Robust Multiuser Detection Based on Least p-Norm State Space Filtering Model

Daifeng Zha

Abstract: **Alpha stable distribution is better for modeling impulsive noises than Gaussian distribution in signal processing. This class of process has no closed form of probability density function and finite second order moments. In general, Wiener filter theory is not meaningful in $S\alpha SG$ environments because the expectations may be unbounded. We proposed a new adaptive recursive least p-norm Kalman filtering algorithm based on least p-norm of innovation process with infinite variances, and a new robust multiuser detection method based on least p-norm Kalman filtering. The simulation experiments show that the proposed new algorithm is more robust than the conventional Kalman filtering multiuser detection algorithm.**

Index Terms: **Alpha stable distribution, CDMA, fractional lower order statistics, Kalman filtering, least p-norm, multiuser detection.**

I. INTRODUCTION

Impulsive noise is known to exist in communication, underwater sonar, submarine communications and radar systems. An impulsive random variable is characterized by a probability distribution function which decays more slowly than in the Gaussian case. Hence, impulsive noise produces more outliers than expected under the Gaussian assumption, degrading the performance of linear filtering. Recent studies [1], [2] show that alpha stable distribution is better for modeling impulsive noise than Gaussian distribution in signal processing, which has some important characteristics and makes it very attractive. Sources that could follow or be modeled by the alpha stable distribution are abundant and include lightning in the atmosphere, switching transients in power lines, static in telephone lines, seismic activity, climatology and weather, ocean wave variability, surface texture, the slamming of a ship hull in a seaway, acoustic emissions from cracks growing in engineering materials under stress, etc. Many sources can exist in the area of target and background signatures that affect detection and classification. In underwater acoustics, examples of these sources could include interference to target detection such as ice cracking, biologics, bottom and sea clutter in active sonar, ocean waves near the surface and in the surf zone.

They could also include target characteristics such as target strength in active sonar and cavitation. Similar sources in radar and infrared can include ocean waves in the form of sea clutter and radar cross section (RCS); see [25].

Stable processes arise as limiting processes of sums of independent, identically distributed random variables via the generalized central limit theorem. Generally, alpha stable process can be described conveniently by its characteristic function [1], [2]

$$\Phi_{\alpha S}(t) = \exp\{j\mu t - \gamma|t|^{\alpha}[1 + j\beta \mathrm{sgn}(t)\varpi(t,\alpha)]\} \quad (1)$$

where

$$\varpi(t,\alpha) = \begin{cases} \tan\frac{\alpha\pi}{2}, & \alpha \neq 1, \\ \frac{2}{\pi}\log|t|, & \alpha = 1, \end{cases}$$

and $-\infty < \mu < \infty,\ \gamma > 0,\ 0 < \alpha \le 2,\ -1 \le \beta \le 1$.

The characteristic exponent α controls the thickness of the tail in the distribution. The Gaussian process is a special case of stable processes with $\alpha = 2$. The dispersion parameter γ is similar to the variance of Gaussian process and β is the symmetry parameter. If $\beta = 0$, the distribution is symmetric and the observation is referred to as the $S\alpha S$ (symmetry α-stable) distribution, i.e., it is symmetrical about the location parameter μ. $S\alpha S$ distribution has been shown to accurately model impulsive interference environments.

Semi-conducting electrical devices in communication and radar systems subject the signal to internal thermal Gaussian noise. Hence, the resultant interference follows the distribution of a sum of independent $S\alpha S$ and Gaussian random variables, i.e., an $S\alpha SG$ distribution [3], [4]. The random variable follows an $S\alpha SG$ distribution, whose probability distribution function does not exist in closed form. The process is, however, easily represented in the characteristic function domain as $\Phi_{S\alpha SG}(t) = \exp\{-\gamma_{S\alpha S}|t|^{\alpha} - \gamma_G t^2\}$, where $\gamma_{S\alpha S} > 0$, $\gamma_G = \sigma_G^2/2 > 0$ are the dispersions of $S\alpha S$ and Gaussian random variables, respectively. σ_G^2 is related to the variance of the Gaussian component . The typical $S\alpha SG$ distribution signals are shown in Fig. 1. Properties of the $S\alpha SG$ distribution are similar to those of the $S\alpha S$ distribution, namely,

1) The median exists for all α while the mean only exists for $\alpha > 1$;
2) For $\alpha = 2$ we have a sum of two independent Gaussian processes;
3) For $\alpha < 2$ all moments of order 2 and above do not exist. In general the fractional lower order moments, $E|X|^p < \infty$, exist for $0 < p < \alpha$.

