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Measurement of Spatial Coherence Function of Laser Beam by using a Sagnac Interferometer

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The spatial coherence function of a laser beam was measured by using a Sagnac interferometer and self referencing technique. For a laser beam passing through a narrow slit, the absolute value of the measured spatial coherence function becomes more symmetric as the slit size is reduced. For diverging beams, the spatial coherence function shows fast oscillations in its real and imaginary parts. We explain this by using a Gaussian Schell-model. One can use this measurement method to study and characterize the property of the light field coming out of a small sample.

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I. INTRODUCTION

Optical techniques are often used to characterize properties of samples in many areas of science and technology. In particular, a non-destructive optical measurement is often preferred for analysis of organic materials and bio-medical samples. For instance, optical coherence tomography is now intensively used for skin disease diagnostics [1,2]. For revealing the internal structure of a biological sample and obtaining a tomographic image of the sample, which is often a highly scattering media, coherence of light should be taken into account [3].

The two-point spatial coherence function (SCF) is related to the structure of the medium where the light beam propagates and scatters, and it gives rich information about the medium. Several studies have been made recently to measure the coherence function using different approaches. One attractive approach is to measure the spatial coherence function directly using a Sagnac type interferometer [4], without using a separate reference beam. One can measure the spatial Wigner function of an optical field by using a Sagnac interferometer[5], and SCF can be obtained from the measured Wigner function. The Wigner function gives us information about the light field in terms of both space and spatial frequency and thus reveals propagation properties of the light beam in an optical sample under investigation [6]. We found that the SCF measured for the optical field coming out of small slit shows more symmetric distribution of its absolute value as the slit size is reduced, and rapid oscillations occur in its real and imaginary part for a fast diverging beam. Our technique may be used to measure the spatial coherence function of a light beam with unknown characteristics emanating from small optical samples and for optical coherence imaging of samples in real time.

II. SCF MEASUREMENT

The two-point spatial coherence function, $\Gamma(x_1,x_2)$, gives us the information of coherence between two points x_1 and x_2 in space. For propagating light, the SCF obeys the wave equation and the coherence of field can be changed during the propagation [7]. We can obtain SCF by making an inverse Fourier transform of the spatial Wigner distribution function, which is a joint function of both position and propagation vector. The Wigner distribution function gives us information about the wavefront and propagation property [8,9]. We can use a Sagnac interferometer to measure the SCF without the need of stable reference light [10-12]. Phase-space tomography for retrieval of phase information from an optical field has been reported [13], and the direct measurement of a two-point correlation function has been reported also by using a similar technique [4].

A Wigner distribution function of a scalar field, $\phi(r,z)$, $r=(x,y)$, at a plane z=constant can be defined as [14,15]

$$W(r,p)=\int dr' e^{ir'\cdot kp}\phi(r+r'/2)\phi^*(r-r'/2). \qquad (1)$$

Here, $k=2\pi/\lambda$. The p = (p_x, p_y) is called the spatial frequency and it is a canonical conjugate to r. The spatial coherence function of the field $\Gamma(r_1,r_2) = <\phi(r_1)\phi^*(r_2)>$ can be obtained from the relation

$$\phi(r_1)\phi^*(r_2) = \frac{k^2}{4\pi^2}\int dp\, W(\frac{r_1+r_2}{2},p)\exp[-i(r_1-r_2)\cdot kp], \tag{2}$$

after taking the ensemble average. For a strong and stable CW laser light, the product $\phi(r_1)\phi^*(r_2)$ does not change much from one measurement to another, so a single measurement is often enough to obtain the SCF. In the case of an optical field, the scalar field is often a cartesian component of the electric field vector of the light wave, and the Wigner function is a function of the position r and the wave-vector k,

$$W(r,k) = \frac{1}{\pi^2}\int ds E(r+s)E(r-s)^* e^{2is\cdot k}. \tag{3}$$

The experimental set-up to measure the Wigner function is shown in Fig. 1. It is similar to the experimental set-up used in previous studies [4,5], with some improvements. In the experiment, we varied only the x and k_x, that is, $r=x$ and $k=k_x$ here.

