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ON INTUITIONISTIC FUZZY w-PRIMARY SUBMODULES

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Abstract. In this paper, we introduce the notions of intuitionistic fuzzy w-primary submodule and intuitionistic fuzzy w-primary ideal. And we give an example of intuitionistic fuzzy w-primary submodule and some related properties are investigated.

1. Introduction

In [11], Zadeh introduced the concept of fuzzy set, and several researchers were conducted on the generalizations of the notion of fuzzy sets. The idea of intuitionistic fuzzy set was first published by Atanassov [1, 2, 3] as a generalization of the notion of fuzzy sets. In [4], Banerjee and Basnet applied the concept of intuitionistic fuzzy sets to the theory of rings, and introduced the notions of intuitionistic fuzzy subrings and intuitionistic fuzzy ideals of a ring. Hur et al. [5] introduced the notions of intuitionistic fuzzy (completely) prime ideals and intuitionistic fuzzy weak completely prime ideals in a ring. Present author et al [8, 9] introduced the notions of intuitionistic product and intuitionistic radical of intuitionistic fuzzy ideals. In this paper, we discuss the generalization of the concepts in the paper [10], we will use the notion of intuitionistic fuzzy sets. We introduce the concept of intuitionistic fuzzy w-primary

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and intuitionistic fuzzy w-primary ideal. And we give a example of intuitionistic fuzzy w-primary submodule and some related properties are investigated. The results of this paper will contribute to the study of intuitionistic fuzzy radicals, intuitionistic fuzzy primary submoudule and intuitionistic fuzzy primary decompostion in modules.

2. Preliminaries

Throughout this paper, R is a commutative ring with unity and M is a unitary R-module. Let S be a proper submodule of M. A submodule S of an R-module M is said to be a *primary submodule* if the condition $rx \in S$, $r \in R$ and $x \notin S$ implies that there exists a positive integer n such that $r^n m \in S$ for all $m \in M$.

A mapping $\mu : X \to [0,1]$, where X is an arbitrary non-empty set, is called a *fuzzy set* in X[11]. As an important generalization of the notion of fuzzy sets in X, Atanassov [1, 2] introduced the concept of an *intuitionistic fuzzy set*(IFS for short) defined on a non-empty set X as objects having the form

$$A = \{\langle x, \mu_A(x), \gamma_A(x)\rangle \mid x \in X\},$$

where the functions $\mu_A : X \to [0,1]$ and $\gamma_A : X \to [0,1]$ denote the *degree of membership* (namely $\mu_A(x)$) and the *degree of nonmembership* (namely $\gamma_A(x)$) of each element $x \in X$ to A respectively, and $0 \leq \mu_A(x) + \gamma_A(x) \leq 1$ for all $x \in X$. Obviously, every fuzzy set A' corresponds to the following intuitionistic fuzzy set:

$$A' = \{\langle x, \mu_{A'}(x), 1 - \mu_{A'}(x)\rangle \mid x \in X\}.$$

For the sake of simplicity, we shall use the symbol $A = (\mu_A, \gamma_A)$ for the intuitionistic fuzzy set $A = \{\langle x, \mu_A(x), \gamma_A(x)\rangle \mid x \in X\}$. Obviously, for an IFS $A = (\mu_A, \gamma_A)$ in M, when

$$\gamma_A(x) = 1 - \mu_A(x), \quad \text{that is,} \quad \mu_A(x) + \gamma_A(x) = 1$$

for every $x \in X$, the IFS A is a fuzzy set. Hence the notion of intuitionistic fuzzy set theory is a generalization of fuzzy set theory.