Conventional Wiener filtering theory describes the behavior of the least-squares (LS) and the least-mean squares (LMS) adaptive filters. In general, Wiener filter theory is not meaningful in $S\alpha SG$ environments because the expectations may be unbounded. Conventional Kalman filtering was based on the second order covariance matrix of process (state) noise and observation (output) noise. Convergence and related issues have

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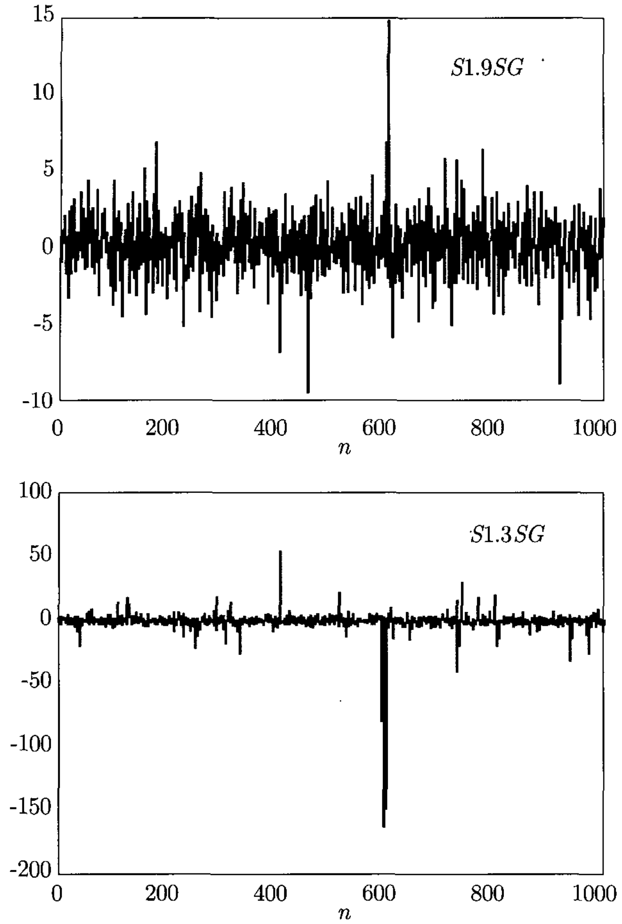

Fig. 1. $S\alpha SG$ distribution signals.

been studied thoroughly. The theory has many applications such as system identification, inverse modeling, prediction, and interference cancellation. Adaptive filters also serve as components in blind equalizers and other systems. These concepts are based on L_2 measurements and are therefore only of extremely limited use when dealing with infinite-variance signals. Filtering techniques for $S\alpha SG$ processes are still at a very early stage [2]. Some theory has been developed for harmonizable stable processes [5]. Stuck modified the Kalman–Bucy filter [6] but did not provide an iterative algorithm. The minimum dispersion (MD) criterion and least p-norm criterion have been used for autoregressive (AR) modeling and linear prediction [7], [8], whereas other research has focused exclusively on the least p-norm criterion for linear prediction [9]–[12]. In this paper, we proposed a new adaptive generalized recursive least p-norm Kalman filtering algorithm based on innovation process with infinite variances and new robust multiuser detection based on least p-norm recursive Kalman filtering.

II. SYSTEM MODEL

Interference noise in the form of sparsely distributed impulses arises frequently in a variety of practical situations, including speech, image, biomedical, and communications applications. Therefore, there is a need for adaptive filtering algorithms that are robust to impulsive interference noise. For discrete dynamic state-space system, we can express it by using process (state) equation and observation (output) equation as follows

$$\begin{aligned}\mathbf{s}(n+1) &= \mathbf{A}(n)\mathbf{s}(n) + \mathbf{v}_1(n)\\ \mathbf{y}(n) &= \mathbf{C}(n)\mathbf{s}(n) + \mathbf{v}_2(n)\end{aligned} \quad (2)$$

where $\mathbf{s}(n) = [s_0(n), s_1(n), \cdots, s_{N-1}(n)]^T$ is state vector at instant n, $(\cdot)^T$ denotes the vector transpose, $N \times N$ matrix $\mathbf{A}(n)$ is state transfer matrix, denoting state transfer from instant n to instant $n+1$, $N \times 1$ vector $\mathbf{v}_1(n)$ is additive noise vector, $\mathbf{y}(n) = [y_0(n), y_1(n), \cdots, y_{M-1}(n)]^T$ is observation (output) vector at instant n, $M \times N$ matrix $\mathbf{C}(n)$ is observation matrix, and $M \times 1$ vector $\mathbf{v}_2(n)$ is additive observation noise vector. For convenience, we can assume that

1) $M = 1$, i.e., only one output $y(n)$;
2) $\mathbf{v}_1(n)$ and $v_2(n)$ are noise vector and scalar following $S\alpha SG$ distribution ($\alpha < 2$).

Kalman filtering can be regarded as estimating $\mathbf{s}(k) = [s_0(k), s_1(k), \cdots, s_{N-1}(k)]^T$ by employing observations $y(1)$, $y(2), \cdots, y(n)$.

