

Pulse Multiplication in Autotransformer Based AC-DC Converters using a Zigzag Connection

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ABSTRACT

This paper deals with pulse multiplication in zigzag connected autotransformer based 12-pulse AC-DC converters feeding vector controlled induction motor drives (VCIMD) for improving the power quality at the point of common coupling (PCC) without using a Zero-Sequence-Blocking-Transformer (ZSBT). The proposed 24-pulse AC-DC converter is based on the principle of DC ripple re-injection technique for pulse multiplication and harmonic mitigation. The design of the autotransformer is carried out for the proposed AC-DC converter and the effect of load variation on VCIMD is also studied to demonstrate the effectiveness of the proposed AC-DC converter. Test results from a laboratory developed prototype, along with simulated results, are presented to validate the design and model of the proposed 24-pulse AC-DC converter.

Keywords: Autotransformer, Multipulse AC-DC converter, Pulse multiplication, Power quality, VCIMD

1. Introduction

The most common industrial motor is an induction motor and its speed control is achieved by employing a voltage source inverter for generating a variable frequency supply. Variable frequency induction motor drives (VFIMDs) are used in numerous applications such as heating, ventilating and air conditioning (HVAC) systems, blowers, fans, cement industries, etc. These VFIMDs are controlled in vector control mode^[1] due to their inherent advantages. These are generally fed from a 6-pulse diode bridge AC-DC converter, which injects current harmonics into AC mains. Fig. 1 shows such a 6-pulse diode bridge

rectifier fed vector controlled induction motor drive (Topology 'A'). These harmonics cause increased losses for the customer and utility power system components, de-rating of distribution transformers and failure of capacitor banks. These current harmonics, while flowing through source impedance, result in voltage distortion at the point of common coupling (PCC), thereby affecting neighboring consumers. Stringent power quality regulation and strict limits on current and voltage distortion have been imposed by various standards such as IEC 61000-3-2^[2] and IEEE 519-1992^[3]. These needs to improve power quality have attracted the attention of researchers.

Power quality can be improved by employing passive or active filters. Passive filters cause additional losses, need additional floor area and are complex in design as their performance is affected by changes in frequency.

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Similarly, active filters are also costly and complex, and have ratings comparable to the load rating. The use of multipulse AC-DC converters is one such technique for harmonic mitigation and is found to be rugged and reliable. This method uses two or more bridge converters, where the harmonics generated by one converter are cancelled by another converter by a proper phase shift. Different topologies of 12-pulse AC-DC converters have been reported in the literature for harmonic mitigation [4-8], but these configurations also fail to comply with the above mentioned power quality standards. Increasing the number of pulses results in further improvement in various power quality indices, but also results in the additional cost of different converters and increased system complexity. To achieve similar performance in terms of various power quality indices at a reduced cost, the DC ripple re-injection technique [9-11] is a viable and cost effective solution.

However, it must be noted that in all these autowound transformer based rectifiers employing ripple reinjection techniques, a Zero-Sequence-Blocking-Transformer (ZSBT) of significant rating (7.5% of DC load power rating) is required. The role of a ZSBT here is to prevent circulating currents between two converters and, thus, prevent unequal conduction of upper and lower diodes in the legs of the converter bridges. This paper presents an autotransformer based 24-pulse AC-DC converter that does not require a ZSBT when the DC ripple re-injection technique is used for pulse doubling. The DC ripple re-injection technique used for harmonic mitigation requires two additional diodes along with a suitable tapped interphase reactor for increasing the number of pulses. A set of tabulated results giving a comparison of different power quality indices such as total harmonic distortion (THD) and crest factor of AC main current (CF), power factor (PF), displacement factor (DPF) and distortion factor (DF), and THD of the supply voltage at PCC is presented for a VCIMD fed from an existing 6-pulse AC-DC converter, 12-pulse AC-DC converter and proposed 24-pulse AC-DC converter. A laboratory prototype of the proposed 24-pulse AC-DC converter is developed to validate its design and simulation model.

2. A Zigzag Transformer Based 24-Pulse AC-DC Converter

The phase shift required for proper harmonic elimination in multipulse AC-DC converters is given by [4]:

$$\text{Phase shift} = 60^\circ / (\text{Number of six-pulse converters}) \quad (1)$$

For achieving 12-pulse AC-DC conversion, the phase shift between the two sets of voltages may be either of 0° and 30° or $\pm 15^\circ$ with respect to the supply voltages. Fig. 2 shows the schematic diagram of a zigzag autotransformer based 12-pulse AC-DC converter [8] with a phase shift of $+15^\circ$ and -15° , referred to as Topology 'B'. Similarly, Fig. 3 shows the transformer winding arrangement and its connection to supply and diode bridge converters DB1 and DB2 along with the phasor diagram. Fig. 4 shows the schematic diagram of a zigzag autotransformer based 24-pulse AC-DC converter with a diode tapped interphase reactor (IPR), referred to as Topology 'C'.

2.1 Design of the Autotransformer for a 12-Pulse AC-DC Converter without ZSBT for Pulse Doubling

In the proposed converter system, three-phase supply voltages are fed to the autotransformer windings connected in zigzag fashion, as shown in Fig. 3(a). It uses four auxiliary windings per phase. From these voltages, two sets of 3-phase voltages (phase shifted through $+15^\circ$ and -15°) are produced. The number of turns required for $+15^\circ$ and -15° phase shift is calculated as follows. Fig. 3(a) shows the schematic diagram of the zigzag-connected autotransformer for a 12-pulse AC-DC converter. The number of turns for its windings is determined as a function of the input phase voltage, V_A using the following relations.