For a subset A of set X and $s, t \in [0,1]$ with $s + t \leq 1$,we denote $\chi_{A^{(s,t)}} = \{\langle x, \mu_{\chi_{A^s}}, \gamma_{\chi_{A^t}} \rangle \mid x \in X\}$ defined by

$$\mu_{\chi_{A^s}}(x) := \begin{cases} s & \text{if } x \in A \\ 0 & \text{otherwise} \end{cases} \text{ and } \gamma_{\chi_{A^t}}(x) := \begin{cases} t & \text{if } x \in A \\ 1 & \text{otherwise} \end{cases}$$

for all $x \in X$. For the sake of simplicity, we shall use the symbol $\chi_{A^{(s,t)}} = (\mu_{\chi_{A^s}}, \gamma_{\chi_{A^t}})$ for the IFS $\chi_{A^{(s,t)}} = \{\langle x, \mu_{\chi_{A^s}}, \gamma_{\chi_{A^t}} \rangle \mid x \in X\}$.

We review some definitions for the later section.

Definition 2.1. [1, 2] Let X be a nonempty set and let $A = (\mu_A, \gamma_A)$ and $B = (\mu_B, \gamma_B)$ be IFS in X. Then

(a1) $A \subseteq B \Leftrightarrow (\forall x \in M)\,(\mu_A(x) \leq \mu_B(x),\ \gamma_A(x) \geq \gamma_B(x))$.

(a2) $A = B \Leftrightarrow A \subseteq B$ and $B \subseteq A$.

(a3) $A \cap B = (\mu_A \wedge \mu_B, \gamma_A \vee \gamma_B)$.

(a4) $A \cup B = (\mu_A \vee \mu_B, \gamma_A \wedge \gamma_B)$.

Definition 2.2. [4, 7, 3] An IFS set $A = (\mu_A, \gamma_A)$ in a ring R is called an *intuitionistic fuzzy ideal* of R if it satisfies the following conditions:

(b1) $(\forall x, y \in R)$ $(\mu_A(x - y) \geq \mu_A(x) \wedge \mu_A(y))$.

(b2) $(\forall a, x \in R)$ $(\mu_A(ax) \geq \mu_A(x))$.

(b3) $(\forall x, y \in R)$ $(\gamma_A(x - y) \leq \gamma_A(x) \vee \gamma_A(y))$.

(b4) $(\forall a, x \in R)$ $(\gamma_A(ax) \leq \gamma_A(x))$.

Definition 2.3. [6] Let M be a module over a ring R. An IFS set $A = (\mu_A, \gamma_A)$ in M is called an *intuitionistic fuzzy submodule* of M if it satisfies the following conditions:

(c1) $(\forall x, y \in M)$ $(\mu_A(x - y) \geq \mu_A(x) \wedge \mu_A(y))$.

(c2) $(\forall x \in M)$ $(\forall r \in R)$ $(\mu_A(rx) \geq \mu_A(x))$.

(c3) $(\forall x, y \in M)$ $(\gamma_A(x - y) \leq \gamma_A(x) \vee \gamma_A(y))$.

(c4) $(\forall x \in M)$ $(\forall r \in R)$ $(\gamma_A(rx) \leq \gamma_A(x))$.

3. Intuitionistic Fuzzy w-Primary submodules

Definition 3.1. Let M be a module over a ring R and $A = (\mu_A, \gamma_A)$ be an IFS in M. An intuitionistic fuzzy submodule $A = (\mu_A, \gamma_A)$ of R-module M is called an *intuitionistic fuzzy w-primary submodule* of M if the following condition:

(d1) $(\forall r \in R)\ (\forall x \in M)\ ((\mu_A(rx) > \mu_A(x))$
$\Rightarrow (\forall m \in M)\ (\exists n \in \mathbb{N})\ (\mu_A(r^n m) \geq \mu_A(rx)))$.

(d2) $(\forall r \in R)\ (\forall x \in M)\ ((\gamma_A(rx) < \gamma_A(x))$
$\Rightarrow (\forall m \in M)\ (\exists n \in \mathbb{N})\ (\gamma_A(r^n m) \leq \gamma_A(rx)))$.