Innovation [13], [15] is proposed by T. Kailath in 1968. For $y(n)$, its innovation process is defined as

$$\theta(n) = y(n) - \hat{y}(n|y(1), \cdots, y(n-1)) \quad (3)$$

where $\hat{y}(n|y(1), \cdots, y(n-1))$ denotes one-step prediction value of $y(n)$. In conventional Kalman filtering algorithm, we must calculate second order moment $E\left\{|\theta(n)|^2\right\}$ of innovation process and second order correlation matrix $E\left\{\mathbf{v}_1(n)\mathbf{v}_1^T(n)\right\}$, $E\left\{|v_2(n)|^2\right\}$ of noise processes, which have no finite value under $S\alpha SG$ noises conditions. Properties of innovation process under $S\alpha SG$ noises conditions

1) $E\{\theta(n)y(k)\} = 0 \ \ (k \le n-1)$
2) $E\{\theta(n)\theta(k)\} = 0 \ \ (k \le n-1)$
3) For $y(n) = \mathbf{C}(n)\mathbf{s}(n) + v_2(n)$ and $v_2(n)$ are $S\alpha SG$ noises ($\alpha < 2$), $E\left\{|\theta(n)|^2\right\} = \infty$, i.e., innovation process has infinite variance.
4) If $0 < p < \alpha$, then $E\{|\theta(n)|^p\} < \infty$, i.e., innovation process has finite p-order moment.

III. RECURSIVE LEAST p-NORM KALMAN FILTERING

Adaptive Kalman filtering is regarded as estimating $\mathbf{s}(n)$ according to $y(1), y(2), \cdots, y(n)$. For innovation process has finite p-order moment, to improve the robustness of algorithm, we can use a new cost function as follows under $S\alpha SG$ distribution noise condition

$$J(n) = \sum_{i=1}^{n} \frac{1}{p}|\theta(i)|^p = \sum_{i=1}^{n} \frac{1}{p}\left|y(i) - \hat{y}(i|y(1), \cdots, y(i-1))\right|^p \quad (4)$$

where $0 < p < \alpha$. Assuming that we obtain $\hat{y}(i)$ by utilizing $\hat{\mathbf{s}}(n|y(1), \cdots, y(n))$ we have $\hat{y}(i|y(1), \cdots, y(i-1)) = \mathbf{C}(i)\hat{\mathbf{s}}(n|y(1), \cdots, y(n))$ and

$$J(n) = \sum_{i=1}^{n} \frac{1}{p}\left|y(i) - \mathbf{C}(i)\hat{\mathbf{s}}(n|y(1), \cdots, y(n))\right|^p . \quad (5)$$

We can obtain instantaneous gradient of $J(n)$ with respect to $\hat{\mathbf{s}}(n)$

$$\begin{aligned}&\sum_{i=1}^{n}|\theta(i)|^{p-1}\text{sign}(\theta(i))\frac{\partial\theta(i)}{\partial\hat{\mathbf{s}}(n)}\\ &=\sum_{i=1}^{n}|\theta(i)|^{p-1}\text{sign}(\theta(i))(-\mathbf{C}^T(i)) \\ &=\sum_{i=1}^{n}|\theta(i)|^{p-1}\text{sign}(\theta(i))\frac{1}{\theta(i)}\theta(i)(-\mathbf{C}^T(i))=0\end{aligned} \tag{6}$$

and

$$\sum_{i=1}^{n}|\theta(i)|^{p-2}(y(i)-\mathbf{C}(i)\hat{\mathbf{s}}(n))\mathbf{C}^T(i)=0 \tag{7}$$

$$\sum_{i=1}^{n}|\theta(i)|^{p-2}y(i)\mathbf{C}^T(i)=\sum_{i=1}^{n}|\theta(i)|^{p-2}\mathbf{C}^T(i)\mathbf{C}(i)\hat{\mathbf{s}}(n). \tag{8}$$

Assuming that

$$\begin{aligned}\mathbf{P}(n)&=\sum_{i=1}^{n}|\theta(i)|^{p-2}y(i)\mathbf{C}^T(i),\\ \mathbf{R}(n)&=\sum_{i=1}^{n}|\theta(i)|^{p-2}\mathbf{C}^T(i)\mathbf{C}(i),\end{aligned}$$

we have

$$\begin{aligned}\mathbf{R}(n)&=\mathbf{P}(n)\hat{\mathbf{s}}(n),\\ \mathbf{R}(n)&=\mathbf{R}(n-1)+|\theta(i)|^{p-2}\mathbf{C}^T(n)\mathbf{C}(n),\\ \mathbf{P}(n)&=\mathbf{P}(n-1)+|\theta(i)|^{p-2}y(n)\mathbf{C}^T(n).\end{aligned}$$

Applying the matrix inversion lemma [14], we have

$$\mathbf{R}^{-1}(n)=\mathbf{R}^{-1}(n-1)-\mathbf{K}(n)\mathbf{C}(n)\mathbf{R}^{-1}(n-1)$$

where gain vector

$$\mathbf{K}(n)=\frac{|\theta(n)|^{p-2}\mathbf{R}^{-1}(n-1)\mathbf{C}^T(n)}{1+|\theta(n)|^{p-2}\mathbf{C}(n)\mathbf{R}^{-1}(n-1)\mathbf{C}^T(n)}$$

can be written as $\mathbf{K}(n)=|\theta(n)|^{p-2}\mathbf{R}^{-1}(n-1)\mathbf{C}^T(n)$. Thus,

$$\begin{aligned}\hat{\mathbf{s}}(n)&=\mathbf{R}^{-1}(n)\mathbf{P}(n)\\ &=\mathbf{R}^{n-1}(n)\mathbf{P}(n-1)+\mathbf{R}^{n}|\theta(n)|^{p-2}y(n)\mathbf{C}^T(n).\end{aligned} \tag{9}$$

We have state vector update of proposed recursive least p-norm Kalman (LP-Kalman) filtering algorithm

$$\hat{\mathbf{s}}(n)=\hat{\mathbf{s}}(n-1)+b(n)\mathbf{K}(n) \tag{10}$$

where $b(n)=y(n)-\mathbf{C}(n)\hat{\mathbf{s}}(n-1)$.

