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Noninformative Priors for the Common Shape Parameter in the Gamma Distributions

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Abstract

In this paper, we develop the noninformative priors for the common shape parameter in the gamma distributions. We develop the matching priors and reveal that the second order matching prior does not exist. It turns out that the one-at-a-time reference prior and the two group reference prior satisfy a first order probability matching criterion. Some simulation study is performed.

Keywords : Common Shape, Gamma Distribution, Matching Prior, Reference Prior

1. Introduction

Consider k independent gamma populations with the shape parameter α and the scale parameter $\beta_i, i=1,\cdots,k.$. Let $X_{ij},\ j=1,\cdots,n_i$ denote observations from the i th gamma population. Then the gamma distribution of X_{ij} is given by

$$f(x_{ij})=\frac{x_{ij}^{\alpha-1}}{\Gamma(\alpha)\beta_i^{\alpha}}\exp\left\{-\frac{x_{ij}}{\beta_i}\right\}, i=1,\cdots,k, j=1,\cdots,n_i, \tag{1}$$

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where $\alpha > 0$, $\beta_i > 0$ and α is the common shape parameter. As noted by various authors (Lawless, 1982; Keating et. al., 1990, etc), the gamma distribution is widely used in reliability and survival analysis. In particular, the shape parameter is of special interest because the shape parameter less than 1, equal to 1 and greater than 1 correspond to a decreasing failure rate, a constant failure rate and an increasing failure rate, respectively. The model (1) can be motivated within competing risks theory (see Wong and Wu, 1998).

The present paper focuses on noninformative priors for the common shape parameter. We consider Bayesian priors such that the resulting credible intervals for the common shape parameter have coverage probabilities equivalent to their frequentist counterparts. Although this matching can be justified only asymptotically, our simulation results indicate that this is indeed achieved for small or moderate sample sizes as well.

This matching idea goes back to Welch and Peers (1963). Interest in such priors revived with the work of Stein (1985) and Tibshirani (1989). Among others, we may cite the work of DiCiccio and Stern (1994), Datta and Ghosh (1995a,b, 1996), Mukerjee and Ghosh (1997) and Mukerjee and Reid (1999).

On the other hand, Ghosh and Mukerjee (1992), and Berger and Bernardo (1989,1992) extended Bernardo's (1979) reference prior approach, giving a general algorithm to derive a reference prior by splitting the parameters into several groups according to their order of inferential importance. This approach is very successful in various practical problems. Quite often reference priors satisfy the matching criterion described earlier (Kim and Sohn, 2004; Kang, 2004).

Wong and Wu (1998) compared the accuracy of tail probabilities obtained by various approximates inference procedure for the common shape parameter of the gamma distributions. They concluded that although the first order methods based on the maximum likelihood estimator and signed square root of the likelihood ratio statistic are the most common approximations used by applied statisticians, they sometimes give unsatisfactory or even misleading approximations, and all the third order methods give very similar results but the approximation using the exact conditional log likelihood function seems to be the best.

The outline of the remaining sections is as follows. In Section 2, we develop first order and second order probability matching priors for the common shape parameter. We revealed that the second order matching prior does not exist. Also we derive the reference priors for the parameters. It turns out that the one-at-a-time reference prior and the two group reference prior satisfy a first order matching criterion. We provide that the propriety of the posterior distribution for the reference priors as well as first order matching prior. In Section 4, simulated frequentist coverage probabilities under the proposed priors are given.

2. The Noninformative Priors

2.1 The Matching Priors

For a prior π, let $\theta_1^{1-\alpha}(\pi;\boldsymbol{X})$ denote the $100(1-\alpha)$th percentile of the posterior distribution of θ_1, that is,

$$P^{\pi}[\theta_1 \le \theta_1^{1-\alpha}(\pi;\boldsymbol{X}) \mid \boldsymbol{X}] = 1-\alpha, \tag{2}$$

where $\boldsymbol{\theta} = (\theta_1, \cdots, \theta_t)^T$ and θ_1 is the parameter of interest. We want to find priors π for which

$$P[\theta_1 \le \theta_1^{1-\alpha}(\pi;\boldsymbol{X}) \mid \boldsymbol{\theta}] = 1-\alpha+o(n^{-u}). \tag{3}$$

for some u, as n goes to infinity. Priors π satisfying (3) are called matching priors. If $u=1/2$, then π is referred to as a first order matching prior, while if $u=1$, π is referred to as a second order matching prior.

