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ON STRONGLY 2-PRIMAL RINGS

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Abstract. We first find strongly 2-primal rings whose subdirect product is not (strongly) 2-primal. Moreover we observe some kinds of ring extensions of (strongly) 2-primal rings. As an example we show that if R is a ring and M is a multiplicative monoid in R consisting of central regular elements, then R is strongly 2-primal if and only if so is RM^{-1}. Various properties of (strongly) 2-primal rings are also studied.

1. Introduction

Throughout this note all rings are associative with identity unless otherwise stated. A given ring R, the prime radical of R, the Jacobson radical of R and the set of all nilpotent elements in R are denoted by $P(R)$, $J(R)$ and $N(R)$, respectively.

Due to Birkenmeier et al.[1], a ring R is called *2-primal* if $P(R) = N(R)$. Hirano [6] used the term N-ring for the condition; while Sun [20] called 2-primal condition *weakly symmetric*. The class of 2-primal rings contains one-sided Artinian local rings and commutative rings. A ring is called *reduced* if it contains no nonzero nilpotent elements. Reduced rings are clearly 2-primal and note that a ring R is 2-primal if and only if $R/P(R)$ is reduced. An ideal I of a ring R is called *2-primal* if $P(R/I) = N(R/I)$, according to [1]. Due to Kim and Y. Lee [9], a ring R is called *strongly 2-primal* if every proper ideal I of R is 2-primal, where the term *proper* means only $I \neq R$. Simple domains are clearly strongly 2-primal. Note that a ring R is 2-primal if and only if the

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zero ideal of R is 2-primal; hence strongly 2-primal rings are clearly 2-primal, but the converse needs not be true by [1, Example 2.7]. A ring is called *right (left) duo* if every right (left) ideal of it is two-sided. Right (left) duo rings are strongly 2-primal by [21, Corollary 4], and so is any local ring in which the Jacobson radical is strongly 2-primal as a ring without identity.

Shin [19, Proposition 1.11] proved that a ring R is 2-primal if and only if every minimal prime ideal of R is completely prime. Hirano [6, Theorem 1] showed that a 2-primal ring R is strongly π-regular if and only if so is the n by n full matrix ring over R for any positive integer n. Note that there is a strongly π-regular (but not 2-primal) ring by Rowen [17, Example 2.3] over which every n by n full matrix ring needs not be strongly π-regular. Sun [20, Theorem 2.3] proved that for a 2-primal ring the prime spectrum is a normal topological space if and only if the maximal ideal spectrum is a continuous retract of the prime spectrum. Hong et al. [8, Theorem 6] showed that a 2-primal ring R is right weakly π-regular if and only if R is strongly π-regular if and only if every prime ideal of R is maximal when every primitive factor ring of R is Artinian. More results, for 2-primal rings and related concepts, can be seen in [1, 2, 6, 8, 10, 12, 13, 14, 15, 16, etc.].

The 2-primal condition is preserved by polynomial rings, direct sums, subrings, triangular matrix rings and Dorroh extensions by Birkenmeier et al. [1]; however it needs not be preserved by direct products and formal power series rings by the examples in [10, 15]. In this note we study subdirect products of (strongly) 2-primal rings with the help of arguments and examples in [1, 14].

A subset I of a ring R is called *right (left) T-nilpotent* provided that for every sequence $a_1, a_2, \ldots$ in I there is a positive integer n such that $a_n \cdots a_2 a_1 = 0$ ($a_1 a_2 \cdots a_n = 0$). A subset J of a ring is called *nilpotent* if $J^n = 0$ for some positive integer n. It is well known that nilpotent subsets of a ring are obviously both right and left T-nilpotent but the converse does not hold in general, and that left (right) T-nilpotent subsets are clearly nil but nil ideals need not be right (or left) T-nilpotent. T-nilpotence is not left-right symmetric by [18, Example 2.7.38]. Recall that the prime radical of a ring is the set of all strongly nilpotent elements in it.

We first note the equivalent conditions of 2-primal rings in the following. An ideal I of a ring R is called *completely prime* if R/I is a domain. Completely prime ideals are clearly prime.

