Classification of Geared Motor Noise Using a Cepstrum and Comb Lifter Analysis

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A gearing system emits inconsistent noises from the impact of gear teeth when defects are present, but it is not easy for a noise inspector on a production line to distinguish defective products objectively. Since customers constantly complain about various noises from geared motors, it is desirable to devise an analytical technique to classify motors. However, it is difficult to separate inconsistent noises due to defective gears from the overall noise produced by a geared motor using a general signal processing method such as a FFT because low frequency impulse signals have a tendency not to appear in the frequency domain. In this paper, we propose a method that can be used to obtain more objective estimates and measurements of inconsistent noises from a gearing system. The proposed method makes use of the cepstrum domain with an applied autocorrelation and comb lifter, followed by a domain inversion.

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NOMENCLATURE

$\overset{\triangledown}{A}(f)$: Modified complex spectrum
$a(t)$: Time signal of the gear inconsistent noise
C_A : Complex cepstrum
C_{AA} : Power cepstrum
$\overset{\triangledown}{C}_A$: Modified power cepstrum
$F\{\}$: Fourier transform
S_{AA} : Autocorrelation

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1. Introduction

Global advancement trends in vehicle technology suggest that the demand for geared DC motors will continue to increase rapidly because of their ability to operate convenient and safe systems for consumers.[1] Because these systems now have improved performance and are faster, more noises are generated by the motors at greater amplitudes. Thus, inspection processes in manufacturing plants are hard pressed to detect defective products. Customer complaints have been increasing due to the subjective nature of noise, increasing the cost of manufacturing the motors.

Since motor noise caused by gear defects is periodic and impulsive, a noise inspector on a production line has great difficulty distinguishing gear noise from noises caused by other mechanical parts. Gear noise has a low frequency component compared to the motor rotational frequency, with high peak values and large vibrations. This means that it is difficult to extract the noise for such a high-amplitude short-duration impulsive signal using a Fourier transform (FT) because a FT is theoretically based on a periodic signal.[2,3] Furthermore, the characteristics of window functions used to sample a signal have large effects in the low frequency range.[5]

Therefore, the cepstrum and kurtosis methods have been widely used to detect impulsive abnormal signals from rotational machines. Both the power cepstrum and the complex cepstrum are useful in detecting periodic impulsive signals. However, because they have poor signal-to-noise ratios for peripheral noise[4,5,8], they are not suitable for detecting gear noise from a motor that also generates many rotational mechanical noises. In addition, kurtosis is only a statistical method, and limitations exist regarding its accuracy and reliability.

In this paper, we propose a signal processing procedure that extracts the gear noise of an automotive DC motor using the power cepstrum method with comb liftering in the quefrency domain, which is the inverse time domain. This signal processing procedure can extract and amplify gear noise from many noise sources, and can also provide an objective and reliable measure of gear noise by calculating its standard deviation or by observing the pattern of the signal. The method can also be used to compare good motors with ones that consumers complain are too noisy.

2. Cepstrum Theory with Comb Liftering

2.1 Cepstrum

The cepstrum and frequency domains are defined differently. The terms used by Bogert *et al.* to describe the cepstrum domain are listed in Table 1.[7]

Table 1 Terms used in the cepstrum and spectrum domains

Spectrum	Cepstrum
Frequency	Quefrency
Harmonics	Rahmonics
Filter	Lifter
Magnitude	Gamnitude
Phase	Saphe

The definition of the power cepstrum is

$$C_{AA} = | F^{-1}\{\log S_{AA}(f)\} |^2 \cdot \qquad (1)$$

An autocorrelation of the time signal is

$$S_{AA} = \overline{| F\{a(t)\} |^2} \cdot \qquad (2)$$

When the cepstrum is defined using Eq. (2), the gamnitude in the low quefrency domain is higher than that in the high quefrency domain. This characteristic often makes it hard to examine the periodicity of a signal because the expected signal is buried in peripheral signals. To overcome this weak point of the cepstrum, Randall[2] proposed a new definition of the power cepstrum:

$$\begin{aligned} &\overset{\vee}{C}_{AA} = F^{-1}\{\Gamma_{AA}(f)\} && (3)\\ &\Gamma_{AA} = 2\log S_{AA}(f) \quad ,f > 0 &&\\ &\Gamma_{AA} = \log S_{AA}(f) \quad ,f = 0 &&\\ &\Gamma_{AA} = 0 \quad ,f < 0 && \end{aligned}$$