We have included a set of lenses in this set-up so that magnified image of the sample (pinhole or slits) could be formed on the mirror M1. The Wigner function of the optical field at the mirror plane is measured in this set-up. A 50:50 beam splitter, BS2, causes the transmitted and the reflected beams to travel in opposite directions in the interferometer. The Dove prism is oriented 45° angle to the plane of the interferometer. When the two beams enter the two ends of the Dove prism, +90° rotation occurs in one beam, and −90° rotation occurs in the other beam, so that the relative 180° rotation required for the Wigner function measurement is achieved. We used a He-Ne laser as a light source and an optical fiber as a spatial filter. The

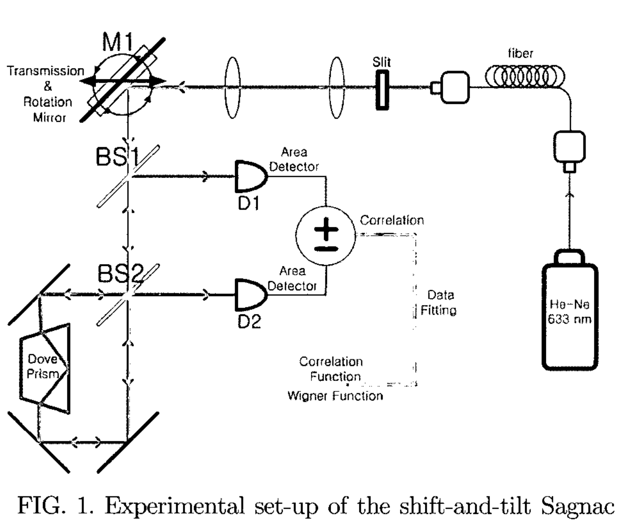

FIG. 1. Experimental set-up of the shift-and-tilt Sagnac interferometer. The fiber acts as a spatial filter.

steering mirror M1 was mounted on a motorized translation-and-rotation combo stage (1 μm/pulse translation, 0.0025°/pulse angular resolution) controlled by a computer. The translation and rotation of the M1 changes $x(r)$ and $k_x(k)$ respectively in the Eq. 3. The detector D2 measures transmitted-transmitted beam (at BS2) interfering with reflected-reflected beam (at BS2), and the detector D1 measures transmitted-reflected (BS2)-reflected (BS1) beam interfering with reflected-transmitted (BS2)-reflected (BS1) beam. The intensity of light measured by the detector D2 is represented by $I_1+I_2+I_{12}$. The I_1 and I_2 are proportional to the integrated intensity of the clockwise and counter-clockwise propagating beams, and they remain constant during the translation and rotation of M1. The third term I_{12} is the integrated interference term that is proportional to the Wigner function :

$$I_{12} = \frac{\pi\eta}{4} W(x,k). \tag{4}$$

Here η is a constant determined by the relative phase between the two interfering beams. Due to the phase relation between the transmission coefficient and reflection coefficient, and because both beam-splitters BS1 and BS2 are 50/50, if the detector D2 output gives $I_{D2} = I_1+I_2+I_{12}$ then the other detector D1 output gives $I_{D1} = (I_1+I_2-I_{12})/2$. Therefore the sum of the two detector photocurrents can give us $(I_{D2}+2I_{D1})/2 = I_1+I_2$, and the difference between the two photocurrents gives us $I_{12} = (I_{D2}-2I_{D1})/2$.

Some examples of the Wigner distribution functions are shown in the Fig. 2. By using the single mode

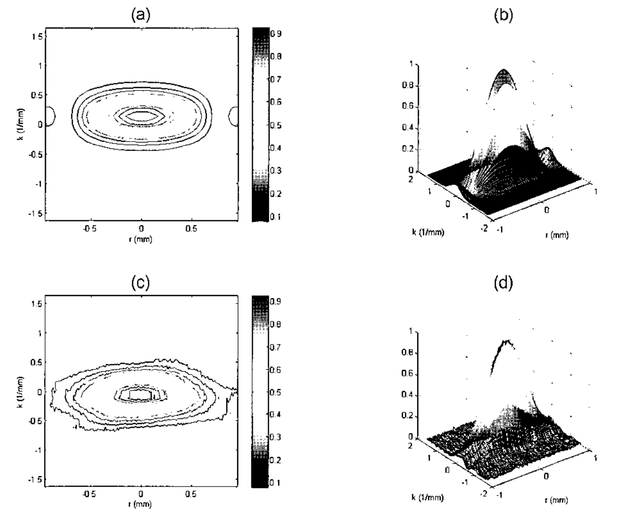

FIG. 2. Plots showing (a) theoretical contour plot of the Wigner function (b) theoretical three dimensional surface plot (c) experimental contour plot (d) experimental surface plot. The plots are made for x and k_x variables.

optical fiber as a spatial filter, the wavefront of the laser beam is found to be in a good Gaussian fundamental mode. The fiber has core diameter of 4um, length 100 m (Model# 630HP, THORLABS).

