Journal of Korean Society of
Coastal and Ocean Engineers
Vol. 19, No. 3, pp. 222~227, June 2007

Nonlinear Focusing Wave Group on Current
흐름의 영향을 받는 파랑 그룹의 비선형 집중

Julien Touboul*, Efim Pelinovsky** and Christian Kharif*
쥬리언 투보울* · 에핌 페리높스키** · 크리스티안 카리프*

Abstract : Formation of freak waves is studied in deep water from transient wave packets propagating on current. Those waves are obtained by means of dispersive focusing. This process is investigated by solving both linear and nonlinear equations. The role of nonlinearity is emphasized in this interaction.

Keywords : dispersive focusing, wave-current interaction, kinematic model, nonlinear equations.

요 지 : 심해에서 생성된 최극해파가 파랑과 상호작용하는 현상에 대한 연구를 수행하였다. 이러한 파랑은 분산집중을 이용하여 산정하였다. 이러한 과정은 선형 및 비선형 방정식의 해를 구하여 얻을 수 있다. 상호작용에서 비선형성의 역할을 강조하였다.

핵심용어 : 분산집중, 파랑-흐름 상호작용, 운동학적 모형, 비선형방정식

1. Introduction

Freak, rogue or giant waves are extreme events. They are characterized by their unpredictability, which explains that they are known as ”waves from nowhere”. As a matter of fact, they are responsible for an important number of large damages, caused to ships or offshore rigs. Over the last twenty years, a large number of events has been reported, and a lot of ship disappearances have been correlated to rogue waves events. Up to now there is no definite consensus about a unique definition of freak wave. The most popular definition is the amplitude criterion: the height of a freak wave should exceed twice the significant height of the background wave field. Several mechanisms have been suggested to explain the formation of freak waves, such as spatio-temporal focusing (Kharif et al., 2001; Johannessen and Swan, 2003; Gibson and Swan), nonlinear or modulational instability (Benjamin-Feir instability) (Benjamin and Feir, 1967; Dyachenko and Zakharov, 2005), envelope soliton and breather interactions (Clamond and Grue, 2002). Those mechanisms have been reviewed by Kharif and Pelinovsky in (2003) and by Dysthe in (2001).

Wave-current interaction contributes also in the freak wave formation and historically, this mechanism was the rst to explain the origin of freak waves (Peregrine, 1976; Smith, 1976). In fact, the vertical shear of the oceanic current is important for short wind waves with length shorter than a few meters (Craik, 1985; Thomas, 1981; Thomas, 1990; Silva and Peregrine, 1988; Shrira and Sazonov, 2001), but can be ignored for typical gravity waves. The horizontal variability of the oceanic current is relatively high (for instance in the vicinity of the Agulhas current off South Africa) and leads to the strong spatial (geometrical) focusing of the swell (Lavrenov, 1998; White and Fornberg, 1998). Meanwhile the current (even uniform) can influence the temporal focusing of wind waves. Recently Wu and Yao (2004) studied the last problem experimentally. They observed a shift of the focusing point, and by analyzing the shape of limiting freak waves, and their spectral evolution, they concluded that the nonlinearity of freak waves is affected by

*Institut de Recherche sur les Phénoménes Hors Equilibre (IRPHE), Technopole de Chateau-Gombert, 49 rue Joliot Curie, BP146, 13384 Marseille cedex 13, France
**Department of Nonlinear Geophysical Processes, Institute of Applied Physics, 46 Uljanov Street, Nizhny Novgorod, 603950 Russia

the presence of current. Indeed, the role of nonlinearity on uniform wave trains propagating on currents has been investigated numerically in by Ryu et al. (2003).

This work presents a series of numerical simulations of a focusing wave train propagated with and without current in infinite depth. The current is assumed to be constant in space. The wave packet with linear frequency modulation is generated at a fixed point. Herein we emphasize the problem we are dealing with is a boundary value problem (BVP) and not an initial value problem (IVP). In this particular case, the Galileo transformation is not used directly. Firstly (section 2), we demonstrate the shifting of focal points in the framework of kinematic approach. Then, the fully nonlinear numerical method is briey introduced (section 3). Numerical results of freak wave formation on the current obtained in the framework of the fully nonlinear model are presented and compared to results given by linear theory, including the kinematic model. A special interest is taken to analyze the evolution of the spectral components present in the focusing wave group.