Lemma 1.1. For a ring R the following conditions are equivalent:

(1) R is 2-primal;

(2) Every minimal prime ideal of R is completely prime;

(3) Every subring (possibly without identity) of R is 2-primal;

(4) $R/P(R)$ is a subdirect product of domains;

(5) $R/P(R)$ is a subdirect product of reduced rings;

(6) $R/P(R)$ is reduced.

Proof. (2)⇒(4) and (4)⇒(5) are obvious. (5)⇒(1) is obtained by the definition. (1)⇒(3) and (3)⇒(1) are proved by [1, Proposition 2.2] and [7, Lemma 1.1], respectively. □

For the strong 2-primalness we have the following equivalent conditions. An ideal I of a ring R is called *completely semiprime* if R/I is a reduced ring.

Lemma 1.2.[9, Proposition 1.2] For a ring R the following conditions are equivalent:

(1) R is a strongly 2-primal ring;

(2) Every prime ideal of R is completely prime;

(3) Every semiprime ideal of R is completely semiprime;

(4) $R/P(R)$ is a strongly 2-primal ring.

Strongly 2-primal rings are clearly 2-primal, but the converse needs not be true by [1, Example 2.7]. In the following we see conditions under which these two concepts are equivalent. A ring R is called *right perfect* if $R/J(R)$ is semisimple Artinian and $J(R)$ is right T-nilpotent. Left perfect rings can be defined similarly.

Proposition 1.3. Let R be a one-sided perfect ring. Then the following conditions are equivalent:

(1) R is 2-primal;

(2) R is strongly 2-primal;

(3) $R/J(R)$ is a finite direct product of division rings.

Proof. (1)$\Rightarrow$(3) is proved by [3, Proposition 3.5], and (2)$\Rightarrow$(1) is trivial. $J(R)$ is right or left T-nilpotent and so $J(R) = P(R)$. Then every prime factor ring of R is a division ring when $R/J(R)$ is a finite direct product of division rings, proving (3)$\Rightarrow$(2) by Lemma 1.2. □

Note that the ring in [1, Example 2.7] is neither right nor left perfect.

A ring R is called *von Neumann regular* if for each $a \in R$ there exists $x \in R$ such that $a = axa$. Von Neumann regular rings are semiprime by [4, Corollary 1.2]. If a given ring is von Neumann regular then the 2-primalness and strong 2-primalness are equal by [9, Proposition 1.6]. But in the following we obtain the same result by another method, adding other conditions.

A ring is called *abelian* if every idempotent is central. A ring R is called *strongly regular* if for each $x \in R$ there exists $y \in R$ such that $x^2y = x$. A ring is strongly regular if and only if it is abelian and von Neumann regular [4, Theorem 3.5]. A given ring R, $r_R(-)$ ($l_R(-)$) is used for the right (left) annihilator in R; an element $a \in R$ called *regular* if $r_R(a) = 0 = l_R(a)$.

Proposition 1.4. Let R be a von Neumann regular ring. Then the following conditions are equivalent:

(1) R is right (left) duo;

(2) R is reduced;

(3) R is abelian;

(4) R is strongly 2-primal;

(5) R is 2-primal;

(6) R is a subdirect product of division rings;

(7) R is a subdirect product of reduced rings.

Proof. The equivalences of the conditions (1), (2), (3), (4) and (5) are obtained by [9, Proposition 1.6]. (6) $\Rightarrow$ (7) and (7) $\Rightarrow$ (2) are obvious. It is sufficient to show that (5) $\Rightarrow$ (6). If R is 2-primal then each minimal prime ideal of R is completely prime by Lemma 1.1; hence R is a subdirect product of domains. Note that each factor ring of R is also von Neumann regular, and that regular elements of von Neumann regular rings are invertible. Thus R/P must be a division ring for each minimal prime ideal P of R. □

A ring R is called *π-regular* if for each $a \in R$ there exist a positive integer n, depending on a, and $b \in R$ such that $a^n = a^n b a^n$. It is easy to show that the Jacobson radical of a π-regular ring is nil. Since von Neumann regular rings are π-regular, one may ask if abelian π-regular rings are (strongly) 2-primal, based on Proposition 1.4. However the answer is negative by the following.