The power cepstrum is often used with a fast Fourier transform (FFT) to provide a signal analysis. This is very useful when analyzing the magnitude of a signal at a given frequency because it gives an absolute quantity. But the technique loses the phase information used for the inverse transform because only absolute quantities are represented. A complex cepstrum can be used to obtain the phase information, but the differences in magnitude at a given frequency are less than those obtained using a power spectrum. Thus, an analysis based on either the power or the complex cepstrum is less capable of detecting gear noise buried in various other noises generated by the rotation of a rotor. Therefore, we combine the two methods to extract gear noise with a much higher degree of accuracy. We can use the power cepstrum to estimate the periodicity of the signal, and the complex cepstrum to obtain the phase information. This process extracts the intended signal, although it does not yield its complete inverse transform.

The complex cepstrum is defined as

$$\begin{aligned} &C_A = F^{-1}\{\log A(f)\} && (4)\\ &A(f) = F\{a(t)\} = | A(f) | e^{j\phi(f)} &&\\ &\log A(f) = \ln | A(f) | + j\phi(f) && \end{aligned}$$

Using the inverse transform of Eq. (4), the impulsive time signal of the gear impact signal can be amplified from the given raw signal.

2.2 Comb Liftering

A theoretical ideal band-pass lifter with a bandwidth Δ and a center quefrency τ_i can be written as

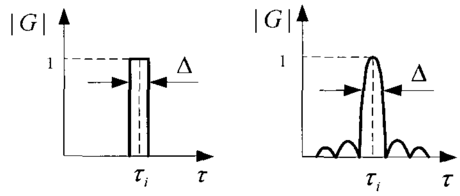
$$| G_i(e^{j\tau}) | = \begin{cases} 1 & ; \ | \tau - \tau_i | < \dfrac{\Delta}{2} \\ 0 & ; \ otherwise \end{cases} \qquad (5)$$

An actual band-pass lifter obtained by mathematic modeling is

$$G_i(e^{j\tau_i}) = \sum_{n=0}^{p} g_i(n) e^{jn\tau_i} = 1 \qquad (6)$$

$g_i(n)$: *Lifter coefficient*

However, this expression produces many small lobes around the center quefrency. These generate a loss of power and increase the deviation by including peripheral signals, as shown in Fig. 1.[5] Therefore, the signals are post-processed to obtain power cepstrum results rather than using a real-time analysis of an actual band-pass lifter. In this study, the ideal lifter was embodied by a conditional equation. Then a comb lifter was realized by repeatedly executing the conditional equation with related rahmonics and bandwidths, which were supplied as input.



(a) (b)

Fig. 1 (a) Ideal band-pass lifter and (b) actual band-pass lifter

3. Cepstrum Measurement and Analysis

3.1 Noise and Vibrations of an Automobile DC Motor

An accelerometer sensor was used for these tests, as shown in Fig. 2. The vibrations of the signal were sampled from 0 to 800 Hz with a resolution of 0.5 Hz. When the time signals were compared with the FFT power spectrum, very similar signal patterns were obtained for the noise and vibrations. This implies that the vibrations directly affect the noise level. Since it is better to measure vibrations to diagnose defects in a system[9], we performed a vibration analysis to quantify the gear noise.