Example 3.2. Let $M = R = \mathbb{Z}$. Define an IFS $A = (\mu_A, \gamma_A)$ in $\mathbb{Z}$ by

$$\mu_A(x) := \begin{cases} 1 & \text{if} \quad x \in < 5^2 > \\ \frac{1}{3} & \text{if} \quad x \in < 5 > \setminus < 5^2 >, \\ 0 & \text{if} \quad x \in \mathbb{Z} \setminus < 5 >, \end{cases}$$

$$\gamma_A(x) := \begin{cases} 0 & \text{if} \quad x \in < 5^2 >, \\ \frac{2}{3} & \text{if} \quad x \in x \in < 5 > \setminus < 5^2 >, \\ 1 & \text{if} \quad x \in \mathbb{Z} \setminus < 5 > \end{cases}$$

for all $x \in \mathbb{Z}$. Then $A = (\mu_A, \gamma_A)$ is an intuitionistic fuzzy w-primary submodule of Z.

Let $A = (\mu_A, \gamma_A)$ be an IFS in M and let $s, t \in [0, 1]$ be such that $s + t \leq 1$. Then the set

$$\mathcal{M}_A^{(s,t)} := \{x \in M \mid \mu_A(x) \geq s,\ \gamma_A(x) \leq t\}$$

is called an *(s, t)-level subset* of $A = (\mu_A, \gamma_A)$.

Theorem 3.3. *Let $A = (\mu_A, \gamma_A)$ be an IFS in M. Assume that $\mathcal{M}_A^{(s,t)}$ is a primary submodule of M for every $s, t \in [0, 1]$ with $s + t \leq 1$. Then $A = (\mu_A, \gamma_A)$ is an intuitionistic fuzzy w-primary submodule of M.*

Proof. Assume that $\mathcal{M}_A^{(s,t)}$ is a primary submodule of M for every $s, t \in [0, 1]$ with $s + t \leq 1$. Let $x, y \in M$ be such that $A(x) = (\mu_A(x), \gamma_A(x)) = (s_1, t_1)$ and $A(y) = (\mu_A(y), \gamma_A(y)) = (s_2, t_2)$ where

$s_i + t_i \leq 1$ for $i = 1, 2$. Then $x \in \mathcal{M}_A^{(s_1,t_1)}$ and $y \in \mathcal{M}_A^{(s_2,t_2)}$. Obviously $x, y \in \mathcal{M}_A^{(s_1 \wedge s_2, t_1 \vee t_2)}$. Since $\mathcal{M}_A^{(s_1 \wedge s_2, t_1 \vee t_2)}$ is a submodule of M, we have $x - y \in \mathcal{M}_A^{(s_1 \wedge s_2, t_1 \vee t_2)}$. It follows that $\mu_A(x-y) \geq s_1 \wedge s_2 = \mu_A(x) \wedge \mu_A(y)$ and $\gamma_A(x-y) \leq t_1 \vee t_2 = \gamma_A(x) \vee \gamma_A(y)$. For any $x \in M$ and $r \in R$,we let $x \in M$ be such that $A(x) = (\mu_A(x), \gamma_A(x)) = (s_3, t_3)$ where $s_3 + t_3 \leq 1$. Then $x \in \mathcal{M}_A^{(s_3,t_3)}$. Since $\mathcal{M}_A^{(s_3,t_3)}$ is a submodule of M, we have $rx \in \mathcal{M}_A^{(s_3,t_3)}$. It follows that $\mu_A(rx) \geq s_3 = \mu_A(x)$ and $\gamma_A(rx) \leq t_3 = \gamma_A(x)$. This prove that $A = (\mu_A, \gamma_A)$ is an intuitionistic fuzzy submodule of R-module M. On the other hand, assume that $\mu_A(rx) > \mu_A(x)$ and $\gamma_A(rx) < \gamma_A(x)$ for any $x \in M$ and $r \in R$. Let $A(rx) = (\mu_A(rx), \gamma_A(rx)) = (s_4, t_4)$ where $s_4+, t_4 \leq 1$. Then $rx \in \mathcal{M}_A^{(s_4,t_4)}$. This implies that $x \notin \mathcal{M}_A^{(s_4,t_4)}$. Since $\mathcal{M}_A^{(s_4,t_4)}$ is a primary submodule of M, then there exits an $n \in \mathbb{N}$ such that $r^n m \in \mathcal{M}_A^{(s_4,t_4)}$. Thus $\mu_A(r^n m) \geq s_4 = \mu_A(rx)$ and $\gamma_A(r^n m) \leq t_4 = \mu_A(rx)$ for every $m \in M$. Consequently $A = (\mu_A, \gamma_A)$ is an intuitionistic fuzzy w-primary submodule of R-module M. □