Consider the following set of three-phase supply voltage applied to the input of autotransformer as:

$$\begin{aligned} V_A &= V_S \angle 0^\circ, V_B = V_S \angle -120^\circ, V_C = V_S \angle 120^\circ \\ V_{AB} &= \sqrt{3} V_S \angle 30^\circ, V_{BC} = \sqrt{3} V_S \angle -90^\circ, V_{CA} = \sqrt{3} V_S \angle 150^\circ \end{aligned} \quad (2)$$

Moreover, from Fig. 3(b), the output voltages of this configuration are expressed as:

$$V_{A1} = K_1(V_{AB} - V_{CA}) - K_2 V_{BC} \quad (3)$$

$$V_{A2} = K_1(V_{AB} - V_{CA}) + K_2 V_{BC} \quad (4)$$

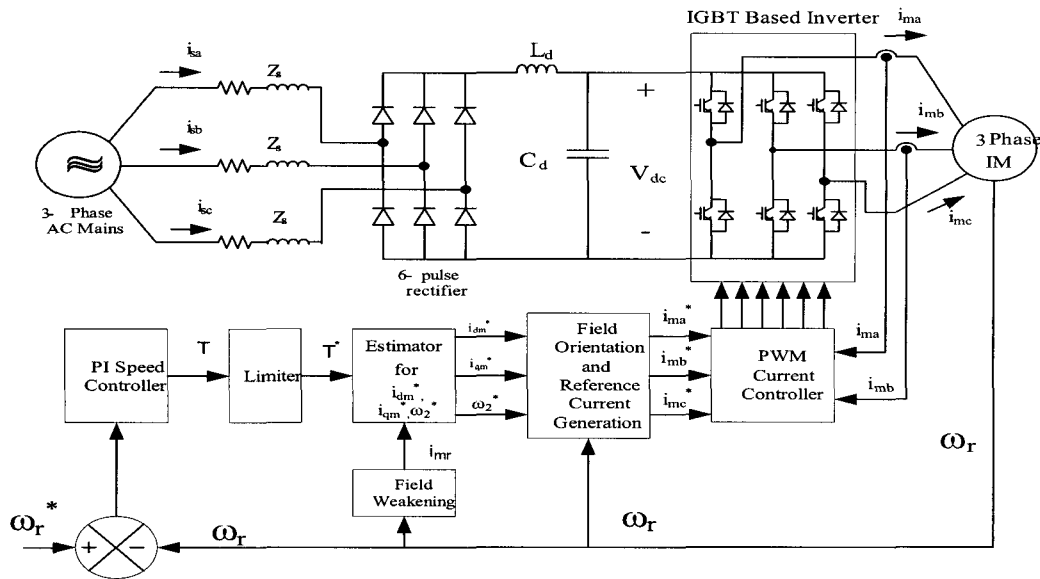


Fig.1 Vector controlled induction motor fed from six-pulse AC-DC converter (Topology A)

The values of these constants K_1 and K_2 determine the winding turns as a fraction of input phase voltage for the 12-pulse AC-DC converter. The values calculated are:

$$K_1=0.5773, \quad K_2=0.2679 \quad (5)$$

The rating of a transformer is dependent on the voltage across each winding and the current through the windings. The winding voltages determine the core size while the currents determine the conductor size and, hence, the two determine its VA rating. The VA rating of these transformers is calculated as:

$$VA \text{ rating} = 0.5 \sum (V_k I_k) \quad (6)$$

where, V_k and I_k are the root-mean-square (r.m.s.) value of the voltage across and the current through the k^{th} winding. The same relation is used for estimating the transformer rating of AC-DC converters.

The total magnetic rating is the sum of the three-autotransformer, the ZSBT and the IPR. In polygon connection for 24-pulse rectifier system the ZSBT rating is 7.5% of the load rating, while the ZSBT is not needed for the proposed configuration, thus, reducing the overall magnetic ratings. In a zigzag connection the 'zig' and 'zag' windings of equal length are connected in series and have 30° phase displaced voltages in such a way that the

inter-bridge currents which have triplen frequency are inherently blocked. The voltages induced in the windings due to the zigzag connection are rich in triplen harmonics (triplen voltage harmonics are not present in the supply line) which prevents triplen current harmonics in the autotransformer winding and rectifier bridges, DB1 and DB2.

2.2 A DC Ripple Re-Injection Technique for 12-Pulse AC-DC Converters

This technique is used for increasing the number of pulses resulting in harmonic reduction on both the AC as well as the DC side. Fig. 4 shows the proposed autotransformer based 24-pulse AC-DC converter fed VCIMD, referred to as Topology 'C'. It inherently exhibits high impedance to zero sequence currents, resulting in 120° conduction for each diode of the bridges,

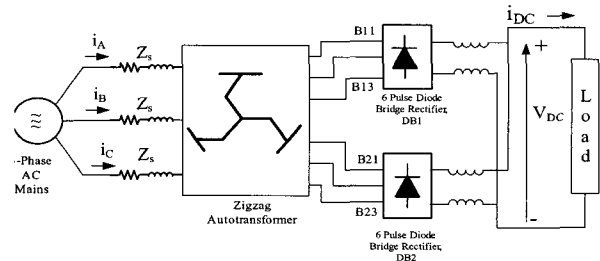


Fig. 2 Twelve-pulse AC-DC converter for VCIMD load using zigzag transformer (Topology B)

and also results in equal current sharing in the output. An interphase reactor tapped suitably to achieve pulse doubling has been connected (Fig. 5) at the output of the DB1 and DB2. Zero sequence currents are blocked by the main autotransformer itself and so a zero sequence blocking transformer (ZSBT) is not required. The two bridge output voltages v_{dc1} and v_{dc2} , shown in Figure 4, are identical except for a phase shift of 30° (required for achieving 12-pulse operation), and these voltages contain a ripple of six times the source frequency.

