

## Energy Harvesting from a Vibrating Piezoelectric Unimorph Bender

### 유니모프 압전 벤더를 이용한 진동에너지의 획득

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#### <Abstract>

이 논문에서는 최근 다시 주목을 끌고 있는 압전소자를 이용한 에너지 획득에 대한 개념을 한 진동원에 응용하여 그 에너지의 이용가능성을 분석해 보고자 한다. 에너지 획득기로는 가장 일반적으로 사용되고 있는 유니모프 압전 캔틸레버 벤더를 사용한다. 먼저 압전 에너지 획득기에 대해 base excitation에서의 거동을 이론적으로 분석하고 실험실내에서 수행할 수 있는 압전 에너지 획득기를 제작하여 가진기 상에서 발생하는 전력을 측정한다. 시뮬레이션과 실험결과를 통해 진동에너지로부터 획득한 전기에너지가 각종 센서는 물론 기계부품들의 진단시스템에 필요한 전원을 공급할 수 있음을 알 수 있다.

**Keywords :** Energy harvesting, Monitoring machinery, Piezoelectricbender, Energy conversion

### 1. INTRODUCTION

Recently, the need for alternative energy sources for wireless electronic devices has led to a considerable amount of research in the area of energy harvesting<sup>1)</sup>. Energy harvesting, in this case, can be defined as converting an ambient energy source into a more useful energy source. Researchers have been investigating methods that could be used to either augment or replace disposable chemical batteries that presently the popular

energy sources for such devices. One method, in particular that has received significant attention is that of converting mechanical vibration energy into electrical energy because of the abundance of vibrations in many applications.

Among the possibilities for energy harvesting devices, cantilever piezoelectric benders have been generally used<sup>2)-6)</sup>. The theory behind cantilever piezoelectric elements is well known, most notably through the substantial literature based on suppression of beam

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vibrations, but not from the context of extracting energy for later use. Piezoelectric materials have the interesting ability to generate an electrical charge when deformed by an applied mechanical load such as pressure<sup>7(8)</sup>, force, and vibration, in addition to exhibit strain under the influence of an applied electrical field<sup>9)</sup>. In this study, a unimorph piezoelectric cantilever bender with a harmonic acceleration at the fixed end is examined as an energy harvesting device.

The interest in piezoelectric energy harvesting is mainly in power generation field for portable and low power consuming devices. The merit of applying piezoelectric power generators to these devices is that they can reduce the battery weight and possibly make the device self-powered by harvesting mechanical energy. To maximize these advantages, there are some issues to resolve such as how to design the energy harvester in order to optimize its electrical energy production.

There have been some prior works on the topic of modeling and predicting energy harvesting capability of various structures. Sodano et al.<sup>2(3)</sup> presented analytical modeling and experimental validation for the 31-type piezoelectric energy harvesting device. In a separate work, experimental results of power generation for three different commercially available piezoelectric power harvesters were presented. The results indicated that power generation was reduced when the interdigitated electrode (IDE) pattern was used. The group also presented a comparison of the performance for charging batteries using two types of piezoelectric energy harvesting devices in another work<sup>4)</sup>. Roundy et al.<sup>10)</sup> presented numerous research articles on vibration-based power generators. Mo et al.<sup>11)</sup> proposed a mathematical model of energy harvesting from a cantilever beam with IDE's to show the analytical solution fairly followed Sodano's experimental results.

As an another related work, analytical results indicate that a perceivable amount of power can be generated when the source of energy is vehicular vibration at the rear axle of a heavy truck.<sup>12)</sup>

This paper presents whether the piezoelectric energy harvester can provide any means either to replace battery or to extend battery life to power sensors for operation and send radio frequency (RF) signals for wireless communication, which is required for the machinery diagnostics application.

## 2. SYSTEM MODELING

A lead zirconate titanate (PZT) unimorph beam is used here to analyze the energy harvesting performance from ambient mechanical vibration. A cantilever bender contains a piezoelectric layer that is bonded to a layer of non-piezoelectric material as shown in Fig. 1.