For any fuzzy set μ in M and any $\alpha \in [0,1]$ we define two sets

$$\mathcal{U}(\mu; \alpha) = \{x \in M \mid \mu(x) \geq \alpha\} \quad \text{and} \quad \mathcal{L}(\mu; \alpha) = \{x \in M \mid \mu(x) \leq \alpha\},$$

which are called an *upper* and *lower level set* of μ.

Theorem 3.4. *Let M be a module over a ring R and $A = (\mu_A, \gamma_A)$ be a IFS in M. Then $A = (\mu_A, \gamma_A)$ is an intuitionistic fuzzy w-primary submodule of M if and only if $\mathcal{U}(\mu_A; \alpha)$ and $\mathcal{L}(\gamma_A; \alpha)$ are primary submodules of M for all $\alpha \in [0,1]$ whenever they are non-empty.*

Proof. Assume that IFS $A = (\mu_A, \gamma_A)$ is an intuitionistic fuzzy w-primary submodule of M. Let $x, y \in \mathcal{U}(\mu_A; \alpha)$ and $r \in R$. We have $\mu_A(x) \geq \alpha$ and $\mu_A(y) \geq \alpha$. It follows that $\mu_A(x-y) \geq \mu_A(x) \wedge \mu_A(y) \geq \alpha$. Therefore $x - y \in \mathcal{U}(\mu_A; \alpha)$. Also we have $\mu_A(rx) \geq \mu_A(x) \geq \alpha$ and so $rx \in \mathcal{U}(\mu_A; \alpha)$. On the other hand, for $rx \in \mathcal{U}(\mu_A; \alpha)$ and $x \notin \mathcal{U}(\mu_A; \alpha)$, we have $\mu_A(rx) \geq \alpha$ and $\mu_A(x) < \alpha$. It follows that $\mu_A(rx) > \mu_A(x)$.