In order to find such matching priors, let

$$\lambda = \alpha \text{ and } \mu_i = \alpha\beta_i, i=1,\cdots,k.$$

With this parameterization, the likelihood function of parameters $(\lambda, \mu_1, \cdots, \mu_k)$ for the model (1) is given by

$$L(\lambda,\mu_1,\cdots,\mu_k) \propto \frac{1}{\Gamma(\lambda)^n}\left[\prod_{i=1}^{k}\left(\frac{\lambda}{\mu_i}\right)^{n_i\lambda}\right]\left[\prod_{i=1}^{k}\prod_{j=1}^{n_i} x_{ij}^{\lambda}\right]\exp\left\{-\lambda\sum_{i=1}^{k}\frac{x_{i\cdot}}{\mu_i}\right\}, \tag{4}$$

where $n = n_1 + \cdots + n_k$ and $x_{i\cdot} = \sum_{j=1}^{n_i} x_{ij}, i=1,\cdots,k$. Based on (4), the Fisher information matrix is given by

$$I = Diag\left\{n[\psi'(\lambda)-\lambda^{-1}], n_1\lambda\mu_1^{-2}, \cdots, n_k\lambda\mu_k^{-2}\right\}, \tag{5}$$

where $\psi'(\cdot)$ is the trigamma function. From the above Fisher information matrix I, λ is orthogonal to $(\mu_1, \cdots, \mu_k)$ in the sense of Cox and Reid(1987). Following Tibshirani(1989), the class of first order probability matching prior is characterized by

$$\pi_m^{(1)}(\lambda, \mu_1, \cdots, \mu_k) \propto [\psi'(\lambda) - \lambda^{-1}]^{1/2} d(\mu_1, \cdots, \mu_k), \tag{6}$$

where $d(\mu_1, \cdots, \mu_k) > 0$ is an arbitrary function differentiable in its arguments.

The class of prior given in (6) can be narrowed down to the second order probability matching priors as given in Mukerjee and Ghosh (1997). A second order probability matching prior is of the form (6), and also d must satisfy an additional differential equation (cf (2.10) of Mukerjee and Ghosh (1997)), namely

$$\frac{1}{6} d(\mu_1, \cdots, \mu_k) \frac{\partial}{\partial \lambda} \left\{ I_{11}^{-\frac{3}{2}} L_{1,1,1} \right\} + \sum_{v=1}^{k} \frac{\partial}{\partial \mu_v} \left\{ I_{11}^{-\frac{1}{2}} L_{11v} I^{vv} d(\mu_1, \cdots, \mu_k) \right\} = 0, \tag{7}$$

where

$$L_{1,1,1} = E[(\frac{\partial \log L}{\partial \lambda})^3] = g(\lambda), \ g(\lambda) = \text{ a function of } \lambda,$$

$$L_{11v} = E[\frac{\partial^3 \log L}{\partial \lambda^2 \partial \mu_v}] = 0, v = 1, \cdots, k \quad \text{and} \quad I_{11} = n[\psi'(\lambda) - \lambda^{-1}].$$

Then (7) simplifies to

$$\frac{1}{6} d(\mu_1, \cdots, \mu_k) \frac{\partial}{\partial \lambda} \{[\psi'(\lambda) - \lambda^{-1}]^{-3/2} g(\lambda)\} \neq 0.$$

Thus the resulting second order probability matching prior does not exist.

Remark 1. In single population $(k=1)$, Garvan and Ghosh (1997) derived the matching prior for the mean parameter and the shape parameter. They revealed that the second order matching prior for the shape parameter does not exist.