2.3 Design of the Interphase Transformer (IPT) for Pulse Doubling

A required condition to achieve the pulse doubling is to ensure that the instantaneous output voltages of the two converters, DB1 and DB2, are the same and displaced by

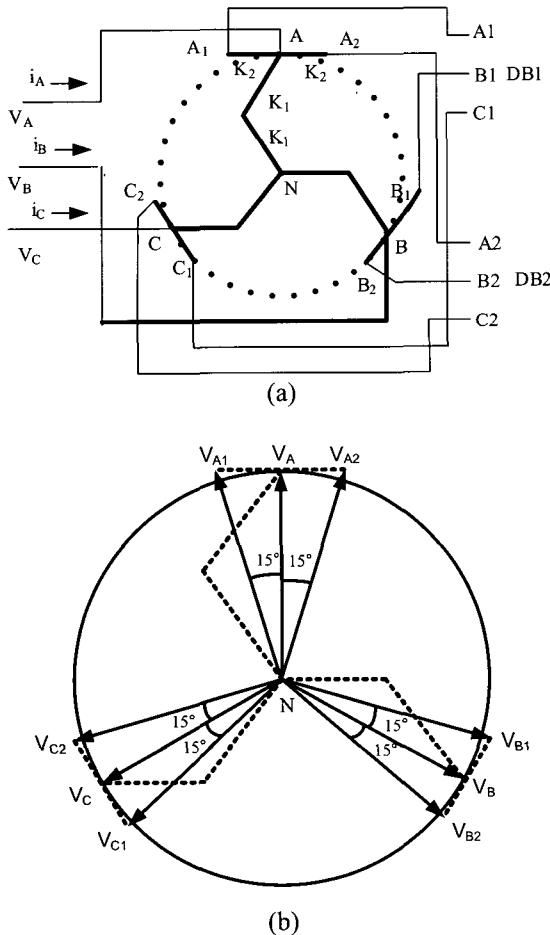


Fig. 3 (a) Proposed autotransformer winding connection diagram. (b) Phasor diagram of voltages for 12-pulse based AC-DC converter

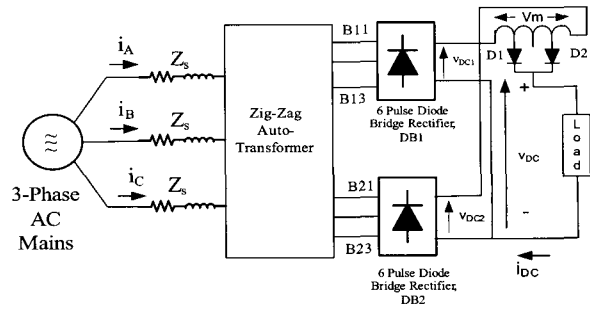


Fig. 4 Twenty-four pulse AC-DC converter for VCIMD load using zigzag transformer (Topology C) without ZSBT

an angle of 30° . It is already known that an IPT or inter-phase reactor (IPR) with suitable diode taps [9-11] can effectively double the pulses in 12-pulse converters where the two converters are fed from 30° phase -shifted voltages. The same concept is used here for achieving the pulse multiplication for line current harmonic reduction. The inter-phase reactor and tapped diodes are arranged in the fashion shown in Fig. 5. The voltage appearing across the inter-phase transformer, V_m , is an AC voltage ripple of six times the source frequency, resulting in smaller size, weight and volume of the interphase transformer. Depending upon the polarity of the impressed voltage V_m across the inter-phase transformer, diodes D_1 and D_2 conduct and result in pulse multiplication. The MMF relationship of the IPR when diode D_1 is conducting is

$$i_{dc1} \cdot N_A = i_{dc2} \cdot N_B \tag{7}$$

where N_A and N_B are the number of turns as shown for the IPR.

Moreover,

$$i_{dc1} + i_{dc2} = i_{dc} \tag{8}$$

From eqn. (7) and eqn. (8), output currents of the two diode converters I and II are given as:

$$i_{dc1} = (0.5 - K_T) i_{dc} \tag{9}$$

$$i_{dc2} = (0.5 + K_T) i_{dc} \tag{10}$$

where $K_T = (N_B - 0.5N_0) / N_0$ and N_0 is the total turns ($=N_A + N_B$).

Similarly, the MMF relationships can be written for the case when diode D_2 is conducting.

$$i_{dc1} = (0.5 - K_T) i_{dc} \tag{11}$$

$$i_{dc2} = (0.5 + K_T) i_{dc} \tag{12}$$

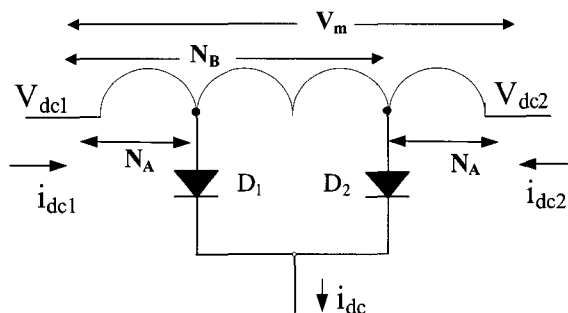


Fig. 5 Tapped interphase reactor circuit

Therefore, depending on the polarity of voltage V_m , the magnitudes of the converter output currents are modulated and this changes the shape of the rectifier input currents and, thereby, doubles the pulses. The turn ratio of the interphase transformer for suppressing the 19th and 21st harmonics is given as: $K_T = 0.2457$. The design parameters of the IPR are given in the Appendix.

3. MATLAB Based Simulation

The proposed AC-DC converter feeding VCIMD is simulated in the MATLAB environment, along with the Simulink and Power System Blockset (PSB) toolboxes. Fig. 6a shows the MATLAB model of the proposed AC-DC converter along with VCIMD to improve various power quality indices. The main autotransformer model is shown in Fig. 6b. The VCIMD consists of a 10 hp, 415 V induction motor drive (detailed data are given in the Appendix) controlled using the indirect vector control technique^[1]. The source impedance has been kept at a practical value of 3% in all the simulations. The load is varied on the VCIMD to study its effect on various power quality indices. The results are tabulated in Table I and II and waveforms are shown in Figs. 7-11.