### 2.1 Cantilever beam model

The Euler-Bernoulli method is used to model the cantilever beam. Modeling is carried out for a harmonic excitation at the clamped base since it is representative of an actual vibration source.<sup>13)</sup>

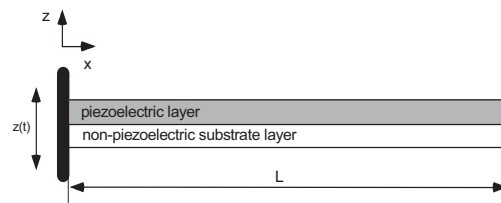


Fig. 1. Unimorph cantilever bender.

The governing equation of the beam is

$$\rho A \frac{\partial^2 w(x,t)}{\partial t^2} + EI \frac{\partial^4 w(x,t)}{\partial x^4} = -\rho A \frac{\partial^2 z(t)}{\partial t^2} \quad (1)$$

where  $w$  is displacement of the beam,  $\rho$  is density,  $A$  is cross-sectional area, and  $z(t)$  is a base-excitation to the beam.

The solution of equation (1) is assumed to

take the form.<sup>14)</sup>

$$w(x,t) = \sum_{i=1}^{\infty} X_i(x)q_i(t) \quad (2)$$

where  $X_i$  is the  $i$ -th mode shape of the beam and  $q_i$  are associated generalized coordinates.

The general mode shapes for the bender are then

$$X_i(x) = \cosh(\beta_i x) - \cos(\beta_i x) - \alpha_i (\sinh(\beta_i x) - \sin(\beta_i x)) \quad (3)$$

where

$$\beta_i^4 = \frac{\omega_n^2}{c^2}, \quad c = \sqrt{\frac{EI}{\rho A}}, \quad \alpha_i = \frac{\sinh(\beta_i L) - \sin(\beta_i L)}{\cosh(\beta_i L) + \cos(\beta_i L)} \quad \text{and}$$

$\omega_n$  is the  $i$ -th natural frequency, is found from the characteristic equation for the fixed-free beam.

$$\cos(\beta_i L) \cosh(\beta_i L) = -1 \quad (4)$$

Using the orthogonality condition, the generalized force by the distributed inertia force can be expressed as

$$F_i(t) = \int_0^L -\rho A \frac{\partial^2 z(t)}{\partial t^2} X_i(x) dx = \int_0^L [-X_i(x)] dx \rho A \frac{\partial^2 z(t)}{\partial t^2} \quad (5)$$

The convolution integral to evaluate  $q_i$  in equation (2) is then in the form

$$q_i(t) = \frac{1}{\omega_d} e^{-\zeta \omega_n t} \cdot \int_0^t F_i(\tau) e^{\zeta \omega_n \tau} \sin(\omega_d(t-\tau)) d\tau \quad (6)$$

where  $\omega_d$  is damped natural frequency and  $\zeta$  is damping ratio.

The deflection  $w(x,t)$  in equation (2) can now be evaluated as

$$w(x,t) = \sum_{i=1}^{\infty} \left[ q_i(t) (\cosh(\beta_i x) - \cos(\beta_i x)) - \alpha_i [\sinh(\beta_i x) - \sin(\beta_i x)] \right] \quad (7)$$

The moment in the cantilever beam induced by external vibration can then be found as;

$$M(x,t) = EI \frac{\partial^2 w(x,t)}{\partial x^2} = \sum_i EI \beta_i^2 \begin{pmatrix} q_i(t) (\cosh(\beta_i x) + \cos(\beta_i x)) \\ -\alpha_i [\sinh(\beta_i x) + \sin(\beta_i x)] \end{pmatrix} \quad (8)$$

## 2.2 Stress and moment in the bender

An alternative way to describe the moment at a certain location in the cantilever bending structure is to integrate the stress multiplied by the distance from the neutral surface along the cross section of the cantilever beam. The neutral surface of a unimorph cantilever beam does not coincide with the center of the cross section because of the properties of each layer, as shown in Fig. 2.

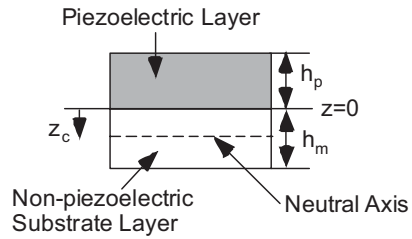


Fig. 2. Cross sectional area of the bender.