2.2 The Reference Priors

Reference priors introduced by Bernardo (1979), and extended further by Berger and Bernardo (1992) have become very popular over the years for the development of noninformative priors. In this section, we derive the reference priors for different groups of ordering of $(\lambda, \mu_1, \cdots, \mu_k)$. Then due to the orthogonality of the parameters, following Datta and Ghosh (1995b), choosing rectangular compacts for each $\lambda, \mu_1, \cdots, \mu_k$ when λ is the parameter of interest, the reference priors are given as follows.

If λ is the parameter of interest, then the reference prior distributions for different groups of ordering of $(\lambda, \mu_1, \cdots, \mu_k)$ are:

Group ordering	Reference prior
$\{(\lambda, \mu_1, \cdots, \mu_k)\}$,	$\pi_1 \propto \lambda^{k/2} [\psi'(\lambda) - \lambda^{-1}]^{1/2} \mu_1^{-1} \cdots \mu_k^{-1}$
$\{\lambda, \mu_1, \cdots, \mu_k\}, \{\lambda, (\mu_1, \cdots, \mu_k)\}$,	$\pi_2 \propto [\psi'(\lambda) - \lambda^{-1}]^{1/2} \mu_1^{-1} \cdots \mu_k^{-1}$

Remark 2. In the above reference priors, the one-at-a-time reference prior and the two group reference prior satisfy a first order matching criterion. But Jeffreys' prior, π_1, is not a first order matching prior.

In the above results, the first order probability matching priors are given by

$$\pi_m^{(1)}(\lambda, \mu_1, \cdots, \mu_k) \propto [\psi'(\lambda) - \lambda^{-1}]^{1/2} d(\mu_1, \cdots, \mu_k),$$

where d is any smooth function of $\mu_1, \cdots, \mu_k$. However every function is not permissible in the construction of priors. For instance, we consider any function of the form$(\mu_1 \cdots \mu_k)^{-a}$. If a is positive integer, then the posterior distribution of λ is proper. But the condition of propriety in this form strongly depend on the a. Moreover there does not seems to be any improvement in the coverage probabilities with this posterior distribution. So we consider a particular first order matching prior where $d = \mu_1^{-1} \cdots \mu_k^{-1}$. Because this matching prior is the one-at-a-time reference prior and the two group reference prior. The matching prior is given by

$$\pi_m^{(1)}(\lambda, \mu_1, \cdots, \mu_k) \propto [\psi'(\lambda) - \lambda^{-1}]^{1/2} \mu_1^{-1} \cdots \mu_k^{-1}. \tag{8}$$

Remark 3. We show that the prior (8) is joint probability matching when λ, $\mu_1, \cdots, \mu_{k-1}$ and μ_k are of interest. Write $\boldsymbol{\theta} = (\lambda, \mu_1, \cdots, \mu_k)$. Let $t_1(\boldsymbol{\theta}) = \lambda$, $t_2(\boldsymbol{\theta}) = \mu_1, \cdots, t_k(\boldsymbol{\theta}) = \mu_{k-1}$ and $t_{k+1}(\boldsymbol{\theta}) = \mu_k$. Following the notation of Datta (1996), $P(\boldsymbol{\theta}) = Diag\{1, 1, \cdots, 1\}$. Thus condition (7) of Datta (1996) is satisfied. Moreover the prior (8) is the unique solution to the equations of (2) of Datta (1996). Thus the prior (8) is joint probability matching prior for $(\lambda, \mu_1, \cdots, \mu_k)$. So this matching prior can be used for the Bayesian inference in reliability and survival analysis.

3. Implementation of the Bayesian Procedure

We investigate the propriety of posteriors for a general class of priors which include the Jeffreys′ prior and the first order matching prior (8). We consider the class of priors

$$\pi_g(\lambda, \mu_1, \cdots, \mu_k) \propto \lambda^a [\psi'(\lambda) - \lambda^{-1}]^b \mu_1^{-1} \cdots \mu_k^{-1}, \tag{9}$$

where $a \geq 0$ and $b > 0$. The following general theorem can be proved.