IV. MODIFICATION AND PERFORMANCES OF LP-KALMAN

To improve robustness and performance of LP-Kalman algorithm, we utilize a robust segmented cost function

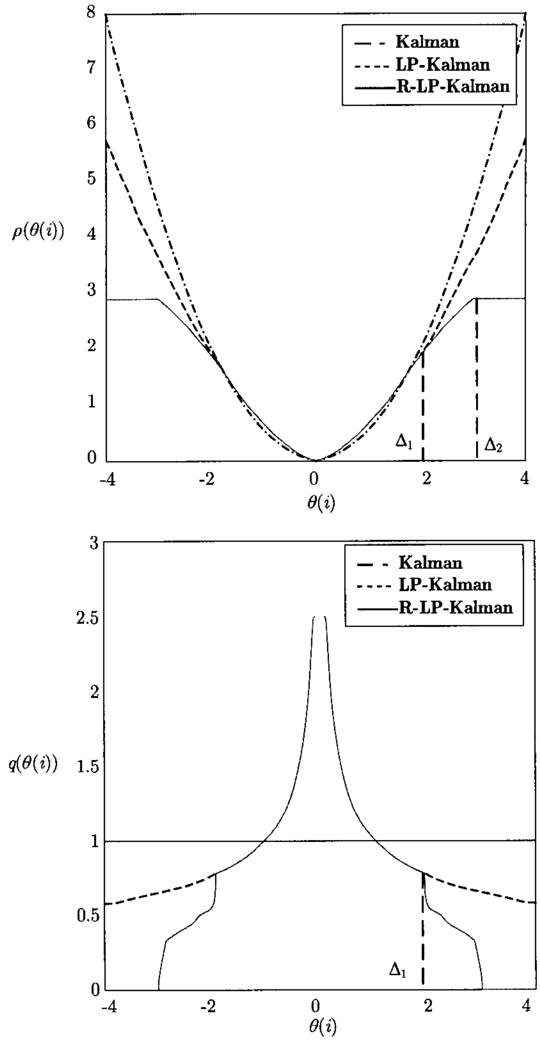

Fig. 2. $\rho(\theta(i))$ and $q(\theta(i))$ for different algorithms.

$$J(n)=\sum_{i=1}^{n}\rho(\theta(i)) \tag{11}$$

where

$$\rho(\theta(i))=\begin{cases}\frac{1}{p}|\theta(i)|^p, & |\theta(i)|\le\Delta_1\\ \frac{2}{p}\Delta_1|\theta(i)|^{p-1}-\frac{1}{p}\Delta_1^p, & \Delta_1\le|\theta(i)|\le\Delta_2\\ \frac{2}{p}\Delta_1\Delta_2^{p-1}-\frac{1}{p}\Delta_1^p, & \Delta_2\le|\theta(i)|\end{cases}$$

and $1<p<\alpha$, $\triangle_1$, and $\triangle_2$ are threshold parameters, thus we can get proposed modified robust LP-Kalman (R-LP-Kalman) algorithm

$$\begin{aligned}\hat{\mathbf{s}}(n)&=\hat{\mathbf{s}}(n-1)+\{(n)-\mathbf{C}(n)\}\\ &\quad\times\frac{q(\theta(n))\mathbf{R}^{-1}(n-1)\mathbf{C}^T(n)}{1+q(\theta(n))\mathbf{C}(n)\mathbf{R}^{-1}(n-1)\mathbf{C}^T(n)}\end{aligned} \tag{12}$$

$$\begin{aligned}\mathbf{R}^{-1}(n)=\mathbf{R}^{-1}(n-1)-&\frac{q(\theta(n))\mathbf{R}^{-1}(n-1)\mathbf{C}^T(n)}{1+q(\theta(n))\mathbf{C}(n)\mathbf{R}^{-1}(n-1)\mathbf{C}^T(n)}\\ &\times\mathbf{C}(n)\mathbf{R}^{-1}(n-1)\end{aligned} \tag{13}$$

and

$$q(\theta(i)) = \frac{\partial \rho(\theta(i))}{\partial \theta(i)} \frac{1}{\theta(i)} = \begin{cases} |\theta(i)|^{p-2}, & |\theta(i)| \leq \triangle_1 \\ \frac{2(p-1)}{p} \triangle_1 |\theta(i)|^{p-3}, & \triangle_1 \leq |\theta(i)| \leq \triangle_2 \\ 0, & \triangle_2 \leq |\theta(i)|. \end{cases} \quad (14)$$

