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# Optimal Design of Fiber-optic Surface Plasmon Resonance Sensors

Jaehoon Jung* and Minwook Kim

*School of Electronics and Computer Engineering, Dankook University,*
*San 8, Hannam-Dong, Yongsan-Gu, Seoul 140-714, Korea*

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We propose a systematic method for design of fiber-optic surface plasmon resonance (SPR) sensors. We used rigorous coupled wave analysis (RCWA) for analysis of the transmission spectrum, and the (1+1) evolution strategy (ES) was employed as an optimization tool. The simulation results show that the optimization method presented here is very useful in designing fiber-optic SPR sensor for strain and temperature measurement. This algorithm can be extended to another objective function with other weighting factors and optical parameters.

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## I. INTRODUCTION

As is well known, the phenomenon of surface plasmon resonance (SPR) in thin metal films is highly sensitive to optical and structural properties of the metal interface. In recent years, there are many sensors using Kretchmann's configuration where the index of refraction of the prism increases the wave vector of the incoming light and surface plasmons are excited at a certain angle for a specific wavelength [1].

As first pointed out by Liedberg in 1983 [2], the phenomenon of optical excitation of surface plasmons is very interesting for optical sensing. After that time, surface plasmon resonance was applied to various chemical and bio sensors [3,4]. Owing to their well-known advantage over the conventional SPR sensors, optical fiber sensors have attracted much attention in the past few years.

However, many researchers proposed only new structures and materials. There is no fundamental design of fiber optic SPR sensors. Until now, many researchers have been analyzing several times or adjusting parameters repeatedly to satisfy standard performance. That is, a design method that satisfied several parameters had not been tried yet.

In this paper, we present an optimization method including 3 dB bandwidth and dip depth of the objective function. We chose the metal thickness of each layer as design parameters. The optimization method is (1+1) evolution strategy (ES) and is applied to a fiber-optic SPR sensor with 3 metal layers as shown in Fig. 1. Thin metal films were composed of Cr, Ag, Au and transmission spectrum analysis was performed through the rigorous coupled wave analysis (RCWA) method. The simulation results show that the optimization method presented here is very useful in designing fiber optic SPR sensors. This algorithm can be extended not only to various configurations of fiber optic SPR sensors, but to other objective functions with other parameters.

## II. RIGOROUS COUPLED WAVE ANALYSIS (RCWA)

A schematic diagram of the proposed SPR sensor is shown in Fig. 2. Where $\Lambda$ is the period of the grating, supposing that the time variation of the electromagnetic field is $\exp(-j\omega t)$ and with the permittivity variation $\varepsilon_r(x)$ along the x direction. TM polarized wave from Maxwell's equation is:

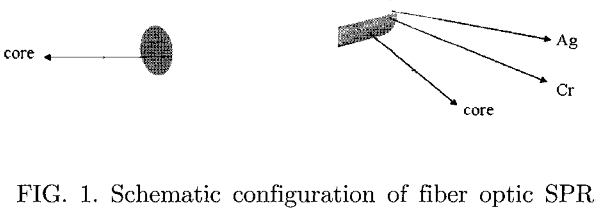

FIG. 1. Schematic configuration of fiber optic SPR.

$$\nabla\times\vec{E}=j\omega\mu_0 H_y \tag{1}$$

$$\nabla\times\vec{H}=-j\omega\varepsilon_0\varepsilon_r(x)\left(\hat{x}E_z+\hat{z}E_z\right) \tag{2}$$

The permittivity is written as a Fourier series expansion:

$$\varepsilon_r(x)=\sum_{k=-M}^{M}\varepsilon_k\exp\left(j\frac{2\pi h}{\Lambda}\right) \tag{3}$$

and inverse permittivity variation $a_r(x)$ to solve the problem on the convergence of RCWA is as follows:

$$\alpha_r(x)=\frac{1}{\varepsilon_r(x)}=\sum_{k=-M}^{M}\alpha_h\exp\left(j\frac{2\pi h}{\Lambda}\right) \tag{4}$$

$M$ is the Fourier truncation order which expresses the structure of the grid.

Traveling light by diffraction is produced along with reflected light by diffraction, when the light comes in according to the Fourier expression. A transverse wave vector $k_{x,m}$ is decided by Floquet condition or Bloch theorem.

$$k_{x,m}=k_{x,0}+m\frac{\lambda}{\Lambda}(-M\le m\le M),\quad k_{x,m}=n_I k_0\sin\theta_{inc} \tag{5}$$

This value is preserved in whole area by phase matching condition [5].