Example 1.5. Let S be a division ring. Denote by U_n the 2^n by 2^n upper triangular matrix ring over a ring S, where n is a positive integer. Define a ring extension of S, that is a subring of U_n,

$$D_n = \{M \in U_n \mid \text{ the diagonal entries of } M \text{ are equal }\}.$$

Define a map $\sigma : D_n \to D_{n+1}$ by $A \mapsto \begin{pmatrix} A & 0 \\ 0 & A \end{pmatrix}$, then D_n can be considered as a subring of D_{n+1} via σ (i.e., $A = \sigma(A)$ for $A \in D_n$). Set R be the direct limit of the direct system (D_n, σ_{ij}) with $\sigma_{ij} = \sigma^{j-i}$. Then R is semiprime but not 2-primal by [5, Theorem 2.2(2)].

Reduced rings are abelian through a simple computation, and so every D_n is abelian by [11, Lemma 2] such that every idempotent in D_n is of the form $\begin{pmatrix} f & 0 & 0 & \cdots & 0 \\ 0 & f & 0 & \cdots & 0 \\ 0 & 0 & f & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & f \end{pmatrix}$ with $f^2 = f \in S$. Thus R is abelian. Since every element of D_n is invertible or nilpotent, D_n is π-regular and thus so is R. □

2. Ordinary Extensions of (Strongly) 2-primal Rings

In this section we first answer a question raised by Birkenmeier et al.[1], in the negative. Next we extend the class of (strongly) 2-primal rings, considering some kinds of ordinary ring extensions. A given ring R, $R[X]$ (resp. $R[[X]]$) denotes the polynomial (resp. power series) ring with X a set of commuting indeterminates over R (possibly infinite). If $X = \{x\}$ then we use $R[x]$ (resp. $R[[x]]$) in place of $R[\{x\}]$ (resp. $R[[\{x\}]]$).

From Lemma 1.1 a ring R is 2-primal if and only if $R/P(R)$ is a subdirect product of reduced rings, and it is obvious that a subdirect product of reduced rings is reduced. Based on these facts, Birkenmeier et al. [1] raised the following natural question:

Is a subdirect product of 2-primal rings also 2-primal?

We will answer this question in the negative. For doing it we first need the following lemma.

Lemma 2.1. Let R be a ring and n be a positive integer.

(1) $R[x]/x^nR[x]$ is (resp. strongly) 2-primal if and only if R is (resp. strongly) 2-primal.

(2) $R[[x]]/x^nR[[x]]$ is (resp. strongly) 2-primal if and only if R is (resp. strongly) 2-primal.

Proof. (1) Let $A = R[x]/x^nR[x]$. Since $xR[x]/x^nR[x]$ is nilpotent, $P(A)$ contains $xR[x]/x^nR[x]$ and so $P(A) = \dfrac{P(R) + xR[x]}{x^nR[x]}$. Thus there is a one to one correspondence between the set of minimal prime ideals of A and the set of minimal prime ideals of R, via $Q' = \dfrac{Q + xR[x]}{x^nR[x]} \mapsto Q$ (resp. there is a one to one correspondence between the set of prime ideals of A and the set of prime ideals of R, via $P' = \dfrac{P + xR[x]}{x^nR[x]} \mapsto P$). Then from the fact $A/Q' \cong R/Q$ (resp. $A/P' \cong R/P$, the proof is completed by Lemmas 1.1 and 1.2. The proof of (2) is same as (1). □

Now we see a counterexample to the question above as follows.

Example 2.2. Let F be a field, V be a infinite dimensional left vector space over F with $\{v_1, v_2, \dots\}$ a basis, and A be the endomorphism ring of V over F. Due to [10, Example 1.1] define $A_1 = \{f \in A \mid \text{rank}(f) < \infty \text{ and } f(v_i) =$

$a_1v_1 + \cdots + a_iv_i$ for $i = 1, 2, \ldots$ with $a_j \in F\}$ and, let R be the F-subalgebra of A generated by A_1 and 1_A. Then R is 2-primal but $R[[x]]$ is not 2-primal by the argument in [10, Example 1.1]. Define a homomorphism

$$\sigma : R[[x]] \to \prod_{n=1}^{\infty} \frac{R[[x]]}{x^n R[[x]]} \text{ with } \sigma(f(x)) = (f(x) + x^n R[[x]])_{n=1}^{\infty}.$$

Since the kernel of σ is zero, $R[[x]]$ is a subdirect product of $\frac{R[[x]]}{x^n R[[x]]}$'s. Since R is 2-primal, so is each $\frac{R[[x]]}{x^n R[[x]]}$ by Lemma 2.1(2); hence $R[[x]]$ is a subdirect product of 2-primal rings. However $R[[x]]$ is not 2-primal. □

Based on Example 2.2 we may also raise the following:

Is a subdirect product of strongly 2-primal rings also (strongly) 2-primal?