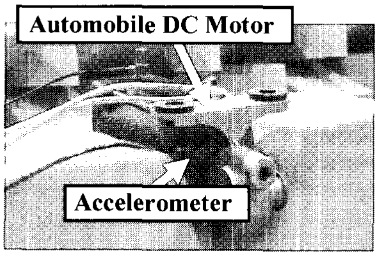
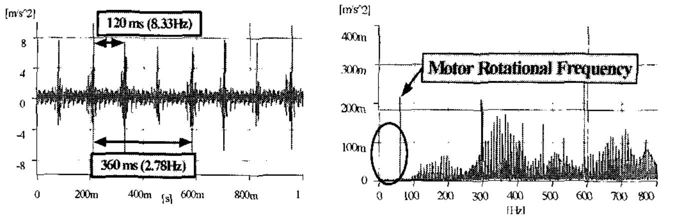


Fig. 2 Setup used to measure the vibrations of the DC motor

3.2 Gear Impulse Signal Analysis

Figure 3(a) shows a time signal that includes the impulsive signal from vibrations of a worm gear. The gear reduction ratio was 7.33:1 and its rotational speed was 3600 rpm. The impulsive signal corresponded to the gear rotational period of the motor. Even though this is an example of a bad motor, it would be very difficult to distinguish the defective gear sounds on a production line when the signal is mixed with many types of other signals coming from the rotors, brushes, and alignment errors. The conversion factor used to obtain the frequency from the rotational speed of the motor was 60 Hz. Applying the gear reduction ratio, we obtained a gear rotational frequency of 8.18 Hz, which corresponds to the impulse frequency visible in Fig. 3(a). Thus, an impact is generated by the gear defects whenever the gear rotates one revolution. Another periodic signal appears in the figure that has a period three times the gear rotational frequency. This implies that one thread of the three-threaded worm screw had some defects. Figure 3(b) shows the FFT of the same time signal. The 60-Hz motor rotational frequency is quite clear, but the gear rotation characteristics are not visible.



(a) Time signal (b) FFT analysis

Fig. 3 FFT analysis of the gear vibration

Figure 4 shows the power cepstrum of the same data illustrated in Fig. 3. The power cepstrum clearly displays impulse signals of 8.33 Hz (120 ms) and 2.78 Hz (360 ms) that were not apparent in the FFT results. Since the power cepstrum was transformed using the autocorrelation given in Eq. (1), the signal amplitude increased at low quefrencies that correspond to the high frequency domain and decreased at high quefrencies that correspond to the low frequency range. Note that the phase information in the power cepstrum was lost when the autocorrelation was applied.

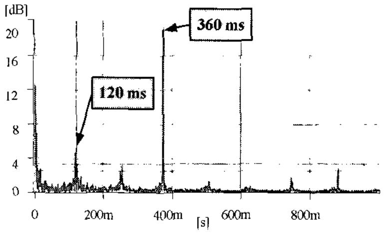


Fig. 4 Power cepstrum analysis of the gear vibration

3.3 Modified Power Cepstrum

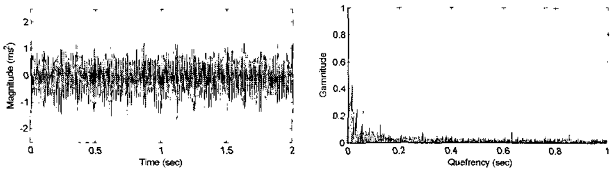
A modified power cepstrum can be used to compensate for the loss of phase information. Thus, we retain the capability to detect impulsive signals while performing inverse transforms. The definition of the modified power cepstrum is

$$\overset{\nabla}{C}_A = F^{-1}\{\log \overset{\nabla}{A}(f)\}$$
$$A(f) = F\{a(t)\} = |A(f)|\, e^{j\phi(f)} \tag{7}$$

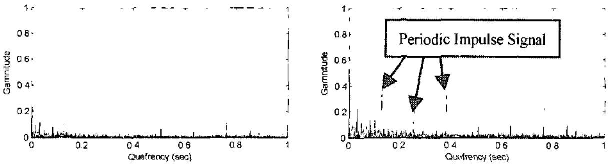
This expression partially adopts some definitions of the complex cepstrum and power cepstrum. To obtain the effect of the autocorrelation, we use the square of the absolute value instead of the absolute quantity of the signal amplitude. We also use the phase of each absolute quantity coming from the complex cepstrum. The procedure is summarized below.

$$\overset{\nabla}{A}(f) = |A(f)|^2\, e^{j\phi(f)} \tag{8}$$
$$\log \overset{\nabla}{A}(f) = 2\ln|A(f)| + j\phi(f)$$

Figure 5 compares the results obtained using the complex cepstrum, power cepstrum, and modified power cepstrum. The complex cepstrum does not clearly show the periodic signal at low quefrencies, unlike the power cepstrum. The modified power cepstrum amplifies the periodic signal level compared to the power cepstrum and also clearly shows the periodic signal at low quefrencies. Therefore, the modified power cepstrum is a more useful method for extracting periodic signals.