Fig. 2 shows function $\rho(\theta(i))$ and $q(\theta(i))$ for different algorithms. Threshold parameters $\triangle_1$ and $\triangle_2$ affect the performances of R-LP-Kalman algorithm. For non-Gaussian alpha stable random process with location $\xi = 0$ and dispersion parameter γ, and when T is great, the following equation holds [3]

$$Prob\{|\theta(n) > T|\} = \gamma \left\{ \int_0^\infty x^{-\alpha} \sin(x) dx \right\}^{-1} T^{-\alpha}. \quad (15)$$

It is well known that, for given $\theta_{\triangle_1}$ and $\theta_{\triangle_2}$, the probability of $|\theta(n)| \in [\triangle_1, \triangle_2]$ and $|\theta(n)| \in [\triangle_1, \infty)$ are $1-\triangle_1$ and $1-\triangle_2$, respectively. Thus, we can obtain $\triangle_1$ and $\triangle_2$.

To discuss the performances of R-LP-Kalman algorithm, we assume that as follows

1) $S\alpha SG$ noise vector $\mathbf{v}_1(n)$ and scalar $v_2(n)$ $(1 < \alpha \leq 2)$ have zero mean, i.e., $E\{\mathbf{v}_1(n)\} = \mathbf{0}$, $\{v_2(n)\} = 0$
2) $\mathbf{C}(n)$, $\mathbf{v}_1(n)$, $v_2(n)$ and $\mathbf{s}(n)$ are statistically independent.

For innovation process,

$$\theta(n) = y(n) - \mathbf{C}(n)\hat{\mathbf{s}}(n) = \mathbf{C}(n)[\mathbf{s}(n) - \hat{\mathbf{s}}(n)] + v_2(n). \quad (16)$$

We have

$$E\{\theta(n)\} = E\{\mathbf{C}(n)[\mathbf{s}(n) - \hat{\mathbf{s}}(n)]\} + E\{v_2(n)\}. \quad (17)$$

For the sake of $1 < \alpha \leq 2$, it holds that $E\{\theta(n)\} = E\{\mathbf{C}(n)[\mathbf{s}(n) - \hat{\mathbf{s}}(n)]\} < \infty$. For the same reason, for innovation process

$$b(n) = y(n) - \mathbf{C}(n)\hat{\mathbf{s}}(n-1). \quad (18)$$

We have

$$\begin{aligned} E\{b(n)\} &= E\{\mathbf{C}(n)[\mathbf{s}(n) - \hat{\mathbf{s}}(n-1)]\} + E\{v_2(n)\} \\ &= E\{\mathbf{C}(n)[\mathbf{s}(n) - \hat{\mathbf{s}}(n-1)]\} < \infty. \end{aligned} \quad (19)$$

From

$$\mathbf{R}(n) = \sum_{i=1}^{n} q(\theta(i))\mathbf{C}^T(i)\mathbf{C}(i),$$

$$\mathbf{P}(n) = \sum_{i=1}^{n} q(\theta(i))y(i)\mathbf{C}^T(i),$$

and

$$\mathbf{R}(n)\hat{\mathbf{s}}(n) = \mathbf{P}(n),$$

we can get

$$\begin{aligned} \hat{\mathbf{s}}(n) &= \left(\sum_{i=1}^{n} q(\theta(i))\mathbf{C}^T(i)\mathbf{C}(i) \right)^{-1} \\ &\quad \times \sum_{i=1}^{n} q(\theta(i))\mathbf{C}^T(i) \left(\mathbf{C}(i)\mathbf{s}(n) + v_2(i) \right) \\ &= \left(\sum_{i=1}^{n} q(\theta(i))\mathbf{C}^T(i)\mathbf{C}(i) \right)^{-1} \\ &\quad \times \left\{ \sum_{i=1}^{n} q(\theta(i))\mathbf{C}^T(i)\mathbf{C}(i)\mathbf{s}(n) + \sum_{i=1}^{n} q(\theta(i))\mathbf{C}^T(i)v_2(i) \right\} \\ &= \mathbf{s}(n) + \left(\sum_{i=1}^{n} q(\theta(i))\mathbf{C}^T(i)\mathbf{C}(i) \right)^{-1} \\ &\quad \times \sum_{i=1}^{n} q(\theta(i))\mathbf{C}^T(i)v_2(i). \end{aligned} \quad (20)$$

If n is sufficiently great, it holds [16] that

$$E\{\mathbf{C}^T(i)\mathbf{C}(i)\} \approx \frac{1}{n} \sum_{i=1}^{n} \mathbf{C}^T(i)\mathbf{C}(i). \quad (21)$$

$\sum_{i=1}^{n} q(\theta(i))\mathbf{C}^T(i)\mathbf{C}(i)$ can be expressed approximately by

$$n \cdot E\left\{ \sum_{i=1}^{n} q(\theta(i))\mathbf{C}^T(i)\mathbf{C}(i) \right\},$$

and we have

$$\begin{aligned} &E\{\hat{\mathbf{s}}(n)\} \\ &\approx \mathbf{s}(n) + E\left\{ \left(n \cdot q(\theta(i))\mathbf{C}^T(i)\mathbf{C}(i) \right)^{-1} n \cdot q(\theta(i))\mathbf{C}^T(i)v_2(i) \right\} \\ &= \mathbf{s}(n) + E\left\{ [\mathbf{C}(i)]^{-1} v_2(i) \right\} = \mathbf{s}(n). \end{aligned} \quad (22)$$

