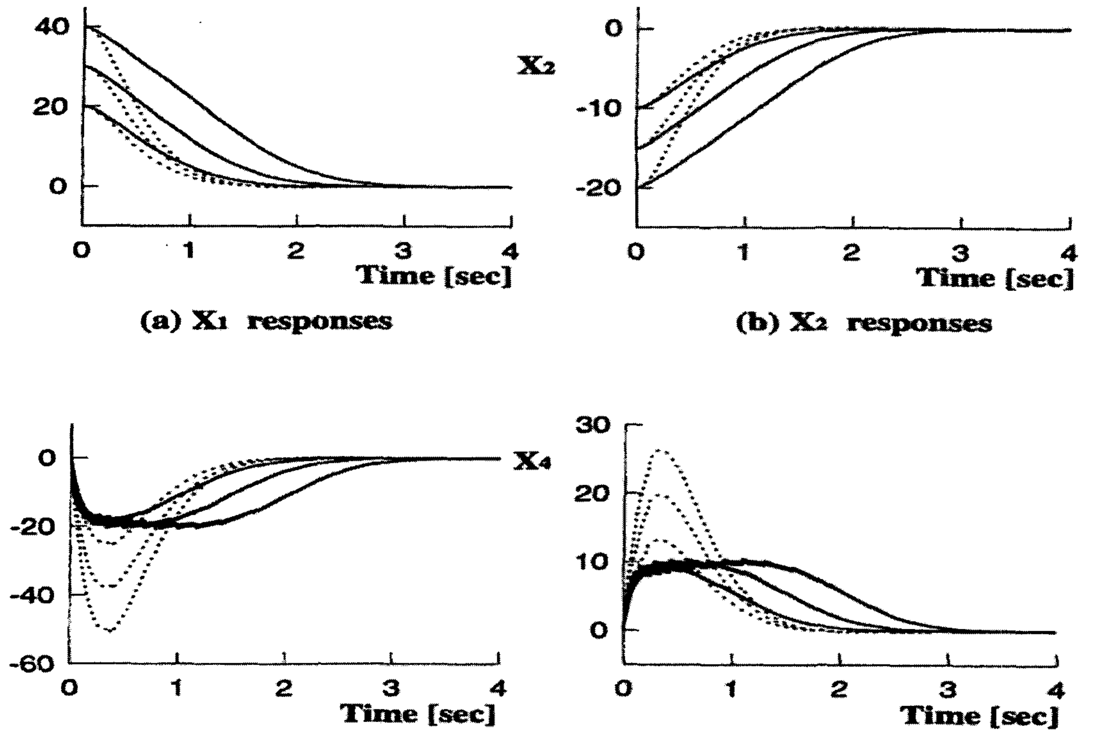
$$\Delta a_{ij} \text{ and } \Delta b_i = 0.5, \quad |d_i(X,t)| = 6, \quad i,j = 1,2. \tag{68}$$

The simulation results for the nonlinear optimal VSS when $M(\cdot)=0$ and $M(\cdot)\neq 0$ are comparatively shown in Fig. 1 through Fig 5. Fig 1 shows each predicted state response of the integral-augmented linear and nonlinear sliding dynamics of (9), i.e., solution of (14) with (16) for the initial condition (67). As designed, the response of x_3 and x_4 by the nonlinear optimal sliding surface are limited by the constraints given in (59), whereas the output response of x_1 and x_2 are retarded compared with those when $G(\cdot)=0$. These features can be also found in phase trajectories of $x_1 vs\ x_3$ and $x_2 vs\ x_4$ in Fig. 2. With the linear dynamics, it is difficult to consider the constraints on the states. Under the uncertainties and disturbance of (68), the real output responses of (3) by the both schemes are given I Fig. 3, which is almost equal to Fig. 1 as can be seen. Thus, the designed nonlinear optimal performance is preserved by the INOVSS algorithm. And the prediction/predetermination of the output is feasible using Fig. 1. The corresponding phase trajectories of $x_1 vs\ x_3$ and $x_2 vs\ x_4$ are presented in Fig. 4. In these figures, the saturation effect on the states x_3 and x_4 can be seen as predicted. In addition, there is no reaching phase, but the sliding mode occurs from the initial conditions. Hence there is no reaching phase problems. The total control inputs of (46) with discontinuity are depicted in Fig. 5 for $X_0^T = [40-20\ 0\ 0RIGHT]$

From the above comparative simulation studies, the potential of the nonlinear optimal VSS is appreciated.