4. Experimentation

To validate the design procedure and simulation model,

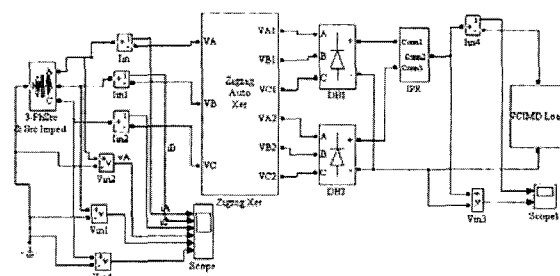


Fig. 6(a) MATLAB model of 24-pulse AC-DC converter with VCIMD load

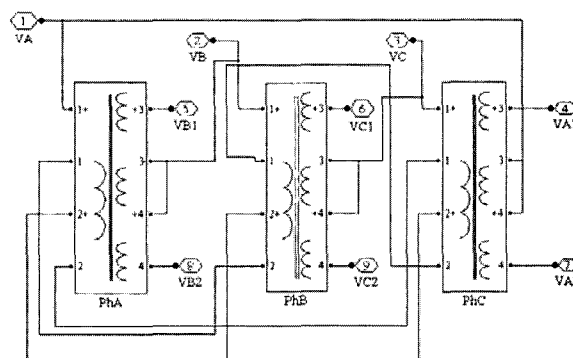


Fig. 6(b) MATLAB model of auto-transformer for the 12-pulse and 24-pulse AC-DC converter simulation

a prototype of the proposed autotransformer based 24-pulse AC-DC converter, as shown in Fig. 3, has been developed in the laboratory. Three single-phase multiwinding transformers have been designed and wound in the laboratory to develop the proposed autotransformer. The design details of the same are given in the Appendix. Based on the voltage across different windings, the number of turns are calculated and based on the current flowing through different windings, the gauge of the wire is calculated. Similarly, an interphase transformer of small rating has been designed and developed (details are given in the Appendix). The experiments have been carried out at a three-phase line voltage of 415 V AC input with an equivalent resistive load.

These test results are given in Table III. The experimental results have been recorded using a Fluke power analyzer model 43B on the developed prototype of the 24-pulse AC-DC converter and shown in Figs. 12-13.

5. Results and Discussion

The proposed 24-pulse AC-DC converter has been modeled and designed along with the VCIMD. The dynamic performance of the drive, along with the load perturbation on the VCIMD fed by a 6-pulse diode bridge rectifier, is shown in Fig. 7. The set of curves consists of the supply voltage v_A , the supply current i_A , the three-phase motor currents i_{abc} , the rotor speed ' ω_r ' (in electrical rad/sec), the motor developed torque ' T_e ' (in N-m), and the DC link voltage v_{dc} (V).

Fig. 8 shows the supply current waveform along with its harmonic spectrum at full load, showing THD of the AC main current as 31.29%, which deteriorates to 74.68% at light load (20%). Moreover, the power factor at full load is 0.927, which deteriorates to 0.893 as the load is reduced to 20%. Thus, there is sufficient scope as well as need for improving the power quality at the main AC using an AC-DC converter, which can easily replace the existing 6-pulse converter.

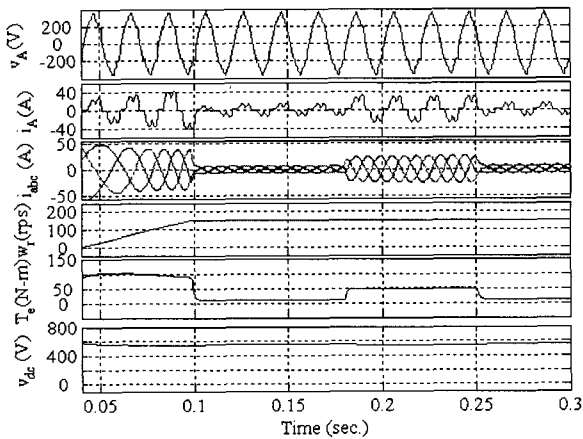


Fig. 7 Dynamic response of 6-pulse AC-DC converter fed VCIMD

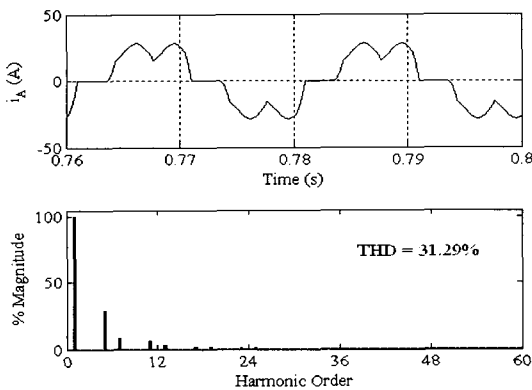


Fig. 8 Input current waveform and harmonic spectrum of 6-pulse AC-DC converter at full-load

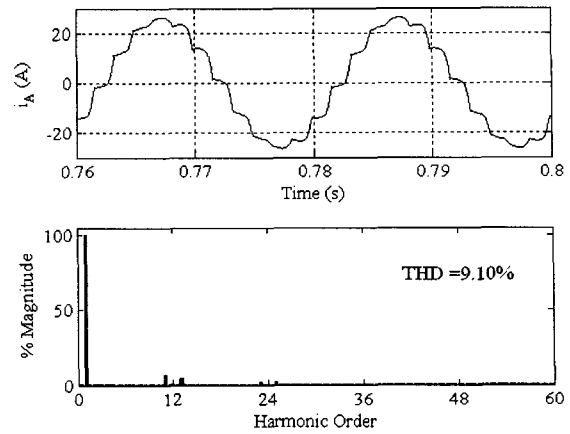


Fig. 9 Input current waveform and harmonic spectrum of 12-pulse AC-DC converter at full-load