But the answer is also negative. For, the ring R in Example 2.2 is moreover strongly 2-primal by the argument in [9, Example 1.8], and so $R[[x]]$ is a subdirect product of strongly 2-primal rings by Lemma 2.1(2).

Birkenmeier et al. showed that the polynomial rings over 2-primal rings are also 2-primal [1, Proposition 2.6]. In the following we see two different proofs of the proposition.

Proposition 2.3. Let R be any ring and X any set of commuting indeterminates over R. If R is 2-primal then $R[X]$ is also 2-primal.

Proof. (Method 1) From the epimorphism $R \to \dfrac{R}{P(R)}$ with $r \mapsto r + P(R)$, we naturally obtain another epimorphism $R[X] \to \dfrac{R}{P(R)}[X]$. This leads us to an isomorphism $\dfrac{R[X]}{P(R)[X]} \cong \dfrac{R}{P(R)}[X]$. But since R is 2-primal, $R/P(R)$ is reduced; hence $\dfrac{R[X]}{P(R)[X]}$ is also reduced and we have $N(R[X]) \subseteq P(R)[X]$. Note that $P(S[X]) = P(S)[X]$ for any ring S; so $N(R[X]) \subseteq P(R[X])$, obtaining $P(R[X]) = N(R[X])$. Thus $R[X]$ is 2-primal.

(Method 2) This is similar to the one of [1, Proposition 2.6], using completely prime ideals. It is well-known that P is a (minimal) prime ideal of R if and only if $P[X]$ is a (minimal) prime ideal of $R[X]$, and that $Q \cap R$ is a prime ideal of R for any prime ideal Q of $R[X]$. Thus each minimal prime ideal of $R[X]$ is of the form $P[X]$ for some minimal prime ideal P of R. If R is 2-primal then every minimal prime ideal of R is completely prime by Lemma 1.1. But we also have that P is completely prime in R if and only if $\frac{R}{P}$ is a domain if and only if $\frac{R}{P}[X] \cong \frac{R[X]}{P[X]}$ is a domain if and only if $P[X]$ is a completely prime ideal of $R[X]$. Hence $R[X]$ is 2-primal by Lemma 1.1. because every minimal prime ideal of $R[X]$ is completely prime. □

However polynomial rings over strongly 2-primal rings need not be strongly 2-primal by [1, Example 3.13]. In the following we consider another extension to which the strong 2-primalness can go up.

Theorem 2.4. Suppose that R is a ring and M is a multiplicative monoid in R consisting of central regular elements. Then R is strongly 2-primal if and only if so is RM^{-1}.

Proof. First we claim that $I \mapsto IM^{-1}$ is a one to one correspondence between the set of all ideals in R and the set of all ideals in RM^{-1}. Let J be an ideal of RM^{-1} and

$$I = \{r \in R \mid rm^{-1} \in J\}.$$

Then since J is an ideal of RM^{-1} we have that $I \subseteq J$ and I is an ideal of R, entailing $IM^{-1} = J$. Conversely, let I be an ideal of R and consider IM^{-1}. Take $a_1m_1^{-1}, a_2m_1^{-2} \in IM^{-1}$. Since I is an ideal of R we have $a_1m_2 - a_2m_1, a_1a_2 \in I$, so that $a_1m_1^{-1} - a_2m_2^{-1} = (a_1m_2 - a_2m_1)(m_1m_2)^{-1}$ and $a_1m_1^{-1}a_2m_2^{-1} = (a_1a_2)(m_1m_2)^{-1}$ are contained in IM^{-1}, concluding that IM^{-1} is an ideal of RM^{-1}. We will use the claim freely.