2. The Kinematic Model

Due to the dispersive behaviour of water waves, when short waves propagate in front of longer waves, they will be overtaken, and large amplitude wave can occur at a fixed point. A linear approach of the problem would lead to consider sea surface as a superposition of linear waves of frequencies $\omega(x, t)$. Following Whitham (1974) and Brown (2001), the spatio-temporal evolution of these components is governed by the following hyperbolic equation

$$\frac{\partial \omega}{\partial t}+c_g(\omega)\frac{\partial \omega}{\partial x}=0 \tag{1}$$

where c_g is the group velocity. The boundary value problem for this equation can be solved by using the method of characteristics. Its solution is

$$\omega(x,t)=\omega_0(\tau),\ \tau=t-\frac{x}{c_g(\omega)} \tag{2}$$

where ω_0 corresponds to the temporal frequency distribution of the wave train at $x = 0$. By differentiating the frequency, it comes

$$\frac{\partial \omega}{\partial t}=\frac{d\omega_0/d\tau}{1+(2x/g)d\omega_0/d\tau} \tag{3}$$

and one can notice that the case $d\omega_0/d\tau < 0$, which corresponds to the case of short waves emitted before longer waves, leads to a singularity. This singularity corresponds to the focusing of several waves at $t = T_f$ and $x = X_f$. For infinite depth, the group velocity of each components is given by $c_g(\omega) = g/2\omega$. As a matter of fact, the frequency to impose to a wave maker located at $x = 0$, and for $0 < t < T$ is given by

$$\omega(0,t)=\frac{g}{2}\frac{T_f-t}{X_f} \tag{4}$$

where g is the acceleration due to gravity. This frequency modulation, varying linearly from ω_{min} to ω_{max}, provides the optimal focusing of the linear wave packets in still water and is very often applied in the laboratory conditions. Components following this law will all merge at the same place X_f, and the same time T_f, coordinates of the focusing point in the (x - t) plane, given by

$$X_f=\frac{gT}{2}\frac{1}{\omega_{\max}-\omega_{\min}},\ T_f=\frac{2\omega_{\max}}{g}X_f \tag{5}$$

In presence of current, equation (1) should be modified, to take care of the Doppler effect. It rewrites

$$\frac{\partial \omega}{\partial t}+[c_g(\omega)+U]\frac{\partial \omega}{\partial x}=0 \tag{6}$$

where U is the current velocity. The solution, previously given by (2), becomes

$$\omega(x,t)=\omega_0(\tau),\ \tau=t-\frac{x}{c_g(\omega)+U} \tag{7}$$

By differentiating the frequency, it comes

$$\frac{\partial \omega}{\partial t}=\frac{d\omega_0/d\tau}{1+\frac{2xg}{(g+2\omega U)^2}\frac{d\omega_0}{d\tau}} \tag{8}$$

One can notice that the dynamics of the wave group is more complicated. The denominator is now a function of time, and is equal to zero for several values of space and time. The waves do not merge at the same place, at the same time. The focusing point is theoretically spread to a focusing area, extending from L_{min} to L_{max}, where

$$L_{\max}=X_f\left(1+\frac{2U\omega_{\max}}{g}\right)^2,\ L_{inx}=X_f\left(1+\frac{2U\omega_{\min}}{g}\right)^2 \tag{9}$$

The kinematic model presented above has some limitations. It demonstrates the shifting of the focusing point, but cannot predict wave amplitudes which are formally infinite at this point. To avoid this problem, a fully linear approach is used based on the Fourier integral. It is based on the linear dispersion relation in infinite depth, in presence of current, that reads

$$(\omega - kU)^2 = gk \tag{10}$$

where $k(\omega)$ is the wave number. This equation is solved iteratively, by means of Newton's method. Solutions preserving the sign of $(kx - \omega t)$ are considered. By introducing this dispersion relation into linear equations, it comes

$$\eta(x,t) = \frac{1}{2\pi}\left[\int_{-\infty}^{+\infty} \eta(0,\tau)\exp(i\omega\tau)d\tau\right]\exp[i(kx-\omega t)]d\omega \tag{11}$$

that can be solved at any place and any time by knowing $\eta(0, t)$.