IV. Conclusions

In this paper, an integral-augmented nonlinear optimal variable structure system is studied for the control of uncertain MIMO plant subjected to persistent disturbances by effectively combing VSS theory and the nonlinear optimal control algorithms. The integral nonlinear sliding surface under consideration can offer significant advantages over the linear one in a variety of circumstances such as state constraint. With the proposed sliding surface, there is no reaching phase problems, Thus the complete robustness for the whole trajectory is obtained. The ideal sliding dynamics of the nonlinear sliding surface is obtained in nonlinear state equation form. Using this ideal sliding dynamics, the nonlinear optimal technique is established for the stable design of the nonlinear sliding surface by minimizing the non-quadratic performance index. The homogeneous $2v(k)$ form is defined in order to easily select the $2v$ or even k-form higher order nonlinear terms in the suggested sliding surface. And using the diagonalization method, the stabilizing input satisfying the existence condition of the sliding mode on the new transformed surface is also designed in order to guarantee the nonlinear optimal performance pre-selected in the sliding dynamics.

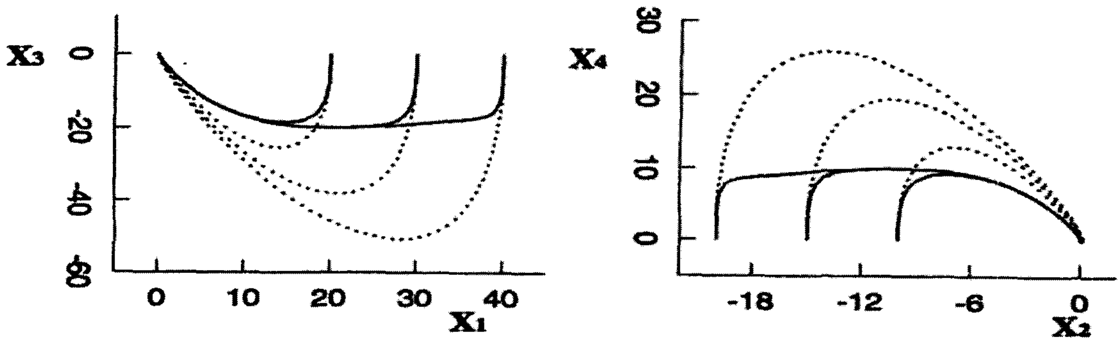


(a) X_1 responses (b) X_2 responses

(c) X_3 responses (d) X_4 responses

Fig 1 Designed outputs by the linear and nonlinear VSS's for

X^{0^T}= [40 −20 0 0], [30 −15 0 0], [20 −10 0 0]
.... linear optimal
—————— nonlinear optimal



(a) X_3 vs. X_1 (b) X_4 vs. X_2

Fig 2 Designed phase trajectories by the linear and nonlinear VSS's for

X^{0^T}= [40 −20 0 0], [30 −15 0 0], [20 −10 0 0]
.... linear optimal
—————— nonlinear optimal