Now suppose that R is strongly 2-primal. Consider a factor ring $A = \frac{RM^{-1}}{IM^{-1}}$ of RM^{-1} for a proper ideal I of R. We will show that A is 2-primal, then RM^{-1} is strongly 2-primal by the definition.

Let $\bar{x}^n = 0$ with $x = am^{-1} \in RM^{-1}$. We have $x^n = a^nm^{-n} \in IM^{-1}$ since M is contained in the center of R; hence $a^n \in I$ because m is regular. But R/I is 2-primal and so every sequence $(a_0, a_1, \dots, a_k, \dots)$ is stationary in R/I (i.e.,

$\bar{a} \in P(R/I))$, where $a_0 = \bar{a}, a_1 \in a_0 \frac{R}{I} a_0, \ldots, a_k \in a_{k-1} \frac{R}{I} a_{k-1}$ for $k = 1, 2, \ldots$. Next consider an arbitrary sequence $(x_0, x_1, \ldots, x_k, \ldots)$ in A such that $x_0 = \bar{x}$, $x_1 = x_0(\bar{b}_0 \bar{u}_0^{-1})x_0 \in x_0 A x_0, \ldots, x_k = x_{k-1}(\bar{b}_{k-1}\bar{u}_{k-1}^{-1})x_{k-1} \in x_{k-1} A x_{k-1}$ for $k = 1, 2, \ldots$. But the sequence

$$(a_0 \bar{b}_0 a_0, a_0 \bar{b}_0 a_0 \bar{b}_1 a_0 \bar{b}_0 a_0, \ldots)$$

is eventually zero in R/I by the argument above; hence so is the sequence $(x_0, x_1, \ldots, x_k, \ldots)$ since M is contained in the center of R. Thus A is 2-primal.

Conversely let RM^{-1} is strongly 2-primal and consider a factor ring R/I of R for a proper ideal I of R. We will show that R/I is 2-primal, then R is strongly 2-primal by the definition. Let $\bar{y}^m = 0$ with $y \in R$ and consider a sequence $(y_0, y_1, \ldots, y_k, \ldots)$, where

$$y_0 = \bar{y}, y_1 \in y_0 \frac{R}{I} y_0, \ldots, y_k \in y_{k-1} \frac{R}{I} y_{k-1}$$

for $k = 1, 2, \ldots$. Note that R/I can be considered as a subring of $A = \dfrac{RM^{-1}}{IM^{-1}}$, and so $y^m \in IM^{-1}$. Thus the sequence can be considered in A, so that it is stationary since A is 2-primal. Thus $y_t \in IM^{-1}$ and then $y_t \in I$, entailing that R/I is 2-primal. □

For 2-primal rings the preceding result also holds by [3, Proposition 3.4]. The ring of *Laurent* polynomials in x, with coefficients in a ring R, consists of all formal sums $\sum_{i=k}^{n} m_i x^i$ with obvious addition and multiplication, where $m_i \in R$ and k, n are (possibly negative) integers. We denote this ring by $R[x; x^{-1}]$.

Corollary 2.5. For a ring R, $R[x]$ is strongly 2-primal if and only if so is $R[x; x^{-1}]$.

Proof. Let $M = \{1, x, x^2, \ldots\}$, then M is a multiplicative monoid in $R[x]$ consisting of central regular elements. Since $R[x; x^{-1}] = M^{-1}R[x]$ we obtain the corollary from Theorem 2.4. □

Polynomial rings over division rings need not be strongly 2-primal by [1, Example 3.13], but we get an affirmative situation for power series rings as follows.

Proposition 2.6. If D is a division ring then $D[[x]]$ is strongly 2-primal.

Proof. Let J be a nonzero proper ideal of $D[[x]]$, then there exists x^k in J with $k \geq 1$ such that k is the least integer with respect to $x^k \in J$, concluding $J = x^k D[[x]]$. If $k \geq 2$ then $D[[x]]/J$ is not a prime ring, so if J is a prime ideal of $D[[x]]$ then k must be 1. Consequently $D[[x]]x$ is the only nonzero prime ideal of $D[[x]]$. It then follows that $D \cong D[[x]]/D[[x]]x$ and $D[[x]] \cong D[[x]]/0$ are all prime factor rings taken from $D[[x]]$. Therefore $D[[x]]$ is strongly 2-primal by Lemma 2.1(2).