(a) Time signal (b) Complex cepstrum



(c) Power cepstrum (d) Modified power cepstrum

Fig. 5 Comparison of the modified power cepstrum with the complex cepstrum and power cepstrum

3.4 Analysis Using a Comb Lifter and Inverse Transform

Figure 6 shows a block diagram that explains the procedure used to obtain the gear impulsive signal using the proposed signal processing of a motor vibration signal. The final extracted signal can be examined objectively to determine whether it is abnormal by converting the pattern to a quantitative value. An inverse transform is performed on the quefrency domain to obtain the time domain so that the gear impulsive signal is more visible and to obtain a more reliable quantitative value. If the signal is quantized in the quefrency domain of a conventional complex cepstrum, the reliability of the results is reduced because other signals are also included. This additional information is also useful to noise inspectors unfamiliar with the cepstrum concept.

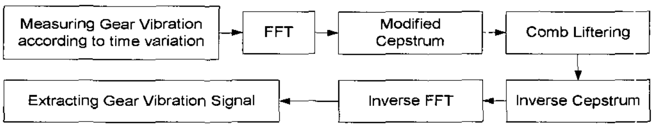
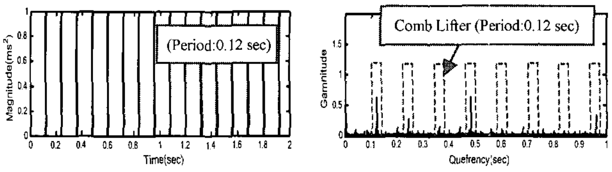
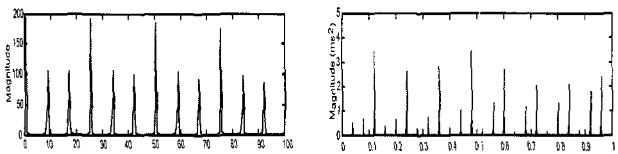


Fig. 6 Signal processing sequence used to extract the gear noise



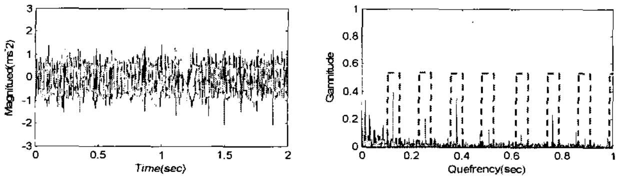
(a) Unit impulse (time signal) (b) Modified power cepstrum



(c) Inverse of the FFT (d) Inverse to the time signal

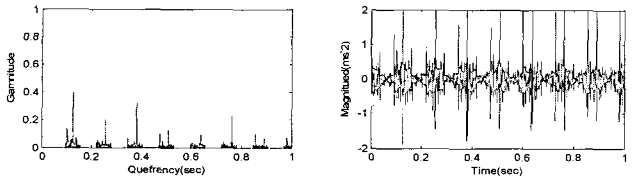
Fig. 7 Signal processing results of a periodic unit impulse

Figure 7 shows the sequence of the analysis stages for a time signal with a unit impulse of 0.12 s. A comb lifter was applied to fit the period of the impulse signal extracted from the modified power cepstrum. Then the final FFT results were obtained by inverting the processed signal again. The FFT results clearly show a harmonic component of 8.33 Hz (1/0.12 s = 8.33 Hz) generated from the impulse period. Note that the valuable time axis was reduced to half its length when the cepstrum transform was executed because of applying the autocorrelation. In the final results, another impulse appears amplified around the original signal. This amplification is the effect of the window function that arises when the modified power spectrum arbitrarily squares the signal. Despite this distortion of the original signal, the extracted signal is sufficient to identify the intended impulse when comparing the final processed signal with the original one.