V. MULTIUSER DETECTION BASED ON R-LP-KALMAN FILTERING

During the past two decades, there has been substantial progress in the development of multiuser detection techniques [17] for enhancing the performance of direct-sequence code-division multiple-access (DS-CDMA) systems. To date, a commonly made hypothesis in the pursuit of multiuser detection has been the Gaussian noise assumption, despite the presence of impulsive noise in many realistic channels [18]. As a result, research in robust multiuser detection has received little attention until recently [19]. In this paper, the impulsive noise is modeled by the class of non-Gaussian stable random variables, which do not include a Gaussian component. It should be noted that the absence of a Gaussian component from the stable model is solely made to utilize the recently proposed geometric signal-to-noise ratio (GSNR) [20], [21], and, therefore, does not reflect realistic channel conditions for all communication systems.

Consider the following discrete-time synchronous CDMA signal model. For a coherent synchronous CDMA system employing BPSK signaling with periodic spreading waveforms, the received signal is first filtered by a chip-matched filter and then sampled at the chip rate $1/T_c$. The discrete-time signal corresponding to i-th symbol is a superposition of K-user signals, plus the ambient noise, given by [22]

$$\begin{aligned} z(i) &= \sum_{k=1}^{K} A_k \cdot b_k(i) \cdot s_k(i) + v(i) \\ &= [A_1, \cdots, A_K] \cdot \mathrm{diag}[b_1(i), \cdots, b_K(i)] \cdot \\ &\quad [s_1(i), \cdots, s_K(i)]^T + v(i) \end{aligned} \tag{23}$$

where K is the number of active users, $v(i)$ is $S\alpha SG$ distribution channel noise, and for the k-th user, A_k, $b_k(i)$, and $s_k(i)$ denote the received amplitude and i-th information bit, normalized signature sequence, respectively. A_k, $b_k(i) \in \{-1, +1\}$ and $s_k(i)$ has unit power ($\sum_{i=1}^{n} |s_k(i)|^2 = 1$). Multiuser detection focuses on estimation of information bit sequences $\mathbf{B}_d = [b_d(1), b_d(2), \cdots, b_d(n)]$ employing received signals vector $\mathbf{Z}(n) = [z(1), z(2), \cdots, z(n)]$ and characteristic waveform $\mathbf{S}_d = [s_d(1), s_d(2), \cdots, s_d(n)]$ of desired user.

For desired user with detector $\mathbf{M}_d$, output is $\mathbf{Z}(n)\mathbf{M}_d(n)$ and estimated information bit is $\hat{b}_d(n) = \mathrm{sign}(\mathbf{Z}(n)\mathbf{M}_d(n))$.

Thus, $\mathbf{M}_d$ can typically be expressed as [23]

$$\mathbf{M}_d = \mathbf{M}_0 - \mathbf{M}_a(n) \tag{24}$$

where $\mathbf{M}_0 = \mathbf{S}_d^T(n)$ denotes fixed part and $\mathbf{M}_a$ denotes adaptive part. For a time invariant CDMA system, its optimal detector $\mathbf{M}_{d-opt}(n)$ is time invariant, too. Let $\mathbf{M}_{a-opt}(n)$ be adaptive part of $\mathbf{M}_{d-opt}(n)$. Thus, we can obtain the following state equation of $\mathbf{M}_{a-opt}(n)$

$$\mathbf{M}_{a-opt}(n+1) = \mathbf{M}_{a-opt}(n). \tag{25}$$

For a time slowly variant system, according to [14], its optimal filter coefficient obeys first order Markov process. For a time slowly variant CDMA system, its multiuser detector satisfies the following state equation

$$\mathbf{M}_{a-opt}(n+1) = u \cdot \mathbf{M}_{a-opt}(n) + \mathbf{v}_1(n)$$

where u is close to 1.

Let us define error $e(n) = \mathbf{Z}(n)\mathbf{M}_d(n) = \mathbf{Z}(n)\mathbf{M}_0 - \mathbf{Z}(n)\mathbf{M}_a(n)$ and observation value $y(n) = \mathbf{Z}(n)\mathbf{M}_0$. When $\mathbf{M}_a(n)$ is close to $\mathbf{M}_{a-opt}(n)$, we have observation equation

$$y(n) = \mathbf{Z}(n)\mathbf{M}_{a-opt}(n) + e_{opt}(n) \tag{26}$$

(25) and (26) constitute dynamic state equations of desired user detector. Using above R-LP-Kalman algorithm, blind multiuser detection can been carried out as follows

$$\begin{aligned} \hat{\mathbf{M}}_{a-opt}(n) =& \hat{\mathbf{M}}_{a-opt}(n-1) \\ &+ \left\{ y(n) - \mathbf{Z}(n)\hat{\mathbf{M}}_{a-opt}(n-1) \right\} \\ &\times \frac{q(\theta(n))\mathbf{R}^{-1}(n-1)\mathbf{Z}^T(n)}{1 + q(\theta(n))\mathbf{Z}(n)\mathbf{R}^{-1}(n-1)\mathbf{Z}^T(n)} \end{aligned} \tag{27}$$

where

$$\begin{aligned} \mathbf{R}^{-1}(n) =& \mathbf{R}^{-1}(n-1) \\ &- \frac{q(\theta(n))\mathbf{R}^{-1}(n-1)\mathbf{Z}^T(n)}{1 + q(\theta(n))\mathbf{Z}(n)\mathbf{R}^{-1}(n-1)\mathbf{Z}^T(n)}\mathbf{Z}(n)\mathbf{R}^{-1}(n-1). \end{aligned} \tag{28}$$