The spatial two-point coherence function is obtained by inverse Fourier transformation of the measured Wigner function based on the Eq. 2. A computer algorithm was used to generate the position vectors $r_1 = x_1$ and $r_2 = x_2$ that satisfy the relation $r_1 + r_2 = 2r$ and $r_1 - r_2 = r'$ in the Eq. 1. Because we have varied the position x in regular steps in the experiment, we can treat the x as a vector (in MATLAB program) and it is not difficult to generate x_1 and x_2 vectors that satisfy the above relations. The SCF obtained for the laser light is shown in Fig. 3.

After verifying the correct operation of the set-up, we measured the SCF for magnified images of three different slits at the same location, with slit widths 50 μm (Fig. 4), 200 μm (Fig. 5), and 400 μm (Fig. 6).

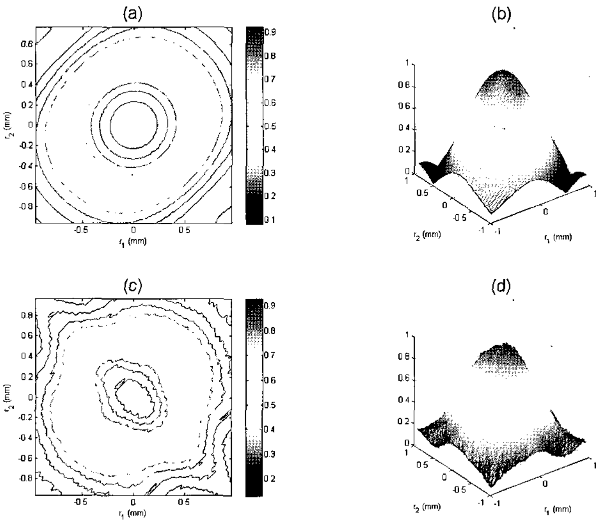

FIG. 3. Experimental data showing the spatial coherence function (SCF) of the laser light after the optical fiber. (a) contour plot of the theoretical SCF (b) three dimensional surface plot (c) experimental contour plot (d) experimental surface plot. All plots are made for x_1 and x_2.

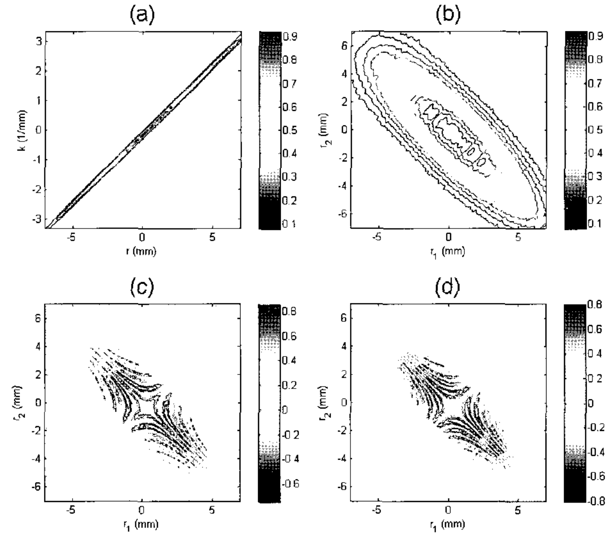

FIG. 4. Experimental data showing the Wigner function (a), absolute value of SCF (b), real part of SCF (c), and imaginary part of SCF (d) for laser light passing through 50 μm slit.

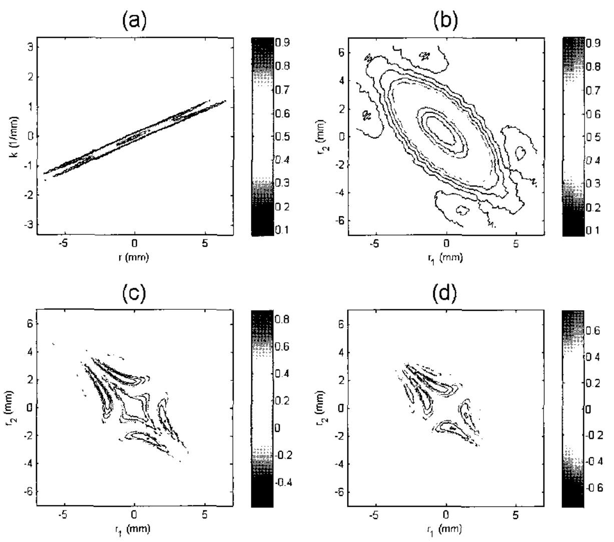

FIG. 5. Experimental data showing the Wigner function (a), absolute value of SCF (b), real part of SCF (c), and imaginary part of SCF (d) for laser light passing through 200 μm slit.