The neutral surface equation for the unimorph cantilever bender using the equivalent area procedure is

$$z_c = \frac{\sum zA}{\sum A} = \frac{z_p A_p - \frac{E_m}{E_p} z_m A_m}{\frac{E_m}{E_p} A_m + A_p} = \frac{h_p^2 s_m^E - h_m^2 s_{11}^E}{2(h_m s_{11}^E + h_p s_m^E)} \quad (9)$$

where,  $A$ 's are areas of each layer's cross section,  $h$ 's are each layer's thickness,  $E$ 's are the Young's moduli,  $s_{11}^E$  and  $s_m$  are the elastic compliance constants of piezoelectric and substrate layer, respectively. The neutral surface is located at a distance  $z_c$  from the interface of the two layers, where  $z=0$ . The subscript  $p$  indicates piezoelectric parameters (upper layer) and  $m$  indicates non-piezoelectric parameters (lower layer).

The strain can then be described in terms of the curvature.

$$\epsilon_1 = -\frac{z-z_c}{R} = -\kappa(z-z_c) \quad (10)$$

where  $R$  is the radius of curvature and  $\kappa$  is the curvature. At any given point along the beam, the moment equation from the deflection (equation (8)) should be the same

as the moment calculated from stresses on a cross-section, that is

$$\begin{aligned} M &= \int_p \sigma(z-z_c)dz + \int_m \sigma(z-z_c)dz \\ &= EI \frac{\partial^2 w(x,t)}{\partial x^2} \end{aligned} \quad (11)$$

The constitutive equations for the piezoelectric layer are<sup>15)</sup>

$$\begin{aligned} \varepsilon_1 &= s_{11}^E \sigma_1 - d_{31} E_3 \\ D_3 &= -d_{31} \sigma_1 + \varepsilon_{33}^T E_3 \end{aligned} \quad (12)$$

where  $\sigma$ : stress,  $\varepsilon_{33}^T$ : permittivity of the piezoelectric,  $d_{31}$ : piezoelectric constant,  $D_3$ : charge density, and  $E_3$ : electric field strength.

From equations (10) and (12), stress equations can be obtained for the piezoelectric layer,

$$\begin{aligned} \sigma_1 &= \frac{1}{s_{11}^E} (\varepsilon_1 + d_{31} E_3) \\ &= \frac{1}{s_{11}^E} (-\kappa(z-z_c) + d_{31} E_3) \end{aligned} \quad (13)$$

and for the non-piezoelectric layer,

$$\sigma_1 = \frac{1}{s_m} \varepsilon_1 = -\frac{1}{s_m} \kappa(z-z_c) \quad (14)$$

Substituting equations (13) and (14) into equation (11), the curvature  $\kappa$  can be found as

$$\begin{aligned} \kappa &= -\frac{12s_m s_{11}^E (s_{11}^E h_m + s_m h_p) EI \frac{\partial^2 w}{\partial x^2}}{bB_{11}} \\ &\quad + \frac{6s_m s_{11}^E d_{31} h_p h_m (h_m + h_p) E_3}{B_{11}} \end{aligned} \quad (15)$$

where  $b$  is width of the beam and  $B_{11} = s_m^2 h_p^4 + 4s_{11}^E s_m h_m h_p^3 + 6s_m s_{11}^E h_p^2 h_m^2 + 4s_{11}^E s_m h_p h_m^3 + s_{11}^E h_m^4$ .

### 2.3 Total system energy and generated voltage

The energy in the unimorph bender can then be described for the piezoelectric layer as

$$\begin{aligned} dU_p &= \frac{1}{2} (s_{11}^E \sigma_1 - d_{31} E_3) \sigma_1 \\ &\quad + \frac{1}{2} (-d_{31} \sigma_1 + \varepsilon_{33}^T E_3) E_3 \\ &= \frac{1}{2} s_{11}^E \sigma_1^2 - d_{31} \sigma_1 E_3 + \frac{1}{2} \varepsilon_{33}^T E_3^2 \end{aligned} \quad (16)$$

and for the non-piezoelectric layer

$$dU_m = \frac{1}{2} s_m \sigma_1^2 \quad (17)$$

The total energy is then found by integrating equations (16) and (17) over the volume of the beam,

$$U_{total} = \int_0^L \int_0^b \left( \int_0^{h_p} dU_p dz + \int_{-h_m}^0 dU_m dz \right) dy dx \quad (18)$$

The converted electrical energy can be obtained from the total energy. The partial derivative of this total energy with respect to the voltage is the charge.