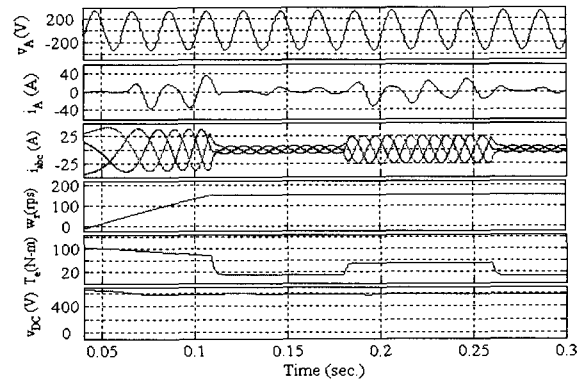


Fig. 10 Dynamic response of 24-pulse AC-DC converter fed VCIMD

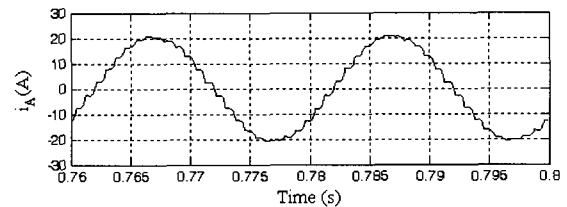


Fig. 11 Input current waveform and harmonic spectrum of 24-pulse AC-DC converter at full-load

5.1 Performance of 12-Pulse and 24-pulse AC-DC Converters

To improve the power quality indices, a 12-pulse

AC-DC converter [8] fed VCIMD has been modeled and simulated. The THD of supply current at full load in Topology 'B' is observed to be 9.10% as shown in Fig. 9, whereas the power factor under this condition is 0.985, as shown in Table I, thus not complying with IEEE Standard 519 [3]. However, the rating of the magnetics is 26.73%, as shown in Table IV.

Fig. 10 shows the dynamic performance of the proposed 24-pulse AC-DC converter (Topology 'C') at starting and load perturbation. The supply current waveform at full load, along with its harmonic spectrum, is shown in Fig. 11 for Topology 'C', which shows that the THD of AC mains current is 5.01%, and the power factor obtained is 0.996. Under light load conditions, the THD of the AC main current is 6.384% and the power factor is 0.996. Table I shows the effect of load variation on the VCIMD to study various power quality indices. It shows that the proposed AC-DC converter is able to perform satisfactorily under load variation on VCIMD with almost unity power factor in the wide operating range of the drive, and THD of supply current always less than 8%. This is within the IEEE Standard 519 [3] limits. A comparison of different power quality indices of a VCIMD fed from different AC-DC converters is shown in Table II, which shows the improvement in these indices with the proposed AC-DC converter.

5.2 Experimental Validation

For experimental validation of the design and simulation model, a laboratory prototype of the proposed 24-pulse AC-DC converter is developed with details given in the previous section and the Appendix. Various tests are conducted on this developed converter, and test results are given in Table III and Figs. 12-13. Figs.12-13 show the tests results of the AC main current waveforms along with its harmonic spectrum at full load. These test and simulated results show a close agreement among them. The experimental results show the THD of the AC main current at full load as 5.2%, displacement factor of 1.00 and power factor near unity, thus confirming the 24-pulse converter behavior of the proposed AC-DC converter. In comparison to the magnetics rating of differential delta [9], polygon [10] and fork [11] connection based AC-DC converters (total magnetic ratings given as 32.15%, 28.8%

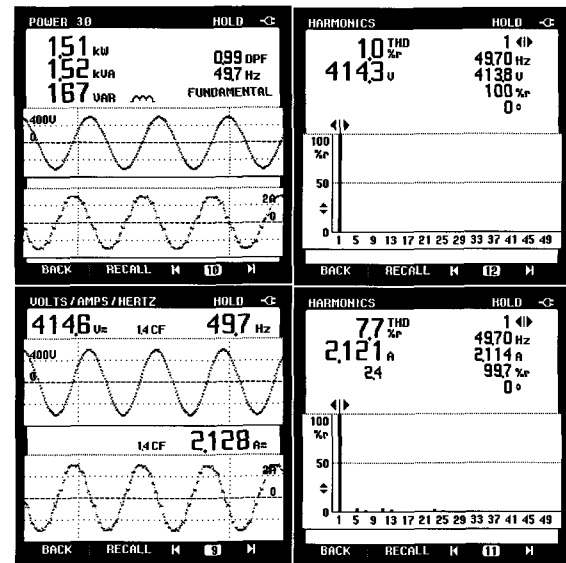


Fig. 12 Test result for 24-pulse non-isolated zigzag transformer based AC-DC converter at 1.51kW load.

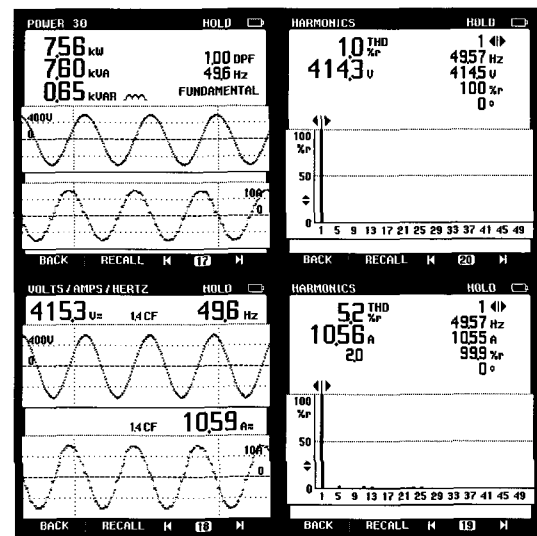


Fig. 13 Test result for 24-pulse non-isolated zigzag transformer based AC-DC converter at 7.56kW load

and 32.84% respectively) the proposed AC-DC converter rating (28.16%) is less and a ZSBT is also not required.