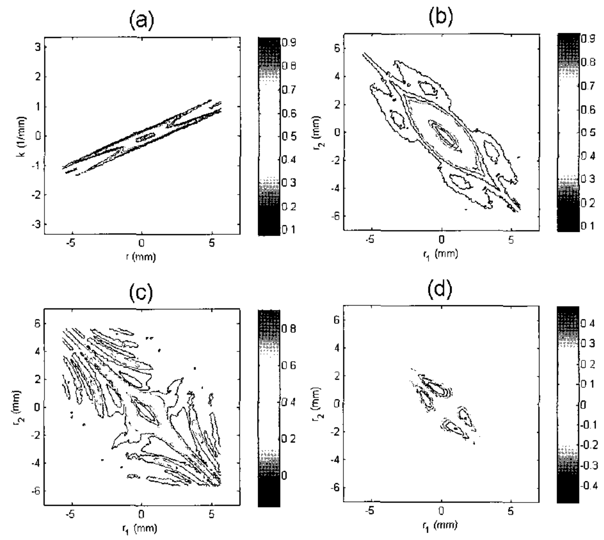

FIG. 6. Experimental data showing the Wigner function (a), absolute value of SCF (b), real part of SCF (c), and imaginary part of SCF (d) for laser light passing through 400 μm slit.

III. DISCUSSION

Because we are dealing with two replicas of the same beam that are rotated 180° with respect to each other, the larger value of SCF along the $x_1 = -x_2$ line means that the relative phase between the two counter-propagating beams are well kept constant. Along the $x_1 = x_2$ direction, the SCF shows coherence within the original laser beam [4]. We observed experimentally that the SCF becomes broader i.e., the SCF value increases along the $x_1 = x_2$ line, for smaller width slit. Considering the fact that the laser beam waist is about 300 μm, The 400 μm slit does not affect the propagation of the laser beam much. The 50 μm slit makes smaller beam cross section that increases the coherence area in the detection plane and therefore the SCF becomes broader. The real and imaginary parts of the SCF become more symmetric along the two diagonal lines too. Theoretically, we could write the SCF for a Gaussian Schell-model beam at z = constant plane as [16]

$$\Gamma(x_1,x_2) = \frac{A}{2\pi\sigma_I^2}\exp[-\frac{1}{4}\frac{x_1^2+x_2^2}{\sigma_I^2}]\exp[-\frac{1}{2}\frac{(x_1-x_2)^2}{\sigma_g^2}] \times \exp[-\frac{ik(x_1^2-x_2^2)}{2R}] \quad (5)$$

for the field that has Gaussian intensity distribution with variance σ_I^2 and the normalized degree of coherence g is given as

$$g(x_1-x_2) = \exp[-\frac{|x_1-x_2|^2}{2\sigma_g^2}]. \quad (6)$$

The absolute value of the SCF indicates the spatial degree of coherence. When $\sigma_g \to \infty$, the laser beam is a perfectly coherent Gaussian beam and the absolute value of SCF should be a completely symmetric function $\exp[-(x_1^2+x_2^2)/(2\sigma_I)^2]$ with respect to $x_1 = x_2$ and $x_1 = -x_2$. One can find that the measured data in Fig. 4 is more symmetric than the data in Fig. 6. $R = R(z)$ is the radius of curvature for the Gaussian wave front. $R > 0$ ($R < 0$) for diverging (converging) beam. The real and imaginary parts become highly oscillatory for highly divergent (convergent) beam ($|R| \to 0$), because of the factor $ik/2R$. One can also see this from the measured data from Fig. 4 to Fig. 6. Thus our experimental results show good agreement with the theory qualitatively.

IV. CONCLUSION

We have shown a method to directly measure the SCF by using a self referencing Sagnac interferometer technique. The optical field coming out of micro-size slits can be magnified and imaged onto a steering mirror and the SCF can be measured. We showed that, because the Wigner distribution function contains not only the information of wavefront but also that of beam divergence, the SCF obtained from the Wigner function contains such information in its absolute value, real and imaginary parts. This technique does not need a stable reference beam so that it can be used for the field that varies slowly in time. The inverse Fourier transform algorithm is very straightforward and it does not need any extra assumptions about the degree of coherence of light or source symmetry. This experimental set-up might be attached to the conventional optical microscope to measure the coherence function of a micro size biological samples in real time.

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