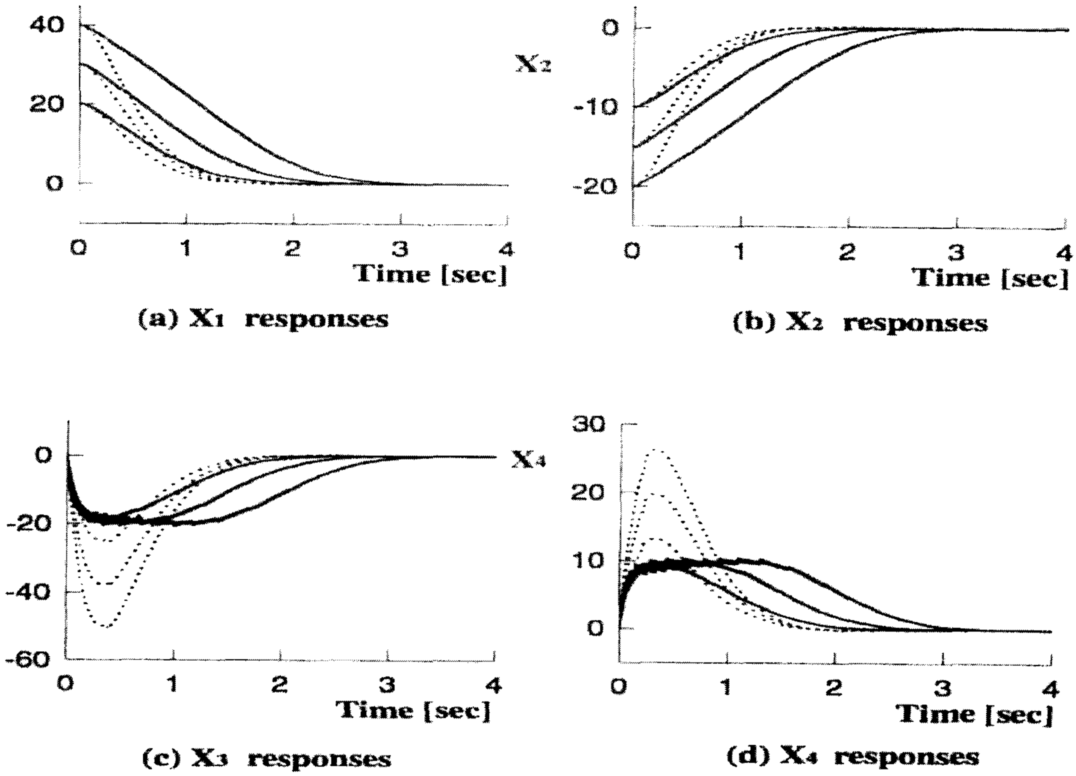


Fig 3 Real outputs by the linear and nonlinear VSS's for

X^{0^T}= [40 −20 0 0], [30 −15 0 0], [20 −10 0 0]

.... linear optimal

—————— nonlinear optimal

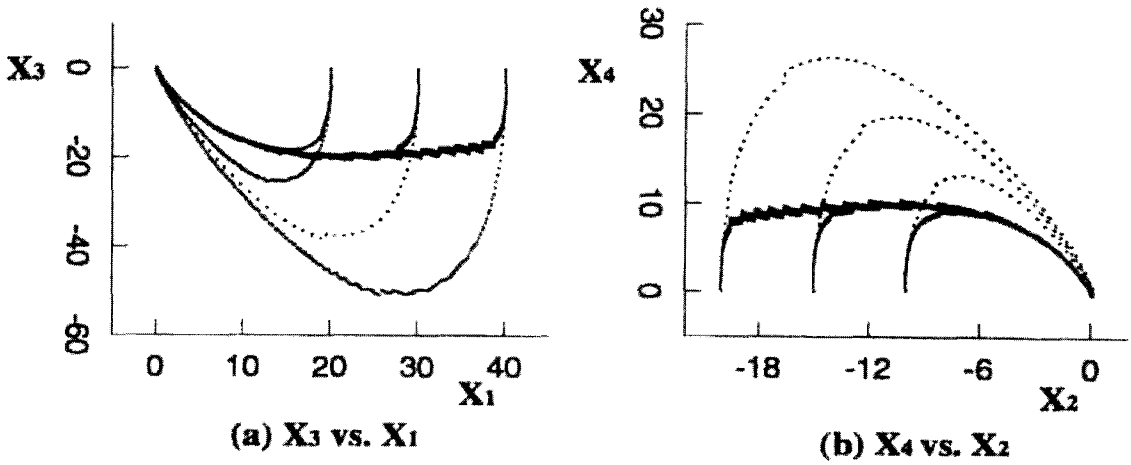


Fig 4 Real phase trajectories by the linear & nonlinear VSS's for

X^{0^T}= [40 −20 0 0], [30 −15 0 0], [20 −10 0 0]