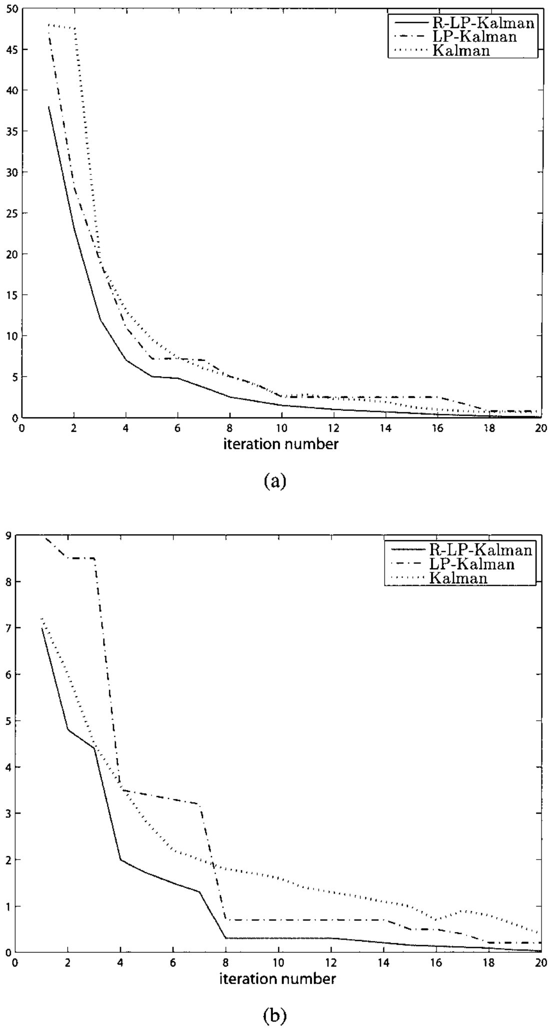

Fig. 3. Error vector norm of $\mathbf{M}_{a-opt}$ as a function of iteration number: (a) $\alpha = 1.8$, $K = 5$, (b) $\alpha = 1.2$, $K = 3$.

VI. EXPERIMENTAL RESULTS

For simulations, we assume a synchronous CDMA system with a processing gain 15, number of users K, the received amplitude $A_k = 1$, $k = 1, 2, \cdots, K$. Channel noise $v(i)$ follows $S\alpha SG$ distribution. User 1 signature sequence is generated randomly and kept fixed throughout simulations. Signature sequences of Users 2 through K are generated by circularly shifting the sequence of User 1. The maximum number of iterations required for the iterative algorithms was constrained to 20 in all simulations. Given an initial value of detector, we can get error vector norm of $\mathbf{M}_{a-opt}(n)$ as a function of iteration number depicted in Fig. 3 for R-LP-Kalman, LP-Kalman, Kalman algorithm. Here, detector based on R-LP-Kalman algorithm has comparable better convergence performance.

Although variance has been accepted in research community as a standard measure associated with defining power, its meaning is not universal and may be misleading when the processes exhibit heavy tails. Since $S\alpha S$ noise has infinite variance, standard SNR based on second-order statistics becomes inconsistent. Several attempts have been made to introduce alternative strength measures. Among them, GSNR [20], based on the the-