Theorem 1. The posterior distribution of $(\lambda, \mu_1, \cdots, \mu_k)$ under the general prior (9) is proper if $n_1 + \cdots + n_k + a - 2b - k - 1 > 0$.

Proof. Under the general prior (9), the joint posterior for $\lambda, \mu_1, \cdots, \mu_k$ given $\boldsymbol{x}$ is

$$\pi(\lambda, \mu_1, \cdots, \mu_k \mid \boldsymbol{x}) \propto \frac{\lambda^{n\lambda + a}}{\Gamma(\lambda)^n} [\psi'(\lambda) - \lambda^{-1}]^b \left[\prod_{i=1}^{k} \prod_{j=1}^{n_i} x_{ij}^{\lambda}\right] \tag{10}$$
$$\times \left[\prod_{i=1}^{k} \mu_i^{-n_i\lambda - 1}\right] \exp\left\{-\lambda \sum_{i=1}^{k} \frac{x_{i\cdot}}{\mu_i}\right\},$$

where $n = n_1 + \cdots + n_k$ and $x_{i\cdot} = \sum_{j=1}^{n_i} x_{ij}, i = 1, \cdots, k$. Integrating with respect to $\mu_1, \cdots, \mu_k$ in (10), we have the posterior

$$\pi(\lambda \mid \boldsymbol{x}) \propto \lambda^a [\psi'(\lambda) - \lambda^{-1}]^b \left[\prod_{i=1}^{k} \prod_{j=1}^{n_i} \frac{x_{ij}}{x_{i\cdot}}\right]^{\lambda} \left[\prod_{i=1}^{k} \frac{\Gamma(n_i\lambda)}{\Gamma(\lambda)^{n_i}}\right]. \tag{11}$$

The marginal posterior (11) is proper if $n + a - 2b - k - 1 > 0$. The proof is omitted because of its similarity to Liseo (1993). This completes the proof. □

Theorem 2. Under the general prior (9), the marginal posterior density of λ is given by

$$\pi(\lambda \mid \boldsymbol{x}) \propto \lambda^a [\psi'(\lambda) - \lambda^{-1}]^b \left[\prod_{i=1}^{k} \prod_{j=1}^{n_i} \frac{x_{ij}}{x_{i\cdot}}\right]^{\lambda} \left[\prod_{i=1}^{k} \frac{\Gamma(n_i\lambda)}{\Gamma(\lambda)^{n_i}}\right], \tag{12}$$

where $n = n_1 + \cdots + n_k$ and$x_{i\cdot} = \sum_{j=1}^{n_i} x_{ij}, i = 1, \cdots, k$.

The normalizing constant for the marginal density of λ requires an one dimensional integration. Therefore we can have the marginal posterior density of λ, and so we can compute the marginal moment of λ. In Section 4, we investigate the frequentist coverage probabilities for Jeffreys' prior and the one-at-a-time reference prior, respectively.

4. Numerical Studies and Discussion

We evaluate the frequentist coverage probability by investigating the credible interval of the marginal posteriors density of λ under the noninformative prior π given in Section 3 for several configurations $(\lambda, \mu_1, \cdots, \mu_k)$ and $(n_1, \cdots, n_k)$. That is to say, the frequentist coverage of a $100(1-\alpha)th$ posterior quantile should be close to $1-\alpha$. This is done numerically. Table 1 and 2 give numerical values of the frequentist coverage probabilities of 0.05 (0.95) posterior quantiles for the proposed prior. The computation of these numerical values is based on the following algorithm for any fixed true $(\lambda, \mu_1, \cdots, \mu_k)$ and any prespecified probability value α. Here α is 0.05(0.95). Let $\theta_1^\pi(\alpha \mid \boldsymbol{X})$ be the posterior α -quantile of λ given x. That is to say, $F(\lambda^\pi(\alpha \mid \boldsymbol{X}) \mid \boldsymbol{X}) = \alpha$, where $F(\cdot \mid \boldsymbol{X})$ is the marginal posterior distribution of λ. Then the frequentist coverage probability of this one sided credible interval of λ is

$$P_{(\lambda, \mu_1, \cdots, \mu_k)}(\alpha; \lambda) = P_{(\lambda, \mu_1, \cdots, \mu_k)}(0 < \lambda \le \lambda^\pi(\alpha \mid \boldsymbol{X})). \quad (13)$$

The estimated $P_{(\lambda, \mu_1, \cdots, \mu_k)}(\alpha; \lambda)$ when $\alpha = 0.05\,(0.95)$ is shown in Table 1 and 2 for the $k=3$ and $k=5$, respectively.