6. Conclusions

The design and model of an autotransformer based 24-pulse AC-DC converter with a VCIMD load, which needs only two additional diodes along with a suitably tapped inductor, has been developed and validated by employing pulse multiplication through a DC ripple

Table 1 Comparison of power quality parameters of 12-pulse and 24-pulse AC-DC converters

Sr. No.	Topology	Load	THD of V_{ac} (%)	AC Mains Current I_{ac} (A)	THD of I_{ac} (%)	Distortion Factor, DF	Displacement Factor, DPF	Power Factor, PF	DC Voltage (V)	Load Current I_{dc} (A)	Ripple Factor, RF (%)
1	B	20%	2.932	8.343	9.743	0.9948	0.9891	0.9840	574.1	10.28	0.0006
		40%	3.487	10.06	9.604	0.9948	0.9898	0.9847	572.7	13.21	0.0008
		60%	4.068	13.12	9.459	0.9947	0.9901	0.9849	571.2	16.48	0.0009
		80%	4.664	15.74	9.291	0.9947	0.9901	0.9848	569.1	19.85	0.0018
		100%	5.269	18.54	9.104	0.9944	0.9901	0.9846	568.0	23.24	0.003
2	C	20%	1.667	8.318	6.384	0.9972	0.9991	0.9963	587.2	10.52	0.0001
		40%	2.221	9.60	5.778	0.9980	0.9988	0.9969	586.7	13.08	0.0003
		60%	2.776	12.81	5.345	0.9981	0.9986	0.9968	586.1	16.36	0.0005
		80%	3.259	15.44	5.345	0.9981	0.9983	0.9965	585.5	19.79	0.0006
		100%	3.693	18.42	5.017	0.9983	0.9981	0.9964	584.8	23.05	0.0006

Table 2 Comparison of power quality parameters of different AC-DC converters

Sr. No.	Topology	% THD of V_{ac}	AC Mains Current I_{ac} (A)		% THD of I_{ac} at		Distortion Factor DF		Displacement Factor DPF		Power Factor PF		DC Voltage (V)	
			Light Load	Full Load	Light Load	Full Load	Light Load	Full Load	Light Load	Full Load	Light Load	Full Load	Light Load	Full Load
1	A	10.58	8.701	19.12	74.68	31.24	0.9110	0.9491	0.9798	0.9768	0.8926	0.9271	552.9	542.8
2	B	5.224	8.343	18.54	9.743	9.104	0.9948	0.9944	0.9891	0.9901	0.9840	0.9846	574.1	568.0
3	C	3.693	4.202	8.318	18.42	4.676	0.9972	0.9983	0.9991	0.9981	0.9963	0.9964	587.2	584.8

re-injection technique in existing 12-pulse AC-DC converters. The proposed converter doesn't use ZSBT, thereby decreasing the component count and, thus, increasing the reliability of the converter. The proposed 24-pulse AC-DC converter has resulted in a reduction in the rating of the magnetics, leading to a savings in the overall cost of the drive. The required level of THD and the crest factor of the AC main current has been achieved at almost close to unity power factor in the wide operating range of the drive.

Appendix

1. Motor and controller specifications:

Three-phase squirrel cage induction motor –10 hp (7.5 kW), 3-phase, 4-pole, Y-connected, 415 V, 50 Hz, $R_s = 1.0$ ohms, $R_r = 0.76$ ohms, $X_{ls} = 0.77$ ohms, $X_{lr} = 0.77$ ohms, $X_m = 18.84$ ohms, $J = 0.1$ kg-m².

PI controller: $K_p = 25$, $K_i = 4.5$

DC link parameters: $L_d = 0.002$ H, $C_d = 3200$ μ F.

2. Design of the Interphase Transformer:

The flux density is taken as 0.8 Tesla and the current density is considered as 2.3 A/mm². The interphase transformer is wound using a core of size No. 3 with E-I laminations of size (76 mm \times 127 mm) and (127 mm \times 19 mm), respectively. Based on the voltage across different windings, the number of turns are calculated and based on the current flowing through different windings, the gauge of wire is calculated and these are:

Winding	Number of turns	Gauge of wire (SWG)
N_A	35	11
N_B-N_A	67	18

Table 3 Comparison of power quality parameters of hardware implementation obtained for 12-pulse and 24-pulse AC-DC converters

Sr. No.	Topology	Input Power (kW)	THD of Vs (%)	AC Mains Current I _s (A)	THD of I _s (%)	Crest Factor, CF	Displacement Factor, DPF	Power Factor, PF	DC Voltage (V)	Load Current I _{dc} (A)	Load (kW)
1	B	1.51	1.0	2.12	13.3	1.4	1.00	0.99	573.0	2.60	1.48
		2.17	1.1	3.04	12.1	1.4	1.00	0.99	570.9	3.77	2.15
		3.10	1.1	4.36	11.8	1.4	1.00	0.99	570.0	5.34	3.04
		4.05	1.1	5.68	11.5	1.4	1.00	0.99	567.4	6.93	3.93
		5.18	1.1	7.27	11.2	1.4	1.00	0.99	566.2	8.84	5.01
		5.98	1.1	8.41	11.1	1.4	1.00	0.99	564.3	10.01	5.65
		6.88	1.1	9.63	11.0	1.4	1.00	0.99	562.4	11.46	6.44
		7.64	1.1	10.83	10.7	1.4	1.00	0.99	561.2	12.24	6.87
2	C	1.51	1.0	2.138	7.7	1.4	0.99	0.99	586.6	2.537	1.49
		2.08	0.8	2.939	7.7	1.4	1.00	0.99	585.1	3.505	2.04
		3.11	0.9	4.358	6.0	1.4	1.00	0.99	580.5	5.292	3.07
		4.18	0.9	5.93	5.6	1.4	1.00	0.99	579.0	7.190	4.13
		5.02	0.9	7.05	5.3	1.4	1.00	0.99	577.5	8.57	4.95
		5.96	0.9	8.33	5.3	1.4	1.00	0.99	575.5	10.14	5.83
		7.03	0.9	9.88	5.3	1.4	1.00	0.99	572.3	12.02	6.88
		7.64	0.9	10.59	5.2	1.4	1.00	0.99	570.5	12.89	7.36