Since $A = (\mu_A, \gamma_A)$ is an intuitionistic fuzzy w-primary submodule of R-module M, then there exists $n \in \mathbb{N}$ such that $\mu_A(r^n m) \geq \mu_A(rx) \geq \alpha$ for every $m \in M$. This implies that there exists $n \in \mathbb{N}$ such that $r^n m \in \mathcal{U}(\mu_A; \alpha)$ for all $m \in M$. Therefore $\mathcal{U}(\mu_A; \alpha)$ is a primary submodule of M. Similary, we can show that $\mathcal{L}(\gamma_A; \alpha)$ is a primary submodule of M. Therefore, the set $\mathcal{U}(\mu_A; \alpha)$ and $\mathcal{L}(\gamma_A; \alpha)$ are primary submodule of M for every $\alpha \in [0, 1]$. Conversely, assume that $\mathcal{U}(\mu_A; \alpha)$ and $\mathcal{L}(\gamma_A; \alpha)$ are primary submodules of M for all $\alpha \in [0, 1]$. Let $x, y \in M$ such that $\alpha_1 = \mu_A(x) \wedge \mu_A(y)$ and $\beta_1 = \gamma_A(x) \vee \gamma_A(y)$ where $\alpha_1, \beta_1 \in [0, 1]$. Then $x, y \in \mathcal{U}(\mu_A; \alpha_1)$ and $x, y \in \mathcal{L}(\gamma_A; \beta_1)$. So $x - y \in \mathcal{U}(\mu_A; \alpha_1)$ and $x - y \in \mathcal{U}(\mu_A; \beta_1)$. It follows that $\mu_A(x - y) \geq \alpha_1 = \mu_A(x) \wedge \mu_A(y)$ and $\gamma_A(x - y) \leq \beta_1 = \gamma_A(x) \vee \gamma_A(y)$. For any $x \in M$ and $r \in R$, we let $x \in M$ be such that $\mu_A(x) = \alpha_2$ and $\gamma_A(x) = \beta_2$ where $\alpha_2, \beta_2 \in [0, 1]$. Then $x \in \mathcal{U}(\mu_A; \alpha_2)$ and $x \in \mathcal{L}(\gamma_A; \beta_2)$. So $rx \in \mathcal{U}(\mu_A; \alpha_2)$ and $rx \in \mathcal{L}(\gamma_A; \beta_2)$. It follows that $\mu_A(rx) \geq \alpha_2 = \mu_A(x)$ and $\gamma_A(rx) \leq \beta_2 = \gamma_A(x)$. This prove that $A = (\mu_A, \gamma_A)$ is an intuitionistic fuzzy submodule of R-module M. On the other hand, assume that $\mu_A(rx) > \mu_A(x)$ and $\gamma_A(rx) < \gamma_A(x)$ for any $x \in M$ and $r \in R$. Let $\mu_A(rx) = \alpha_3$ and $\gamma_A(rx) = \beta_3$ where $\alpha_3, \beta_3 \in [0, 1]$. Then $rx \in \mathcal{U}(\mu_A; \alpha_3)$, $rx \in \mathcal{L}(\gamma_A; \beta_3)$, $\mu_A(x) > \alpha_3$ and $\gamma_A(x) < \beta_3$. This imply that $x \notin \mathcal{U}(\mu_A; \alpha_3)$ and $x \notin \mathcal{L}(\mu_A; \beta_3)$ for every $\alpha_3, \beta_3 \in [0, 1]$. Since $\mathcal{U}(\mu_A; \alpha)$ and $\mathcal{L}(\gamma_A; \alpha)$ are primary submodule of M for all $\alpha \in [0, 1]$ then there exits an $n \in \mathbb{N}$ such that $r^n m \in \mathcal{U}(\mu_A; \alpha_3)$ and $r^n m \in \mathcal{L}(\gamma_A; \beta_3)$ for all $m \in M$. Thus $\mu_A(r^n m) \geq \alpha_3 = \mu_A(rx)$ and $\gamma_A(r^n m) \leq \alpha_3 = \mu_A(rx)$ for every $m \in M$. Consequently $A = (\mu_A, \gamma_A)$ is an intuitionistic fuzzy w-primary submodule of R-module M. □

Theorem 3.5. *Let S be a nonempty subset of an R-module M. If an IFS $A = (\mu_A, \gamma_A)$ defined by*

$$\mu_A(x) := \begin{cases} 1 & \text{if } x \in S \\ \alpha & \text{otherwise} \end{cases} \quad \textit{and } \gamma_A(x) := \begin{cases} 0 & \text{if } x \in S \\ \beta & \text{otherwise} \end{cases}$$

for all $x \in M$ *where* $0 \leq \alpha < 1$, $0 \leq 0 < \beta$ *and* $\alpha + \beta \leq 1$ *for* $i = 1, 2$, *then* $A = (\mu_A, \gamma_A)$ *is an intuitionistic fuzzy w-primary submodule of M if and only if S is a primary submodule of M.*