3. Mathematical Formulation of the Fully Nonlinear Problem

The problem is solved numerically by assuming valid the potential theory. Hence, ϕ, the velocity potential, satises the Laplace's equation $\Delta\phi = 0$. By introducing the decomposition

$$\phi(x,z,t) = Ux + \phi(x,z,t) \tag{12}$$

where Ux is the potential due to the presence of current, and φ can be understood as a perturbed potential, one can notice that φ is also solution of the Laplace's equation $\Delta\varphi = 0$. Classical free surface conditions, respectively kinematic and dynamic, reads

$$\frac{\partial\eta}{\partial t} = \frac{\partial\varphi}{\partial z} - \frac{\partial\varphi}{\partial x}\frac{\partial\eta}{\partial x} - U\frac{\partial\eta}{\partial x}$$

$$\frac{\partial\varphi}{\partial x} = \frac{(\nabla\varphi)^2}{2} - U\frac{\partial\varphi}{\partial x} - g\eta \tag{13}$$

The solution of the Laplace's equation for j, with the above boundary conditions can be obtained by using Green's second identity. A mixed Euler-Lagrange description of the problem is adopted, meaning that a particular description of the surface is used. More details can be found in Brown (2006). This method has been checked by comparison with numerical simulations by Zhu and Zhang (1997), and a good agreement has been found.

4. Results and Discussion

Numerical simulations presented here show the interaction of the focusing wave packet and current. The focusing wave group has a frequency varying from 1.3 to 0.7 Hz. It is propagated numerically with, and without a current varying from U/c_g = -0.25 to U/c_g = + 0.25. If the mean steepness, during simulations, is of order $\varepsilon \sim 0.1$, it can locally reach 0.35 for large wave events corresponding to an important nonlinearity.

Figure 1 presents the free surface obtained at each focusing point by solving the nonlinear equations, without current, and with current velocities of U/c_g = - 0.125 and U/c_g = + 0.125. Here c_g is the mean group velocity of the frequency modulated wave group. The focusing point is defined as the location where maximum elevation is reached. Differences appear between those profiles. The group propagated with a co-current focuses further (and later) than the one propagated freely, while its elevation is lower. On the other hand, the group propagated in a counter-current focuses closer (and faster), and its amplitude is larger than the amplitude of the group propagated freely. Variation of freak wave amplitude is of order 10%. The evolution of the focusing

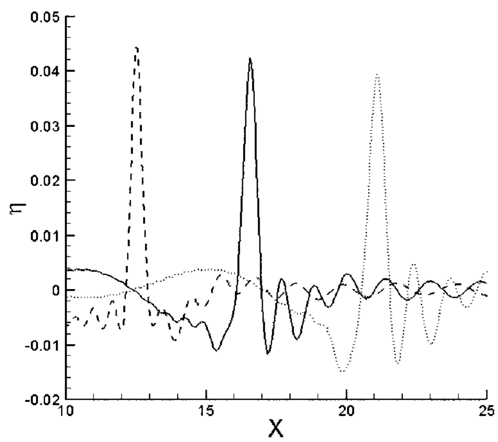

Fig. 1. Free surface elevation at each focusing time for a group propagated freely (solid line) with counter-current of velocity U/c_g = -0.125 (dashed line), and with co-current of velocity U/c_g = +0.125 (dotted line).