Table 3 Comparison of active-magnetic power ratings in different AC-DC converters

Sr. No.	Topology	Main Transformer rating (% of load)	Interphase transformer rating (% of load)	Total magnetic rating (% of load)
1	12-pulse	19.31	7.42	26.73
2	24-pulse	26.44	1.72	28.16

3. Design of the Autotransformer:

Autotransformer rating 2.684 kVA (26.44%)

Interphase transformer 0.175 kVA. (1.72%)

Transformer design details:

Flux Density: 0.8Tesla, Current Density: 2.3A/mm²,

Core size: No. 7B

E-Laminations: Length=190mm, Width=124mm

I-Laminations: Length=190mm, Width= 32mm

Area of cross-section of core=48.76cm² (6.40 cm X 7.62cm)

Autotransformer winding details-

Winding Voltage	Number of turns	Gauge of wire (SWG)
K ₁ *V _A	241	22
K ₂ *V _A	108	18

4. Derivations for kVA ratings of transformers:

The autotransformer employed for the 24-pulse AC-DC converter is redrawn in Fig, A1 and various currents are marked for estimating its power rating. The MMF balance equations for the A, B and C phases of the autotransformer are:

$$K_1(i_1 - i_3) + K_2(i_{B2} - i_{B1}) = 0 \tag{A1}$$

$$K_1(i_2 - i_1) + K_2(i_{C2} - i_{C1}) = 0 \tag{A2}$$

$$K_1(i_3 - i_2) + K_2(i_{A2} - i_{A1}) = 0 \tag{A3}$$

$$\text{Also } i_1 + i_2 + i_3 = 0 \tag{A4}$$

The equations A1-A4 give the value of currents i₁ to i₃ as:

$$i_1 = \frac{K_2}{3K_1}(i_{B1} - i_{B2} + i_{C2} - i_{C1}) \tag{A5}$$

$$i_2 = \frac{K_2}{3K_1}(i_{C1} - i_{C2} + i_{A2} - i_{A1}) \tag{A6}$$

$$i_3 = \frac{K_2}{3K_1}(i_{A1} - i_{A2} + i_{B2} - i_{B1}) \tag{A7}$$

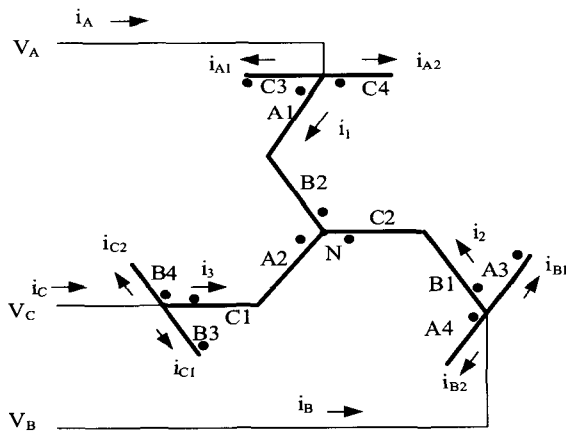


Fig. A1 The zigzag autotransformer winding arrangement and the currents for MMF balance

Moreover,

$$i_A = i_{A1} + i_{A2} + i_1 \tag{A8}$$

$$i_B = i_{B1} + i_{B2} + i_2 \tag{A9}$$

$$i_C = i_{C1} + i_{C2} + i_3 \tag{A10}$$

The input current i_A , i_B and i_C are similar and phase displaced by 120° from one another. The waveform obtained theoretically for i_A from eqn. (A8) is shown in Fig. A2.

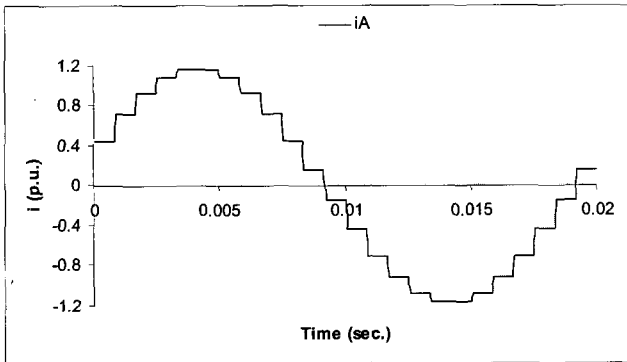


Fig. A2 Input current waveform of 24-pulse AC-DC Converter

The currents I_{A1} , I_{A2} , I_{B1} , I_{B2} , I_{C1} , I_{C2} are input currents to the diode bridge rectifiers which have similar shape as shown but phase shifted. The waveform of currents i_{A1} and i_{B1} are shown in Fig. A3. The rms value of current i_{A1} is:

$$I_{A1} = \sqrt{\frac{0.25^2 + 0.75^2}{2}} I_d = 0.4564 I_d \tag{A11}$$

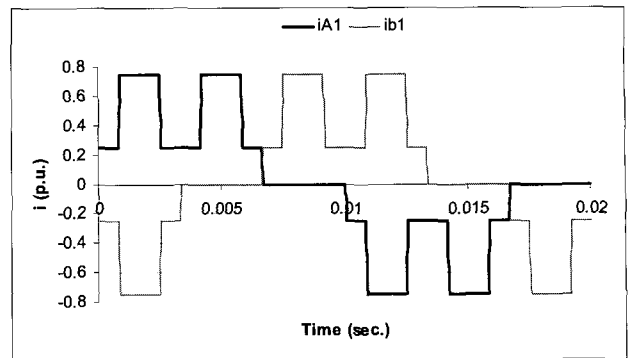


Fig. A3 Waveform of bridge input currents

Similarly the value of rms current for i_1 , i_2 and i_3 can be obtained from their waveform that is described by eqns. (A5-A7). The waveform for current i_1 is shown in Fig. A4 that gives rms value of i_1 as:

$$I_1 = \sqrt{\frac{0.07755^2 X 4 + 0.1547^2 X 4 + 0.03868^2 X 2 + 0.1934^2 X 2}{12}} I_d = 0.1283 I_d$$