We applied the RCWA method to analysis the SPR sensor. We took the fabrication resolution of the actual manufacturing into account and interpreted the thickest model Cr=6 nm. Fig. 3 is to compare two kinds of methods, RCWA and the admittance method. RCWA is more correct compared with the admittance method because the RCWA method does analysis of the finite core and cladding thickness.

## III. OPTIMIZATION OF FIBER OPTIC SPR USING (1+1) EVOLUTION ALGORITHM

### A. (1+1) Evolution Strategy

In this paper, (1+1) ES is employed as a main optimization tool. Among several stochastic methods, ES uses the principle of organic evolution for searching for the optimal values. The ES is widely used because it can find the global optimal solution, the algorithm is simple, and the convergence speed is fast. The algorithm roughly consists of four parts: reproduction, mutation, competition, and selection [6,7].

The (1+1) ES is a simple mutation-selection scheme called two-membered ES. The "population" consists of one parent only, determined by a certain parameter configuration, creating one descendant by means of adding a normally distributed random vector (mutation) to the parameter values. The 'fitter' of both individuals, obtained by evaluating the objective function, serves as the ancestor of the following iteration (selection). The step width is adjusted periodically (e.g. after 10*$np$ function calls, where *np* is the number of optimization parameters) in such a way that the ratio of successful mutations over all mutations becomes *p*. This strategy parameter is usually set to *p*=0.2. In this paper, an annealing factor is set to be 0.85 and a shaking process is also considered to prevent a solution from converging to a local minimum.

### B. Objective Function

Generally, objective function of the optimal design can contain all parameters for analysis. In this paper, the objective function of the optimal design of SPR was chosen, the 3 dB bandwidth and depth of dip. The design parameters are the thicknesses of Ag, Au. The optimization problem and the objective function are defined as follows:

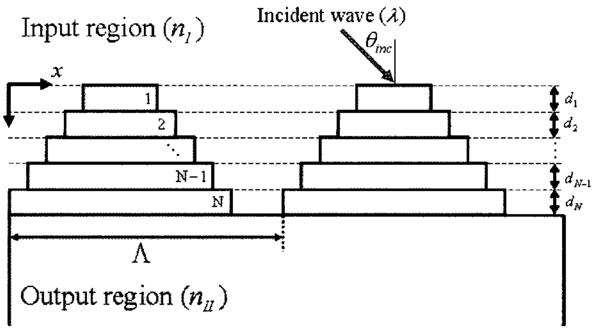

FIG. 2. Schematic diagram of a multi-grating.

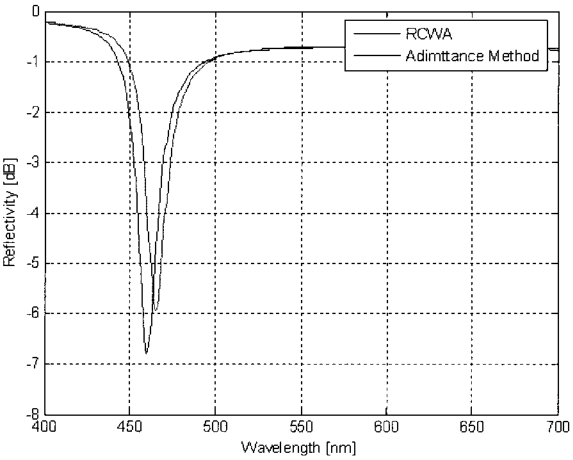

FIG. 3. Comparison of admittance method with RCWA.

$$\sqrt{\sum_{i=1}^{2} a_i x_i} \text{ Minimize objective function,}$$

$$x_i = \left(\frac{y_i - y_{it}}{y_{it}}\right)^2, \quad \sum_{i=1}^{2} a_i^2 = 1 \qquad (6)$$

where $y_1$ is the bandwidth of the dip, $y_2$ is the depth of the dip, $y_{it}$ is each objective value and $a_i$ is a weighting factor. Fig. 4 is the flow chart of optimization designing.