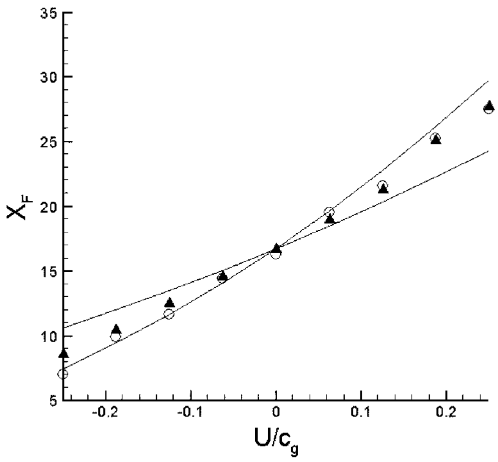

Fig. 2. Numerical focusing point as a function of the current velocity, for both linear (circles) and nonlinear models (triangles), plotted together with the theoretical extreme values of the focusing area L_{min} and L_{max} from the kinematic model.

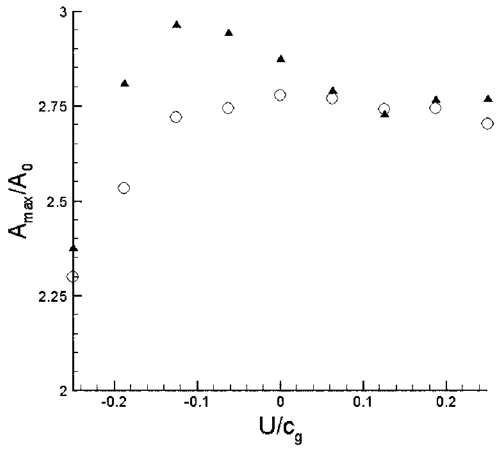

Fig. 3. Amplitude at the focal point as a function of the current velocity, for both linear (circles) and nonlinear models (triangles).

point corresponds to linear prediction. Indeed, one can notice from equation (9) that linear theory predicts an area spreading closer than X_f for counter-stream, and further for co-stream. On the other hand, results concerning wave amplitudes are surprising. Actually, linear theory predicts a spreading of the focusing area in both counter and co-stream, which should result in a decay of the wave amplitude in both cases. Quantitative comparisons can be obtained with linear approach presented above.

Figure 2 presents the location of the focusing point obtained from both linear and nonlinear methods, plotted as a function of current velocity. Theoretical values obtained from the kinematic model also appear. One can notice a very good agreement between linear and nonlinear methods. Focusing appears in an area spreading from $L_{\max}$ to $L_{\min}$, as predicted by the kinematic model. Very weak differences can be observed between linear and nonlinear methods, especially in strong countercurrents, where the waves become steeper. This shows that from a kinematic point of view, the linear representation of the problem is a relevant approach.

Figure 3 shows the maximum elevation reached at the focusing point as a function of the current velocity, for both linear and nonlinear models. Differences are more important here. For the linear model, the maximum is reached without any current. This can be understood by considering that the focal point turns into a focusing area in both cases of counter and co-current. The energy is spread, and the resulting wave is lower. The influence of current, through Doppler effect, leads to non-optimal focusing, as it was shown in the framework of the kinematic model. Dispersion parameter, $d^2k/d\omega^2$ increases for counter-current more than for co-current, which results in a decay of the focal amplitude for counter-current. For the nonlinear case, the observation is different. If the behavior in co-currents is very similar to the behavior presented by the kinematic model, the evolution in counter-currents is different. These differences have a pronounced maximum in a counter-current of $U/c_g = -0.125$. The amplification of the wave amplitude on counter-current is due to two effects. Firstly, wave dispersion (deviation from optimal focusing) is decreased in counter-currents when nonlinearity is taken into account, as it is emphasized later. This mechanism tends to keep coherence of components, and maintain focusing on counter-currents. Secondly, the role of the modulational (Benjamin-Feir) instability increases while the wave steepness is enlarged in counter-currents. Thus, the nonlinear parameter *ak* is increased, and the growth rate of modulational instability is also increased. This mechanism also tends to increase wave amplitude on focusing point. Development of this mdoulational instability in frequency modulated wave groups propagated has been observed in by Brown and Jensen (2001). One can notice that this phenomenon should disappear for two dimensional waves propagating in finite depth,