Table 1: Frequentist Coverage Probabilities of 0.05 (0.95) Posterior Quantiles for λ

λ	μ_1, μ_2, μ_3	n_1, n_2, n_3	π_1	π_2
0.5	1, 1, 1	5, 5, 5	0.130(0.980)	0.050(0.950)
		5, 5, 10	0.122(0.978)	0.054(0.950)
		10, 10, 10	0.094(0.975)	0.047(0.952)
		10, 10, 15	0.093(0.972)	0.048(0.948)
	1, 1, 3	5, 5, 5	0.136(0.983)	0.049(0.957)
		5, 5, 10	0.117(0.976)	0.050(0.949)
		10, 10, 10	0.098(0.974)	0.051(0.950)
		10, 10, 15	0.095(0.972)	0.051(0.945)
	0.5, 1, 3	5, 5, 5	0.139(0.981)	0.055(0.947)
		5, 5, 10	0.117(0.979)	0.050(0.950)
		10, 10, 10	0.094(0.974)	0.048(0.951)
		10, 10, 15	0.092(0.971)	0.052(0.948)
1	1, 1, 1	5, 5, 5	0.145(0.983)	0.049(0.949)
		5, 5, 10	0.129(0.980)	0.054(0.954)
		10, 10, 10	0.101(0.975)	0.048(0.951)
		10, 10, 15	0.097(0.974)	0.051(0.949)
	1, 1, 3	5, 5, 5	0.142(0.981)	0.051(0.948)
		5, 5, 10	0.125(0.981)	0.046(0.951)
		10, 10, 10	0.106(0.978)	0.053(0.954)
		10, 10, 15	0.098(0.973)	0.049(0.950)
	0.5, 1, 3	5, 5, 5	0.145(0.983)	0.049(0.953)
		5, 5, 10	0.125(0.978)	0.051(0.947)
		10, 10, 10	0.102(0.974)	0.050(0.948)
		10, 10, 15	0.101(0.973)	0.051(0.949)
2	1, 1, 1	5, 5, 5	0.151(0.984)	0.052(0.953)
		5, 5, 10	0.134(0.980)	0.055(0.950)
		10, 10, 10	0.108(0.975)	0.051(0.947)
		10, 10, 15	0.103(0.976)	0.051(0.953)
	1, 1, 3	5, 5, 5	0.148(0.983)	0.047(0.948)
		5, 5, 10	0.126(0.980)	0.052(0.948)
		10, 10, 10	0.106(0.977)	0.045(0.952)
		10, 10, 15	0.102(0.978)	0.049(0.951)
	0.5, 1, 3	5, 5, 5	0.153(0.983)	0.056(0.951)
		5, 5, 10	0.128(0.978)	0.050(0.951)
		10, 10, 10	0.110(0.976)	0.052(0.951)
		10, 10, 15	0.105(0.975)	0.051(0.951)

In particular, for fixed $(\lambda, \mu_1, \mu_2, \mu_3)$ and $(\lambda, \mu_1, \cdots, \mu_5)$, we take 10,000 independent random samples of $\boldsymbol{X}$ from the model (1).

For the cases presented in Table 1 and 2, we see that the one-at-a-time reference prior π_2 matches the target coverage probability much more accurately than the Jeffreys' prior π_1 for small values of n_i, and values of λ. Note that the one-at-a-time reference prior satisfies a first order matching criterion but Jeffreys' prior is not matching prior. Thus we recommend to use the one-at-a-time reference prior π_2 in the sense of asymptotic frequentist coverage property.