$$\begin{aligned} Q &= \frac{\partial U_{total}}{\partial V} = Q_{ext} + Q_{voltage} \\ &= \sum_i -6 \frac{s_{11}^E s_m d_{31} h_m (h_m + h_p) M_i}{B_{11}} \\ &\quad + \frac{\varepsilon_{33}^T bL}{h_p} \left( 1 + \left( 3 \frac{s_m s_{11}^E h_p h_m^2 (h_m + h_p)^2}{s_h B_{11}} - 1 \right) K_{31}^2 \right) V \end{aligned} \quad (19)$$

where  $M_i = EI \beta_i q_i(t) \begin{pmatrix} \alpha_i (\cosh(\beta_i L) - \cos(\beta_i L)) \\ -\sin(\beta_i L) - \sinh(\beta_i L) \end{pmatrix}$ ,

$s_h = s_{11}^E h_m + s_m h_p$  and  $K_{31} = \frac{d_{31}}{\sqrt{\varepsilon_{33}^T s_{11}^E}}$ .

The charge equation (equation (19)) consists of two parts. The first term is derived from the external excitation and the second term is from a pure electric relation. The voltage relation with the charge is the capacitance, so the term in front of the voltage parameter in equation (19) shows the capacitance of the piezoelectric power generator structure. Therefore, the generated charge from the excitation is the first term of the charge equation.

$$Q_{Gen} = Q_{ext} = \sum_i -6 \frac{s_{11}^E s_m d_{31} h_m (h_m + h_p) M_i}{B_{11}} \quad (20)$$

The capacitance is described as

$$C_{free} = \frac{Q_{voltage}}{V} = \frac{e_{33}^T bL}{h_p} \cdot \left( 1 + \left( 3 \frac{s_m s_{11}^{E2} h_p h_m^2 (h_m + h_p)^2}{s_h B_{11}} - 1 \right) K_{31}^2 \right) \quad (21)$$

Using the capacitance and the generated charge from external excitation, the voltage that appearson the electrode of the piezoelectric layer can be calculated as

$$V_{Gen} = \frac{Q_{ext}}{C_{free}} = \frac{\sum_i -6 \frac{s_{11}^E s_m d_{31} h_m (h_m + h_p) M_i}{B_{11}}}{\frac{e_{33}^T bL}{h_p} \left( 1 + \left( 3 \frac{s_m s_{11}^{E2} h_p h_m^2 (h_m + h_p)^2}{s_h B_{11}} - 1 \right) K_{31}^2 \right)} \quad (22)$$

The generated energy from the external vibration is then

$$U_{Gen} = \frac{1}{2} C_{free} V_{Gen}^2 = \frac{1}{2} Q_{Gen} V_{Gen} \quad (23)$$

### 3. EXPERIMENTAL VERIFICATION

#### 3.1 Experimental setup

The unimorph PZT energy harvesting bender was built. The PZT layer was attached the aluminum substrate layer using a two-part conductive epoxy. The thickness of the aluminum substrate is 0.127 mm and PZT-5H with the same thickness is used for the piezoelectric layer. The length and width of the bender are 68.6mm and 9.8mm respectively. The actual dimensions are shown in Table 1.

Table 1. Dimensions of the energy harvesting beam

	Width (mm)	Thickness (mm)	Length (mm)
PZT layer	9.8	0.127	68.6
Aluminum layer	9.8	0.127	68.6

The first five mode shapes for the fixed-free designed beam, based on the analytical results are shown in Fig. 3.

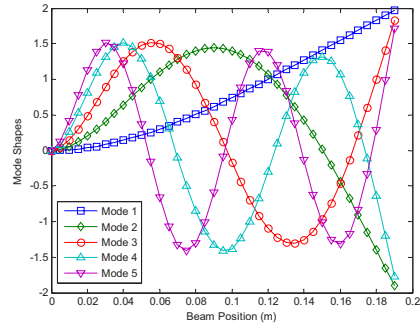


Fig. 3. First five mode shapes of the bender.

Table 2 shows the first three natural frequencies of the bender.

Table 2. Natural frequencies of the bender

Mode	1	2	3
Natural freq. (Hz)	31.8	195.6	542.0

To test the piezoelectric bender, it was clamped on the shaker as shown in Fig. 4. The lead wires were also attached to the bender with a two part conductive epoxy to measure charge output. The test rig include associated instruments which include power supply for the shaker, oscilloscope, function genertaor, and pico ammeter.