(a) Time signal with a gear impulse

(b) Modified power cepstrum and comb lifter



(c) Comb liftering

(d) Extracted time signal

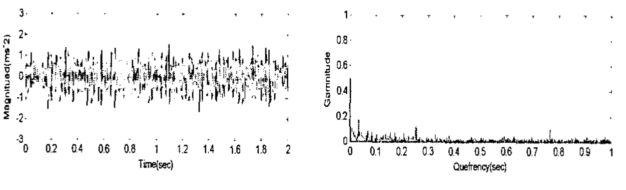
Fig. 8 Signal processing used to extract a gear impulse signal buried in motor noise

Figure 8 shows the signal processing procedures used to extract a gear impulse signal buried in motor noise. In the figure, (a) shows a time signal in which a gear impulse is buried in motor noises, (b) shows the modified cepstrum and comb liftering applied according to the gear rotational frequency, (c) gives the liftered cepstrum from the comb lifter, and (d) is the final gear impulse signal obtained from the inverse of the liftered cepstrum. The bandwidth of the comb lifter was chosen in consideration of the mechanical load and input power variation that could be expected on a production line. The bandwidth was 0.05 s with a center quefrency of 0.12 s. In this case, the frequency range of the gear rotation was 6.9–10.5 Hz.

3.5 Verification of the Proposed Method

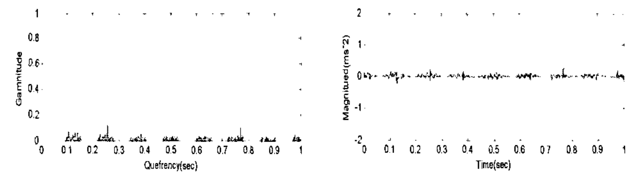
3.5.1 Classification of Motors with Similar Signal Patterns

The proposed analysis was performed on good motors and bad motors that were either rejected by a noise inspector working on a production line or returned by a customer because of excessive noise. These types of motors are very difficult to analyze on a production line because the sorting results are affected by the subjective analysis of the noise inspectors. No large differences existed between the time signals of the two motors. Figures 9 and 10 show the signal processing results for a good motor and a motor with gear noise, respectively. The extracted gear impulse in the bad motor is quite obvious, providing a means to objectively reject it.