Proof. Let $A = (\mu_A, \gamma_A)$ is an intuitionistic fuzzy w-primary submodule of R-module M. Let $x, y \in S$ and $r \in R$. We have $\mu_A(x - y) \geq \mu_A(x) \wedge \mu_A(y) = 1$. It follows that $\mu_A(x - y) = 1$. Hence $x - y \in S$. Also we have $\mu_A(rx) \geq \mu_A(x) = 1$. It follows that $\mu_A(rx) = 1$. Hence $rx \in S$. This prove that S is a submodule of M. On the other hand, for $rx \in S$ and $x \notin S$, we have $\mu_A(rx) = 1 > \alpha = \mu_A(x)$. Since $A = (\mu_A, \gamma_A)$ is an intuitionistic fuzzy w-primary submodule of R-module M, then there exists $n \in \mathbb{N}$ such that $\mu_A(r^n m) \geq \mu_A(rx)$ for every $m \in M$. This implies that there exists $n \in \mathbb{N}$ such that $r^n m \in S$ for all $m \in M$. Therefore S is a primary submodule of M. Conversey,assume that S is an primary submodule of M. Let $x, y \in M$. If $x, y \in S$, then $\mu_A(x-y) = 1 = \mu_A(x) \wedge \mu_A(y)$ and $\gamma_A(x-y) = 0 = \gamma_A(x) \wedge \gamma_A(y)$. If any one of x and y dosenot belongs to S, then $\mu_A(x-y) \geq \alpha = \mu_A(x) \wedge \mu_A(y)$ and $\gamma_A(x - y) \leq \beta = \gamma_A(x) \vee \gamma_A(y)$. Also, let $x \in M$ and $r \in R$. If $x \notin S$,then $\mu_A(rx) \geq \alpha = \mu_A(x)$ and $\gamma_A(rx) \leq \beta = \gamma_A(x)$. If $x \in S$, then $\mu_A(rx) = 1 = \mu_A(x)$ and $\gamma_A(rx) = \beta = \gamma_A(x)$. This prove that $A = (\mu_A, \gamma_A)$ is an intuitionistic fuzzy submodule of R-module M. On the other hand, assume that $\mu_A(rx) > \mu_A(x)$ and $\gamma_A(rx) < \gamma_A(x)$ for any $x \in M$ and $r \in R$. Then $\mu_A(rx) = 1$, $\mu_A(x) = \alpha$,$\gamma_A(rx) = 0$ and $\gamma_A(x) = \beta$. This implies that $rx \in S$ and $x \notin S$. Since S is primary submodule of M, then there exits an $n \in \mathbb{N}$ such that $r^n m \in S$ for all $m \in M$. Thus $\mu_A(r^n m) \geq \mu_A(rx)$ and $\gamma_A(r^n m) \leq \mu_A(rx)$ for every $m \in M$. Consequently $A = (\mu_A, \gamma_A)$ is an intuitionistic fuzzy w-primary submodule of R-module M. □

Corollary 3.6. *Let A be a submodule of R-module M. Then the function $\chi_{A^{(1,0)}}$ is a intuitionistic fuzzy primary w-submodule of M if and only if A is a primary submodule of M.*

If $A = (\mu_A, \gamma_A)$ is an intuitionistic fuzzy subset of M then $\mathcal{Q}_A = (\mu_{\mathcal{Q}_A}, \gamma_{\mathcal{Q}_A})$ is intuitionistic fuzzy subsets of M defined as follows for all $r \in R$:

$$\mu_{\mathcal{Q}_A}(r) = \bigwedge_{m\in M} \mu_A(rm) \text{ and } \gamma_{\mathcal{Q}_A}(r) = \bigvee_{m\in M} \gamma_A(rm).$$

Definition 3.7. Let R be a commutative ring. An intutionistic fuzzy ideal $A = (\mu_A, \gamma_A)$ is called an *intuitionistic fuzzy w-primary ideal* of R if the following condition:

(e1) $(\forall x, y \in R)\ (\mu_A(xy) > \mu_A(x)) \Rightarrow (\exists n\mathbb{N})\ (\mu_A(y^n) \geq \mu_A(xy))$,

(e2) $(\forall x, y \in R)\ (\gamma_A(xy) < \gamma_A(x)) \Rightarrow (\exists n\mathbb{N})\ (\gamma_A(y^n) \leq \gamma_A(xy))$.