## IV. OPTIMIZATION RESULTS

Fig. 5 is the convergence of the objective function with iterations at Cr=6 nm. The fact that the convergence value is small means that we draw more near to a target. Fig. 4 is reflectance spectrum at initial design variable and optimized value for the case Cr=6 nm. Initial thickness of Ag and Au is 15 nm and optimize thickness is Ag=7.9966 nm, Au=10.1651 nm. Fig. 6 shows 3 dB bandwidth and dip depth for initial and optimized cases. 3 dB bandwidth is 15 nm, dip depth is 6.6 dB at initial thickness. This is optimized to 3 dB bandwidth of 6.75 nm, dip depth of 11.8 dB.

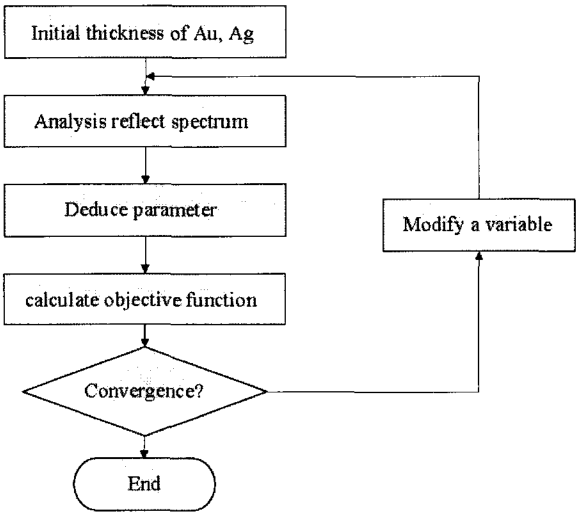

FIG. 4. Evlolution Strategy.

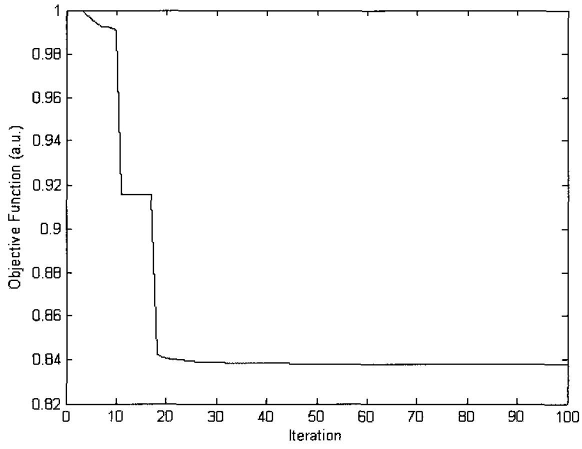

FIG. 5. Convergence process of objective function with iterations.

We could obtain a more improved value if we selected more parameters. (e.g. incident angle, etc.) The optimization method which was presented in this paper does not have a restriction on the number of design variables. Therefore, this method can be applied to other optimization courses and can offer a systematic tool for new SPR sensor design.

## V. CONCLUSIONS

This paper presents a systematic optimum design method of fiber optic SPR. The reflectivity spectrum is calculated by the RCWA and (1+1) ES is employed as an optimization tool. The designing variables are the thicknesses of Ag and Au. The objective function of the optimal design of SPR is 3 dB bandwidth and depth of dip. The 3 dB bandwidth and dip depth were optimized to 6.75 nm and 11.8 dB. When the bandwidth is 15 nm, dip depth is 6.6 dB at initial design parameters. This algorithm can be extended to another objective function with other weighting factors and optical parameters.

## ACKNOWLEDGEMENT

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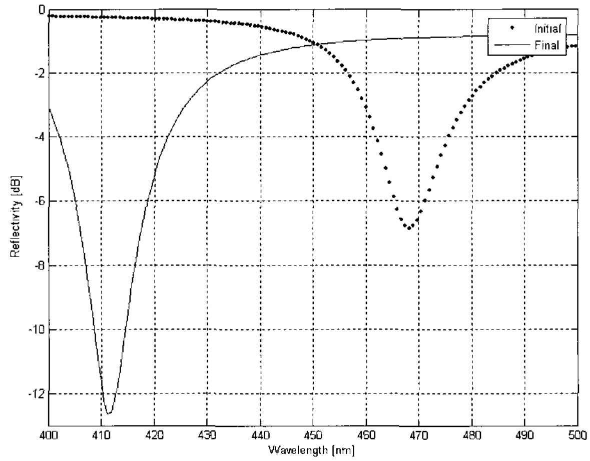

FIG. 6. Transmission spectra at the initial and final states.

*Corresponding author: andyjung@dku.edu

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