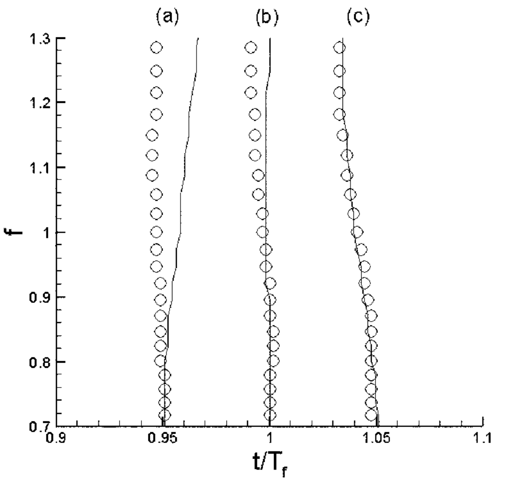

Fig. 4. Location of the components in the $(t\text{-}f)$ plane at the focusing point, from both linear (solid line) and nonlinear models (circles). (a): $U/c_g = -0.125$, (b): $U/c_g = 0$, and (c): $U/c_g = +0.125$. For sake of clarity, curves are artificially shifted from $t/T = \pm 0.05$.

since modulational instability is not relevant in this case.

The time frequency representation of the focused wave group, obtained by means of wavelet analysis, is presented on Figure 4 for freak waves obtained on several current velocities U/c_g. A perfect focusing should be represented as a vertical straight line. Results obtained with the linear model are plotted together with results of the fully nonlinear simulations. As expected, the agreement is excellent for a freak wave obtained on a co-stream (c). It emphasizes the linear behavior of the waves during the focusing process in a co-stream. It can be understood by the low steepness of waves involved in the process. In both linear and nonlinear simulations, the focusing obtained is not perfect, illustrating the spread area of focusing. Weak differences appear for the case without current. Those differences concern high frequencies, corresponding to the most nonlinear waves. Components are almost perfectly focused in the $(t\text{-}f)$ plane, explaining the high amplitude reached for that current velocity. As mentioned earlier, the differences between nonlinear and linear simulations are larger in the case of $U/c_g = -0.125$. Figure 4 confirms that high frequencies propagate faster in the nonlinear simulation, as predicted by first-order correction of the linear dispersion relation (see Whitham (1974)). Therefore, high frequencies components focus on longer distances in this case, than in the linear one. Nonlinearity seems to balance the linear effect, by accelerating slowest components of the group, and maintaining focusing on larger locations. This explains partially the differences observed in amplitudes between kinematic model and nonlinear one.

5. Conclusion

The interaction between a focusing wave group and current is studied numerically. Two methods are used. In linear theory we use an approximated kinematic approach as well as full linear solution based on the Fourier integral. Another approach is to solve numerically the nonlinear equations. The results obtained for both models are compared. The global kinematic behavior (locations of the focal points) is found to be similar. In terms of maximum amplitude, some differences are obtained. In the linear description, maximal amplitudes of freak waves are obtained without current, while in the nonlinear case, maximal amplitude of freak waves occurs in a weak opposing current. The presence of a maximum of the amplitude curve in linear theory is related with the shift from optimal conditions of linear focusing. In nonlinear theory, waves propagated on counter-current suffer a decay of wavelength resulting in an increase of their steepness. Increasing nonlinearity results in two phenomena. First, the dispersion relation is changed, leading nonlinear waves to propagate faster than the group velocity predicted by the linear dispersion relation. Secondly, the modulational instability can develop. These phenomena result in larger values of the wave amplitude at the focal point.

Acknowledgement

Particularly, this study is supported by INTAS grant (06-1000013-9236), and also for EP - by grants from INTAS (03-51-4286) and RFBR (05-05-64265).

References

Kharif, C., Pelinovsky, E., Talipova, T. and Slunyaev, A. (2001). Focusing of nonlinear wave groups in deep water, *JETP Letters,* vol. 73, No. 4, 170-175.