In the gamma populations, we have found a prior which is a first order matching prior and reference prior for the common shape parameter. We revealed that the second order matching prior does not exist. It turns out that the one-at-a-time reference prior satisfies the first order matching criterion. Also we revealed that the one-at-a-time reference prior is a joint probability matching prior for $(\lambda, \mu_1, \cdots, \mu_k)$. Thus we recommend the use of the one-at-a-time reference prior for the Bayesian inference in reliability and survival analysis.

Table 2: Frequentist Coverage Probabilities of 0.05 (0.95) Posterior Quantiles for λ

λ	$\mu_1, \mu_2, \mu_3, \mu_4, \mu_5$	n_1, n_2, n_3, n_4, n_5	π_1	π_2
0.5	1, 1, 1, 1, 1	5, 5, 5, 5, 5	0.163(0.984)	0.050(0.945)
		5, 5, 5, 10, 10	0.135(0.983)	0.047(0.952)
		10, 10, 10, 10, 10	0.124(0.981)	0.052(0.951)
		10, 10, 10, 15, 15	0.109(0.977)	0.051(0.951)
	1, 1, 1, 3, 3	5, 5, 5, 5, 5	0.161(0.985)	0.051(0.946)
		5, 5, 5, 10, 10	0.133(0.985)	0.049(0.952)
		10, 10, 10, 10, 10	0.117(0.980)	0.053(0.951)
		10, 10, 10, 15, 15	0.107(0.977)	0.052(0.949)
	0.1, 0.5, 1, 3, 5	5, 5, 5, 5, 5	0.166(0.986)	0.052(0.951)
		5, 5, 5, 10, 10	0.136(0.981)	0.052(0.943)
		10, 10, 10, 10, 10	0.113(0.983)	0.048(0.954)
		10, 10, 10, 15, 15	0.109(0.977)	0.052(0.948)
1	1, 1, 1, 1, 1	5, 5, 5, 5, 5	0.174(0.987)	0.049(0.949)
		5, 5, 5, 10, 10	0.145(0.984)	0.049(0.949)
		10, 10, 10, 10, 10	0.123(0.980)	0.046(0.950)
		10, 10, 10, 15, 15	0.112(0.977)	0.051(0.947)
	1, 1, 1, 3, 3	5, 5, 5, 5, 5	0.175(0.989)	0.050(0.950)
		5, 5, 5, 10, 10	0.147(0.984)	0.050(0.952)
		10, 10, 10, 10, 10	0.126(0.982)	0.053(0.951)
		10, 10, 10, 15, 15	0.117(0.980)	0.050(0.952)
	0.1, 0.5, 1, 3, 5	5, 5, 5, 5, 5	0.182(0.988)	0.051(0.950)
		5, 5, 5, 10, 10	0.139(0.982)	0.051(0.947)
		10, 10, 10, 10, 10	0.125(0.982)	0.049(0.951)
		10, 10, 10, 15, 15	0.110(0.980)	0.048(0.952)
2	1, 1, 1, 1, 1	5, 5, 5, 5, 5	0.190(0.990)	0.050(0.952)
		5, 5, 5, 10, 10	0.156(0.984)	0.051(0.951)
		10, 10, 10, 10, 10	0.129(0.983)	0.051(0.952)
		10, 10, 10, 15, 15	0.122(0.980)	0.051(0.952)
	1, 1, 1, 3, 3	5, 5, 5, 5, 5	0.182(0.989)	0.050(0.952)
		5, 5, 5, 10, 10	0.160(0.987)	0.055(0.950)
		10, 10, 10, 10, 10	0.125(0.983)	0.047(0.949)
		10, 10, 10, 15, 15	0.120(0.981)	0.049(0.949)
	0.1, 0.5, 1, 3, 5	5, 5, 5, 5, 5	0.186(0.990)	0.052(0.952)
		5, 5, 5, 10, 10	0.150(0.983)	0.050(0.947)
		10, 10, 10, 10, 10	0.126(0.984)	0.050(0.950)
		10, 10, 10, 15, 15	0.114(0.979)	0.049(0.947)

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