.... linear optimal

—————— nonlinear optimal

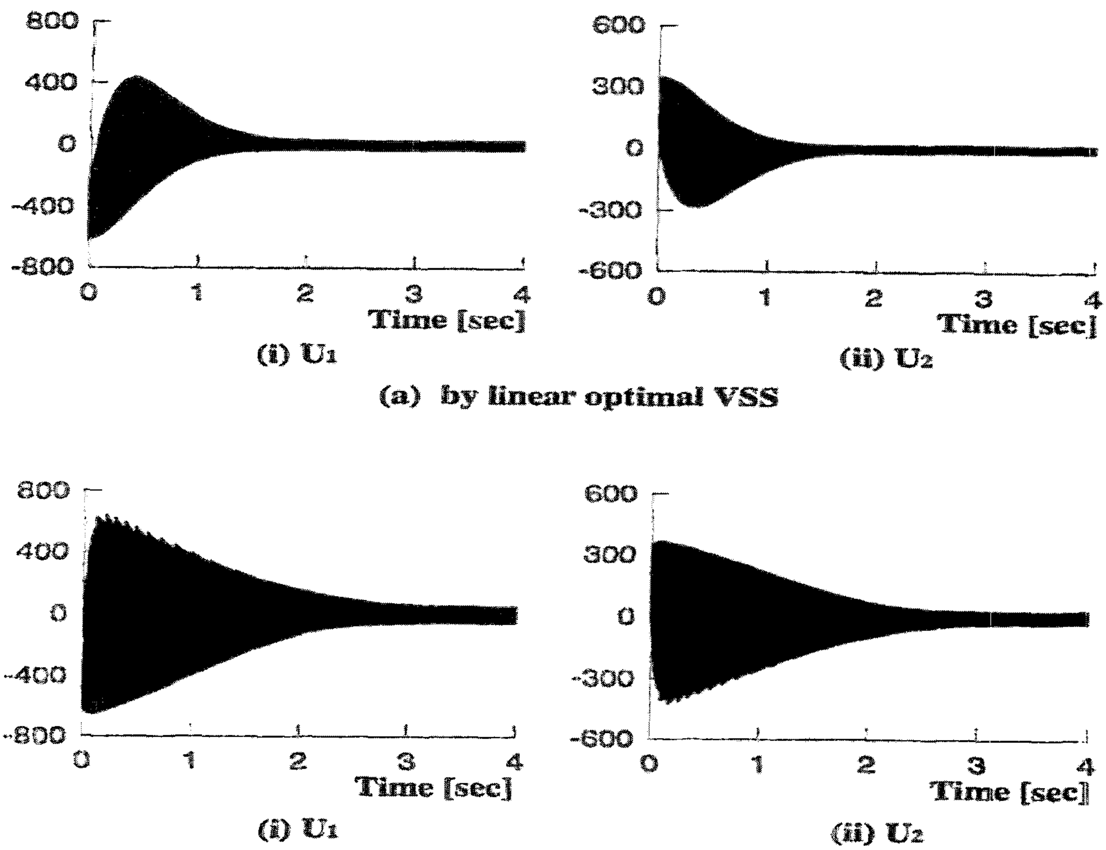


Fig. 5 Real inputs under $X^{0^T} = [40\ -20\ 0\ 0]$

The asymptotic stability of the closed loop system with respect to the transformed new nonlinear surface including the origin is investigated together with the existence condition of the sliding mode. An example is be given for showing the usefulness of the algorithm. The nonlinear optimal VSS possesses the potential of the application to the control problems of large scale systems. Finally, the attractive performance of the INOVSS are pointed out in view of no reaching phase, complete robustness fro whole trajectory, output prediction/predetermination, separation of the performance design and robustness problem, the better performance by the integral nonlinear sliding surface, and etc.

References

[1]V. I. Utkin, Sliding modes and their application in variable structure systems. Moscow, 1978.

[2] U. Itkis, Control systems of variable structure. New York, John Wiley & Sons, 1976.

[3]V. I. Utkin, “Variable structure systems with sliding mode,” *IEEE Trans. Automat. Control*, Vol. AC-22, No.2, pp. 212-222, 1977.

[4]R. A. Decarlo, S. H. Zak, and G. P. Mattews, "Variable structure control of nonlinear multivariable system:A tutorial," Proc. IEEE, pp.212-232, Vol. 76, 1988.

[5]J. Y. Hung, W. Gao, and J. C. Hung, "Variable structure control: A survey," IEEE Trans. Industrial Electronics, Vol. 40, no.1, Feb. pp.2~22, 1993.

[6]V. I. Utkin and K. D. Yang, "Methods for constructing discontinuity planes in multidimensional variable structure systems," Auomat. Remote Control, Vol.39, no.10 pp.1466-1470, 1978.