□

Let R, S be rings and $f : R \to S$ be a ring homomorphism. Consider a ring extension

$$T(R, S) = \left\{ \begin{pmatrix} a & b \\ 0 & a \end{pmatrix} \mid a \in R \text{ and } b \in S \right\}$$

with the usual addition and the multiplication

$$(r_1, s_1)(r_2, s_2) = (r_1 r_2, f(r_1)s_2 + s_1 f(r_2)).$$

As a similar construction to Example 1.5, denote by $U_k(S)$ the k by k upper triangular matrix ring over a ring S, where k is a positive integer. Define a ring extension of S, that is a subring of U_k,

$$D_k(S) = \{M \in U_k(S) \mid \text{ the diagonal entries of } M \text{ are equal } \}.$$

Birkenmeier et al. showed that a ring S is strongly 2-primal if and only if so is $U_k(S)$ [1, Proposition 3.11]. The following retains similar results.

Proposition 2.7. (1) Let R and S be rings. If R is (resp. strongly) 2-primal then $T(R, S)$ is (resp. strongly) 2-primal.

(2) A ring S is strongly 2-primal if and only if so is $D_k(S)$.

Proof. (1) Let R be (resp. strongly) 2-primal and set $A = T(R, S)$. Note that $P(A) = \left\{ \begin{pmatrix} a & b \\ 0 & a \end{pmatrix} \mid a \in P(R) \text{ and } b \in S \right\}$ and $A/P(A) \cong R/P(R)$, thereby A is (resp. strongly) 2-primal by Lemma 1.1 (resp. Lemma 1.2).

(2) Let $R = D_k(S)$. Since

$$P(R) = \left\{ \begin{pmatrix} a & a_{12} & a_{13} & \cdots & a_{1n} \\ 0 & a & a_{23} & \cdots & a_{2n} \\ 0 & 0 & a & \cdots & a_{3n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & a \end{pmatrix} \mid a \in P(S), a_{ij} \in S \right\},$$

we have $R/P(R) \cong S/P(S)$. Thus we are done by Lemma 1.2. □

Subrings of 2-primal rings are 2-primal by [1, Proposition 2.2]. However subrings of strongly 2-primal rings need not be strongly 2-primal by [1, Example 3.13]. But some kind of subrings may have an affirmative situation as follows.

Proposition 2.8. A given ring R the following conditions are equivalent:

(1) R is strongly 2-primal;

(2) eR and $(1-e)R$ are strongly 2-primal for every nonzero central idempotent e of R;

(3) eR and $(1-e)R$ are strongly 2-primal for some nonzero central idempotent e of R.

Proof. (1)⇒(2): Suppose that R is strongly 2-primal. Let A be a proper ideal of $S = eRe = eR$ and consider $T = \frac{S}{A}$, where e is a nonzero central idempotent in R. Then A is also a proper ideal of R since e is central. Let $\bar{x} \in N(T)$, then clearly $\bar{x} \in N\left(\frac{R}{A}\right)$; hence $\bar{x} \in P\left(\frac{R}{A}\right)$ since R is strongly 2-primal. Next consider a sequence $(x_1, x_2, \dots)$ with

$$x_0 = \bar{x}, x_1 = x_0 y_0 x_0 \in x_0 T x_0, \dots, x_{k+1} = x_k y_k x_k \in x_k T x_k$$

for $k = 0, 1, 2, \dots$ and $y_j = \bar{e}\bar{r}_j\bar{e}$ for $j = 0, 1, 2, \dots$, where r_j's are taken arbitrarily in R. But

$$x_k y_k x_k = x_k \bar{r}_k x_k \in x_k \frac{R}{A} x_k \text{ and } \bar{x} \in P\left(\frac{R}{A}\right),$$

it follows that the sequence $(x_1, x_2, \dots)$ is eventually zero; hence $\bar{x} \in P(T)$ and T is 2-primal. Thus eR is strongly 2-primal, and so is $(1-e)R$.

(2)⇒(3) is trivial.

(3)⇒(1): Suppose that eR and $(1-e)R$ are strongly 2-primal for a nonzero central idempotent e of R. Then since $R = eR \oplus (1-e)R$, R is strongly 2-primal by [1, Proposition 3.10]. □

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