Johannessen, T.B. and Swan, C. (2003). On the nonlinear dynamics of wave groups produced by the focusing of surface water waves, *Proc. R. Soc. London,* vol. A459, 1021-1052.

Gibson, R.S. and Swan, C. The evolution of large ocean waves: the role of local and rapid spectral changes, submitted to *Proc. Royal Soc. London.*

Benjamin, T. and Feir, J. (1967). The desintegration of wave train on deep water, *J. Fluid Mech.* vol. 27, 417-430.

Dyachenko, A.I. and Zakharov, V.E. (2005). Modulation instability of Stokes waves - freak wave, *JETP Letters,* vol. 81, 6, 255-259.

Clamond, D. and Grue, J. (2002). Interaction between envelope solitons as a model for freak wave formations. Part I: Long time interaction *C. R. Mecanique,* vol. 330, 575-580.

Kharif, C. and Pelinovsky, E. (2003). Physical mechanisms of the rogue wave formation, *European J. Mech. B/Fluids,* vol. 22, 603-634.

Dysthe, K.B. (2001). Modelling a Rogue Wave - Speculations or a realistic possibility?, *Rogue Waves*, Brest, Ed. by M. Olagnon, and G.A. Athanassoulis, 255-264.

Peregrine, D.H. (1976). Interaction of water waves and currents, *Adv. Appl. Mech.,* vol. 16, 9-117.

Smith, R. (1976). Giant waves, *J. Fluid Mech.* vol. 77, 417-131.

Craik, A.D.D. (1985). Wave interactions and fluid flows, *Cambridge University Press.*

Thomas, G.P. (1981). Wave-current interactions: an experimental and numerical study. Part I: linear waves, *J. Fluid Mech.*, vol. 110, 457-474.

Thomas, G.P. (1990). Wave-current interactions: an experimental and numerical study. Part II: nonlinear waves, *J. Fluid Mech.*, vol. 216, 505-536.

Teles da Silva, A.F. and Peregrine, D.H. (1988). Steep, steady surface waves on water of finite depth with constant vorticity, *J. Fluid Mech.*, vol. 195, 281-302.

Shrira, V.I. and Sazonov, I.A. (2001). Quasi-modes in boundary-layer-type flows. Part 1. Inviscid two-dimensional spatially harmonic perturbations, *J. Fluid Mech.*, vol. 446, 133-171.

Lavrenov, I.V. (1998). The wave energy concentration at the Agulhas current of South Africa, *Natural Hazards,* vol. 17, 117-127.

White, B.S. and Fornberg, B. (1998). On the chance of freak waves at sea, *J. Fluid Mech.,* vol. 355, 113-138.

Wu, C.H. and Yao, A. (2004). Laboratory measurements of limiting freak waves on current, *J. Geophys. Res.,* vol. 109, C12002, doi:10.1029/2004JC002612.

Ryu, S., Kim, M.H. and Lynett, P.J. (2003). Fully nonlinear wave-current interactions and kinematics by a BEM-based numerical wave tank, *Comp. Mech.,* vol. 32, 336346.

Brown, M.G. and Jensen, A. (2001). Experiments on focusing unidirectional water waves, *J. Geophys. Res.*, vol. 108, No. C8, 16917-16928.

Whitham, G.B. (1974). Linear and nonlinear waves, *Wiley.*

Brown, M.G. (2001). Space-time surface gravity wave caustics: structurally stable extreme events, *Wave Motion,* vol. 33, 117-143.

Touboul, J., Giovanangeli, J.P., Kharif, C. and Pelinovsky, E. (2006). Freak waves under the action of wind: experiments and simulations, *European J. Mech. B/Fluids,* vol. 25, 662-676.

Zhu, S. and Zhang, Y. (1997). On nonlinear transient free-surface flows over a bottom obstruction, *Phys. Fluids,* vol. 9, 2598-2604.

Received April 11, 2007
Accepted May 29, 2007