[7]D. M. E. El-Ghezawi, A. S. I. Zinober, D. H. Owens, and S. A. Bilings, "Computation of the zeros and direction of linear multivariable systems," Int. J. Control, vol.36, no.5, pp.833-843, 1982.

[8]D. M. E. El-Ghezawi, A. S. I. Zinober, and S. A. Bilings, "Analysis and design of variable structure system using a geometric approach," Int. J. Control, vol. 38, no.3, pp.657-671, 1983.

[9]S. R. Hebertt, "Differential geometric method in variable structure control," Int. J. Control, vol. 48, 1988.

[10]J. Sander and Williamson. "Design of nonlinear regulators for linear plants," IEEE Trans. Autom. Control, Vol.AC-22, no.1, pp.47-50, 1977.

[11] D. S. Lee and M. J. Youn, "Controller design of variable structure system with nonlinear sliding surface," Electronics Letter, Dec. vol. 25, no.25, pp1715-1717, 1989.

[12]D. S. Lee, Controller design of variable structure system with nonlinear sliding surface, Ph. D. Dissertation, KAIST, 1991.

[13]J. J. Slotine and S. S. Sastry, " Tracking control of nonlinear systems using sliding surface, with application to robot manipulators," Int. J. Control, vol. 38, no.2, pp.465-492, 1983.

[14] K. K. D. Young P. V. Kokotovic, and V. I. Utkin, "A singular perturbation analysis of high-gain feedback systems," IEEE Trans. Autom. Control, vol. AC-22, no.6, pp.931-938, 1977.

[15]S. B. Ceong, C. C., and D. W. Park, "Moving switching surfaces for robust control second-order variable structure system," Int. J. Control, vol. 58, no.1 pp.229-245, 1993.

[16]F. Harashima, H. Hashimoto, and S. Kondo, "MOSFET converter-fed position servo system with sliding mode control," IEEE Trans. Ind. Electronics , vol.IE-32, no.3, 1985.

[17]T. H. Chang and Y. Hurmulzlu, "Sliding control without reaching phase and its application to bipdedal locomotion," Trans. of ASME, vol.115, pp.447-455, 1993.

[18]R. W. Bass and R. F. Webber, "Optimal nonlinear feedback control derived from quartic and higher order performance index," IEEE Trans. Autom. Control, Vol.AC-11, pp.448-455, 1966.

[19]R. Mekel and P. Peruo, "Design of controller for a class of nonlinear control systems, IEEE Trans. Autom. Control, Vol.AC-17, no.2, pp.206-213, 1972.

[20]J. F. Frankena and R. Sivar, "A nonlinear optimal control law for linear system," Int. J. Control, vol.30, no.1 pp.159-178,1979.

[21]K. Poolla and T. Ting, "Nonlinear time-varying controllers for robust stabilization," IEEE Trans. Autom. Control, Vol.AC-32, no.3, pp.195-200, 1987.

[22]L. R. Hunt, B. Su, and G. Meyer, "Global transformation of nonlinear systems," IEEE Trans. Autom. Control, Vol.AC-28, no.1, pp.24-31, 1983.

[23]A. G. Luk'yanov and V. I. Utkin, "Methods for reducing equations for dynamic system to a regular form," Automation and Remote Control, vol.42, no.4, pp.413-420, 1981.

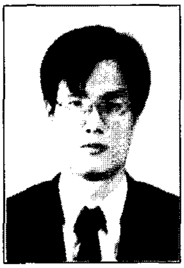
[24]J. W. Brewer, "Kronecker products and matrix calculus in system theory," IEEE Trans. Circuit and System, Vol.CAS-25, no.9, pp.772-781, 1978.

[25]S. V. Yallapragada and S. Heck, "Reaching conditions in variable structure systems for output feedback control," ACC'91, pp.32-36, 1991.

[26]J. H. Lee and M. J. Youn, "A integral-augmented optimal variable structure control for uncertain dynamical SISO system," Transaction of KIEE, vol.44,no.8,pp.1332-1351,1994

BIOGRAPHY

Jung Hoon Lee (Nember)



1988 : BS degree in Electronics Engineering,, Kyeongbook National University.
1990 : MS degree in Control and power electronics Engineering, KAIST.
1995 : PhD degree in Control and power electronics Engineering, KAIST.
1995~2007 ; Associate Professor, Gyeongsang National University.