Theorem 3.8. *If $A = (\mu_A, \gamma_A)$ is a intuitionistic fuzzy w-primary submodule of M, then $\mathcal{Q}_A$ is an intuitionistic fuzzy w-primary ideal of R.*

Proof. For any $u, w \in R$, we have

$$\begin{aligned}\mu_{\mathcal{Q}_A}(u-w) &= \bigwedge_{x\in M} \mu_A(ux - wx)\\ &\geq \bigwedge_{x\in M} (\mu_A(ux) \wedge \mu_A(wx))\\ &\geq \left(\bigwedge_{x\in M} \mu_A(ux)\right) \wedge \left(\bigwedge_{x\in M} (\mu_A(wx)\right)\\ &= \mu_{\mathcal{Q}_A}(u) \wedge \mu_{\mathcal{Q}_A}(w)\end{aligned}$$

and

$$\begin{aligned}\gamma_{\mathcal{Q}_A}(u-w) &= \bigvee_{x\in M} \gamma_A(ux - wx)\\ &\leq \bigvee_{x\in M} (\gamma_A(ux) \vee \gamma_A(wx))\\ &\geq \left(\bigvee_{x\in M} \gamma_A(ux)\right) \vee \left(\bigvee_{x\in M} (\gamma_A(wx)\right)\\ &= \gamma_{\mathcal{Q}_A}(u) \vee \gamma_{\mathcal{Q}_A}(w)\end{aligned}$$

Also, we have

$$\mu_{\mathcal{Q}_A}(uw) = \bigwedge_{x\in M} \mu_A(uwx) \geq \bigwedge_{x\in M} \mu_A(wx) = \mu_{\mathcal{Q}_A}(w)$$

and

$$\gamma_{\mathcal{Q}_A}(uw) = \bigvee_{x\in M} \gamma_A(uwx) \leq \bigvee_{x\in M} \gamma_A(wx) = \gamma_{\mathcal{Q}_A}(w).$$

This prove that Q_A is an intuitionistic fuzzy ideal of R. On the other hand, assume that $\mu_{Q_A}(uw) > \mu_{Q_A}(u)$ and $\gamma_{Q_A}(uw) < \gamma_{Q_A}(u)$. Then $\bigwedge_{x\in M} \mu_A(uwx) > \bigwedge_{x\in M} \mu_A(ux)$ and $\bigvee_{x\in M} \gamma_A(uwx) < \bigvee_{x\in M} \gamma_A(ux)$. It follows that there exists $y \in M$ such that $\mu_A(uwy) > \mu_A(uy)$ and $\gamma_A(uwy) < \gamma_A(uy)$. Since $A = (\mu_A, \gamma_A)$ is an intuitionistic fuzzy w-primary submodule of M, there exits an $n \in \mathbb{N}$ such that $\mu_A(w^n m) > \mu_A(uwx) \geq \mu_{Q_A}(uw)$ and $\gamma_A(w^n m) < \gamma_A(uwx) \leq \gamma_{Q_A}(uw)$ for all $m \in M$. This implies that $\bigwedge_{m\in M} \mu_A(w^n m) > \mu_A(uwx) \geq \mu_{Q_A}(uw)$ and $\bigvee_{x\in M} \gamma_A(w^n m) < \gamma_A(uwx) \leq \gamma_{Q_A}(uw)$ for all $m \in M$. Hence $\mu_{Q_A}(w^n) \geq \mu_{Q_A}(uw)$ and $\bigvee_{x\in M} \gamma_A(w^n) \leq \gamma_{Q_A}(uw)$. Therefore Q_A is an intuitionistic fuzzy w-primary ideal of R.

□

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