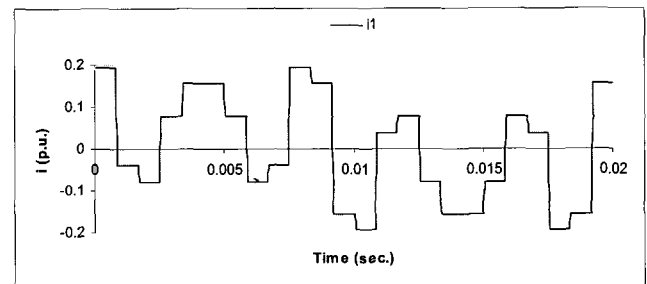


Fig. A4 Waveform of current through zigzag winding of autotransformer

Therefore, the kVA rating of autotransformer is =

$$\left[\frac{6 \times 0.4564 \times 0.2679 + 6 \times 0.1283 \times 0.5337}{2} \right] \frac{I_{dc} V_{dc}}{1.398 \sqrt{3}} = 0.2432 I_{dc} V_{dc} = 24.32\% \text{ of load rating}$$

The IPR Rating can be expressed mathematically as:

$$VA(IPR) = (V_1 \times I_{dc1} + V_2 \times I_{dc2} + V_3 \times I_{dc3})/2 \tag{A12}$$

Equations 9-12 give waveforms of i_{dc1} , i_{dc2} and i_{dc3} and their rms values are given [9] as follows.

$$I_{dc1} = I_{dc2} = 0.567 I_{dc}$$

$$I_{dc3} = 0.232 I_{dc}$$

The rms voltage across IPR is due to the conduction of the positive and negative group of diodes in two diode bridges. The difference in the voltage at positive terminals of the two bridges is triangular with frequency twelve times the supply frequency. Its rms value is determined as:

$$V_m = \sqrt{\frac{1}{T} \int_0^T v_m^2 dt}$$

$$= \sqrt{\frac{12}{\pi} \int_0^{\pi/12} \left(\left(\frac{\sqrt{2}}{1+\sqrt{3}} \right) \sqrt{2} V_{LL}^2 \sin \omega t \right) d(\omega t)} \quad (A13)$$

$$= 0.0814 V_{dc}$$

The voltages across the IPR sections can be written^[9] in terms of the voltage V_m as

$$\frac{V_1}{V_m} = \frac{V_2}{V_m} = \frac{N_A}{N_A + N_B} = 0.2543 \quad (A14)$$

$$\frac{V_3}{V_m} = \frac{N_B - N_A}{N_A + N_B} = 0.4914 \quad (A15)$$

Therefore, the rms values of V_1 , V_2 , V_3 are given as

$$V_1 = V_2 = 0.2543 V_m = 0.021 V_{dc}$$

$$V_3 = 0.4914 V_m = 0.04 V_{dc}$$

VA rating of IPR is:

$$= (0.021 \times 0.567 + 0.021 \times 0.567 + 0.04 \times 0.232) V_{dc} I_{dc} / 2$$

$$= 0.0165 P_{dc}$$

$$= 1.65\% \text{ of load rating}$$

Therefore, theoretically the total magnetic rating of the proposed configuration is:

$$0.2432 P_{dc} + 0.0165 P_{dc} = 25.97\% \text{ of load power.}$$

References

- [1] P. Vas, *Sensorless Vector and Direct Torque Control*, Oxford University Press, 1998.
- [2] Limits for Harmonic Current Emissions, International Electrotechnical Commission Standard 61000-3-2, 2004.
- [3] IEEE Guide for Harmonic Control and Reactive Compensation of Static Power Converters, IEEE Standard 519-1992.
- [4] D. A. Paice, *Power Electronic Converter Harmonics: Multipulse Methods for Clean Power*, New York: IEEE Press, 1996.
- [5] Bin Wu, *High-Power Converters and AC Drives*, IEEE Press, Wiley-Interscience, 2006.
- [6] S. Choi, P. N. Enjeti, and I. J. Pitel, "Polyphase transformer arrangements with reduced kVA capacities for harmonic current reduction in rectifier type utility interface", *IEEE Trans. on Power Electronics*, Vol. 11, No. 5, pp. 680–689, September 1996.
- [7] P. W. Hammond, "Autotransformer", U.S. Patent No. 5619407, April 8, 1997.
- [8] D. W. Owens, "Autotransformer for Use with Multiple Phase Rectifiers", US Patent 7049921, May 23, 2006.
- [9] S. Choi, B. S. Lee, and P. N. Enjeti, "New 24-pulse diode rectifier systems for utility interface of high power AC motor drives", *IEEE Trans. on Industry Applications*, Vol. 33, No. 2, pp. 531–541, March/April 1997.
- [10] B. Singh, G. Bhuvaneswari, V. Garg, "Power-quality improvements in vector-controlled induction motor drive employing pulse multiplication in AC-DC converters", *IEEE Trans. on Power Delivery*, vol. 21 no. 3, pp. 1578 – 1586, July 2006,
- [11] B. Singh, G. Bhuvaneswari, V. Garg, "24-Pulse AC-DC Converter for Power Quality Improvement in Vector Controlled Induction Motor Drives", *Electric Power Systems and Components*, vol. 34, no. 10, pp. 1077 - 1098, October 2006.



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