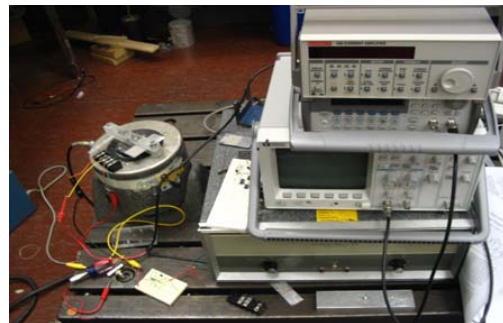


Fig. 4. Test rig with the mounted piezoelectric bender on the shaker.

#### 3.2 Experimental results and analysis

The bender was excited at a vibration magnitude of 1.5g at 32 Hz, which is the bender's fundamental frequency. Power was

dissipated through a simple resistive load. To be more detailed, the open circuit voltage rectified through a rectifying bridge and dumped into 2.2  $\mu\text{F}$  capacitor. The assembled simple harvesting circuit is shown in a breadboard in Fig. 5.

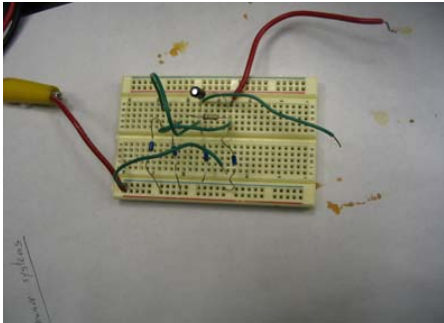


Fig. 5 Assembled harvesting circuit.

Fig 6. depicts schematic of the circuit. The voltage across a resistor in parallel with the capacitor was measured.

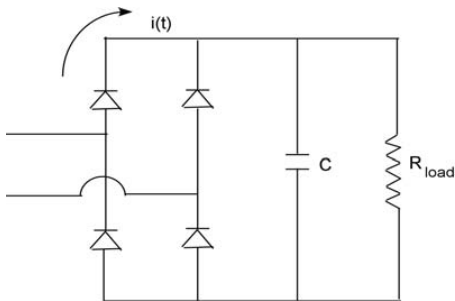


Fig. 6 Schematic of the harvesting circuit.

From the test rig (Fig. 4) the measured power output versus load resistance can be obtained. As a result of the test, the source resistance can be assumed to be around 220 kohm from the measurement. The simulated power output versus load resistance can then be calculated by theoretical analysis in the previous section. About 1% damping coefficient  $\zeta$  is also assumed for Eq. (6). The simulated and measured power outputs versus load resistance are depicted in Fig. 7. The analytical prediction of power output was about 23% off comparing to the

measured, the exact damping ratio and source impedance may reduce the difference.

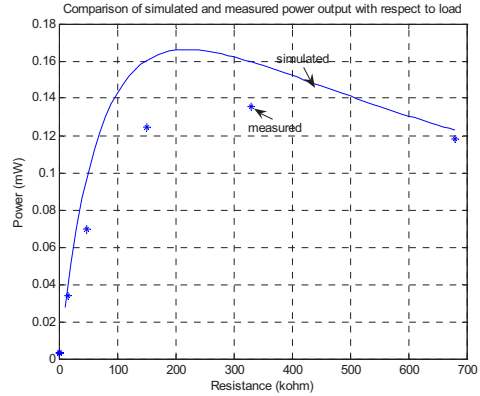


Fig. 7. Simulated and measured power output with respect to load resistance.

It is also noted that the power generation performance depends on material properties, shape of the structures, harvesting circuits, etc. Accordingly, optimal design of the bender for the given vibration source will also increase the performance. Above all, the generated power with a unimorph piezoelectric bender is sufficient considering the machinery diagnostic system that can be required to power a microprocessor-based sensor and RF communications module.

#### 4. CONCLUSION

This paper presents adoptability of the piezoelectric self-powered sensors and wireless communication devices for machinery monitoring system. The power generating performance for a unimorph piezoelectric cantilever bender with harmonic ambient vibration at the fixed end was first analyzed and then tested. The simulated and experimental results indicate that ambient vibration offers sufficient power sources for those systems.

As future research, the baseline findings should be steadily improved through optimization of the design.

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