(a) Time signal with a gear impulse

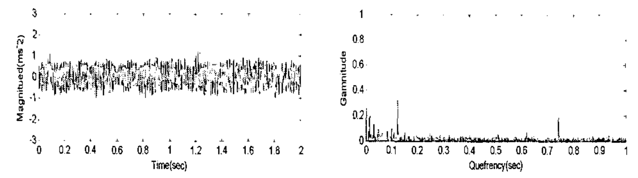
(b) Modified power cepstrum



(c) Comb liftering

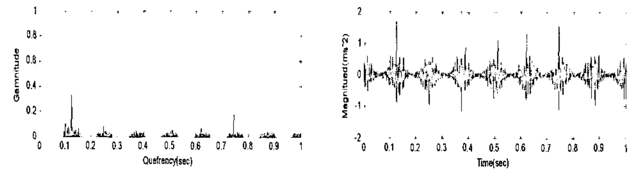
(d) Extracted time signal

Fig. 9 Signal processing results of a good geared motor



(a) Time signal for a gear impulse

(b) Modified power cepstrum



(c) Comb liftering

(d) Extracted time signal

Fig. 10 Signal processing results of a bad geared motor

Table 2 Test results from noise inspector and the proposed cepstrum analyses

No.	Dev. square	Noise Inspector Analysis	Cepstrum Analysis
1	9.8516	High Pitch Noise (Motor)	Good
2	22.9883	Good	Good
3	88.6697	Gear Noise	Gear Noise
4	472.0182	Bad Noise (Motor & Gear)	Gear Noise
5	99.0336	Gear Noise	Gear Noise
6	126.1834	Gear Noise	Gear Noise
7	151.3267	Gear Noise	Gear Noise
8	133.8939	Gear Noise	Gear Noise
9	202.8854	Gear Noise	Gear Noise
10	42.2487	Gear Noise	Gear Noise
11	72.2365	Gear Noise	Gear Noise
12	21.4828	Good	Good
13	46.8199	Gear Noise	Gear Noise
14	25.0813	Good	Good
15	122.6450	Gear Noise	Gear Noise
16	9.1291	Good	Good
17	17.5222	Good	Good
18	77.4510	Gear Noise	Gear Noise
19	314.0853	Gear Noise	Gear Noise
20	21.4001	Good	Good
21	9.3477	Good	Good
22	13.1928	Good	Good
23	22.4019	Alignment error	Good
24	44.0258	Good	Gear Noise
25	274.8901	Gear Noise	Gear Noise
26	12.5311	Good	Good

3.5.2 Comparison of Noise Inspector Findings and the Proposed Signal Processing Analysis

The results of the proposed analysis are compared to the findings of noise inspectors in Table 2. The selection criterion for the proposed analysis was the deviation square, defined by Eq. (9). The deviation square is similar to the standard deviation or variance used in statistics, but the population is omitted from the denominator to increase the differences in the calculated data. Motors with deviation squares greater than 40 were deemed to be bad; otherwise, they were deemed to be good. This criterion was chosen statistically as the most meaningful value from 300 sets of test samples. The noise inspectors were accurate 88.5% of the time while the cepstrum analyses were accurate 96.2% of the time. In addition, the cepstrum analyses did not rely on subjective findings, providing greater objective reliability.

$$\sum_{i=0}^{n}(x_i-\bar{x})^2 \tag{9}$$

4. Conclusions

A modified power cepstrum was proposed to identify bad motors with impulsive gear noise. The following conclusions were drawn based on our findings.

1. Our signal processing method successfully extracted gear impulses using a comb lifter in the quefrency domain of the cepstrum.
2. We used a modified power cepstrum, obtained from the conventional complex cepstrum, to perform an inverse transform of the domain while preserving the advantages offered by a power cepstrum analysis.
3. The deviation square was defined and used to provide an objective criterion to either reject or accept motors using the proposed signal processing procedure.
4. The proposed method proved to be very effective at estimating gear impulsive noises. The reliability of the method was verified by comparing the results with the analyses of noise inspections working on a production line.

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REFERENCES

1. Lee, M. H., Kang, D. B., Kim, H. Y. and Ahn, J. H., "Development of worm machining method to improve noise and vibration of automotive DC motor," Proc. of the KSPE Spring Conference, pp. 777-780, 2004.

2. Randall, R. B., "Frequency Analysis," B&K, pp. 16-77, 271-304, 1987.

3. Taylor, J. I., "The Gear Analysis Handbook," VCI, pp. 156-228, 2000.

4. Choi, Y. C. and Kim, Y. H., "Detection of impulse signals in noise using a minimum variance cepstrum theory," Journal of KSNVE, Vol. 10, No. 4, pp. 642-647, 2000.

5. Choi, Y. C. and Kim, Y. H., "Detection of impulse signals in noise using a minimum variance cepstrum application on faults detection in a bearing system," Journal of KSNVE, Vol. 10, No. 6, pp. 985-990, 2000.

6. Cunningham, E. P., "Digital Filtering," Houghton Mifflin, pp. 236-402, 1992.

7. Bogert, B. P., Healy, M. J. R. and Tukey, J. W., "The quefrency analysis of time series for echoes: Cepstrum, pseudo-autocovariance, cross-cepstrum and shape cracking," Proceedings of the Symposium on Time Series Analysis, pp. 209-243, 1963.

8. Wismer, N. J., "Gearbox Analysis Using Cepstrum Analysis and Comb Liftering," Application Note, B&K, 1994.

9. Adams, M. L., "Rotating Machinery Vibration," Marcel Dekker Inc., pp. 225-290, 2001.