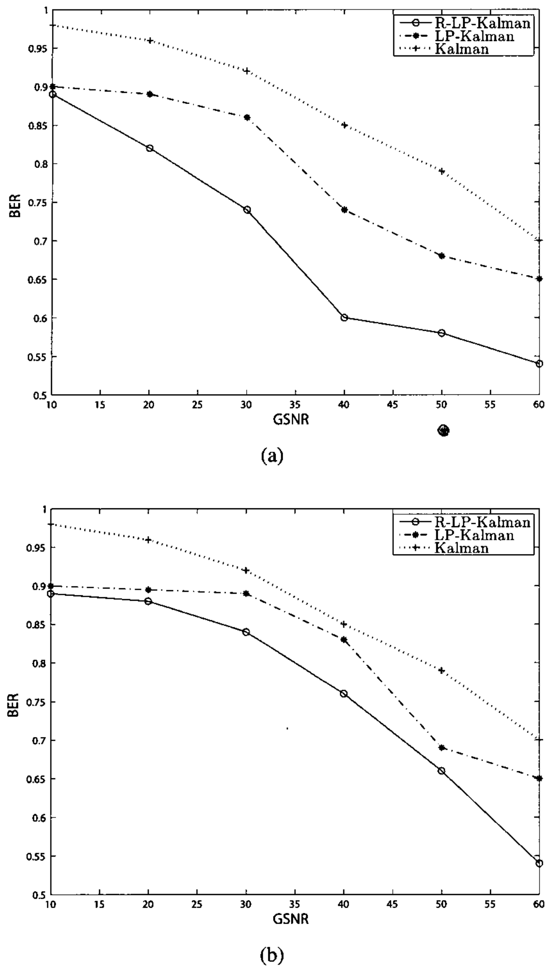

Fig. 4. Bit error rate (BER) of information bit as function of GSNR: (a) $\alpha = 1.8$, $K = 5$, (b) $\alpha = 1.2$, $K = 3$.

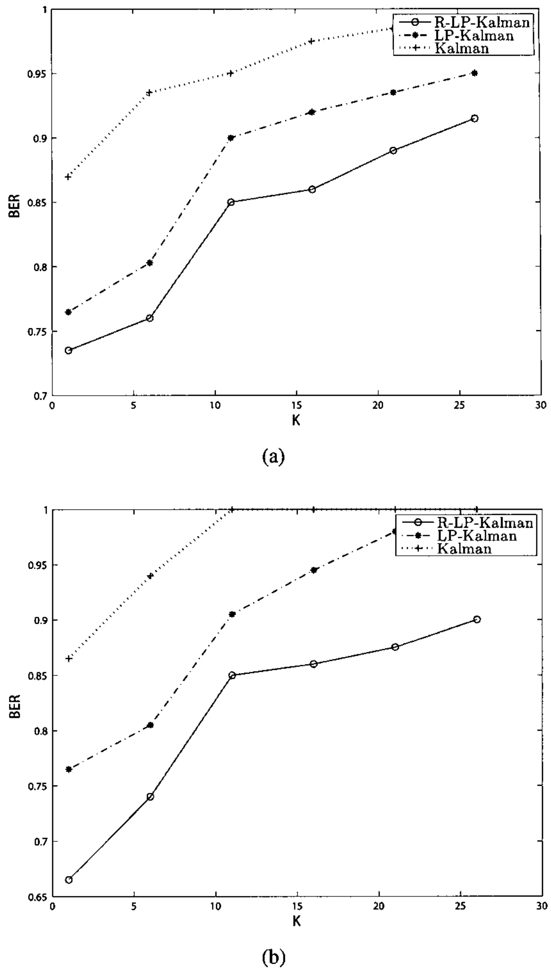

Fig. 5. Bit error rate (BER) of information bit as function of K: (a) $\alpha = 1.8$ GSNR=20 dB, (b) $\alpha = 1.2$, GSNR=20 dB.

ory of zero-order statistics (ZOS) [21], is evidently the first approach toward a mathematically and conceptually valid characterization of the relative strength between information-bearing signal and channel noise with infinite variance. The use of stable random processes and the GSNR for the study of linear decorrelator in impulsive noise has recently been reported in [24]. The *geometric power* of an $S\alpha S$variable is defined in [21] as follows

$$S_0 = (C_g \gamma)^{1/\alpha} / C_g \tag{29}$$

where $C_g \approx 1.78$, is the exponential of the Euler constant. The GSNR per symbol, is given as

$$\text{GSNR} = \frac{1}{2C_g} \left(\frac{A_k}{S_0} \right)^2 \tag{30}$$

where A_k is described in (23). Given $n = 40$ and different GSNR, we can get bit error rate (BER) of information bit $\hat{b}_d(n) = \text{sign}(\mathbf{Z}(n)\mathbf{M}_d(n))$ as a function of GSNR depicted in Fig. 4 for R-LP-Kalman, LP-Kalman, Kalman algorithm. Here, we can see that detector based on R-LP-Kalman algorithm has comparable smaller BER.

Since BER of information bit $\hat{b}_d(n) = \text{sign}(\mathbf{Z}(n)\mathbf{M}_d(n))$ can be affected by interference users and their number, given $n = 40$ and GSNR $= 20$ dB and different the number of users K, we can get bit error rate of information bit $\hat{b}_d(n) = \text{sign}(\mathbf{Z}(n)\mathbf{M}_d(n))$ as a function of K depicted in Fig. 5 for R-LP-Kalman, LP-Kalman, Kalman algorithm. We can see that detector based on R-LP-Kalman algorithm has comparable smaller BER.

VII. CONCLUSION

Alpha stable distribution is better for modeling impulsive noises than Gaussian distribution in signal processing. This class of process has no close form of probability density function and finite second order moments. In general, Wiener filter theory is not meaningful in $S\alpha SG$ environments because the expectations may be unbounded. We proposed a new adaptive generalized recursive least p-norm Kalman filtering algorithm based on innovation process with infinite variances, and improved and analyzed its robustness and performances, and a new robust multiuser detection method based on least p-norm Kalman filtering is also proposed. The simulation experiments show that the proposed new algorithm is more robust than the conventional Kalman filtering algorithm. Detector based on R-LP-Kalman algorithm has comparable better convergence performance, smaller BER.

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