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Weight Reduction Design and the Applications

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하중 감량 설계와 적용

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Abstract

The geometry in the weight reduction design is very required to be started from the conceptual design with low cost, high performance and quality. In this point, a topological shape for optimization concerned with conceptual design of structure is important. The method used in this paper combines three optimization techniques, where the shape and physical dimensions of the structure and material distribution are hierachically optimized, with the maximum rigidity of structure and lightweight. As the applications, the technology of weight reduction design is applied on designs of aluminum control arm and inner panel of hood.

Key Words : Weight reduction design(하중 감량 설계), Size optimization(크기 최적화), Shape optimization(형상 최적화), Topology optimization(위상 최적화)

1. Introduction

Recently, developing a design configuration that fulfills various performance requirements, such as strength, stiffness and cost, must be necessary in an extensive amount of structural designs. Thus, it become important that the concept design takes into account a minimum weight structure with maximum or feasible performance based on the given constraints. Optimization techniques are useful design tools, in this point. Structural optimization can be categorized into the following three classes. First is referred to as sizing optimization, which chooses the sizes of structure as design variables, such as cross sectional dimensions of members (thickness, width, height, moment of inertia, torsional constant) in the given domain. The next important design is the shape optimization, in which the geometry of structure is varied to obtain the optimal structural shape. In shape optimization,

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the boundary of structure is variable, so parametrization of geometry is the most important aspect[1,2], In both sizing and shape optimization, the topology(connectivity and hole of element in a microstructure) is predefined. In other words, topology optimization is to find a preliminary structural configuration that meets a predefined criterion. Topology optimization can be identified into two general approached. The first approach(microstructure approach) is to find the microstructure parameters (size and orientation of hole) of each designed element in a finite element model[2]. The second approach is find the material properties of each discretized part of design domain[3,4]. Traditional shape optimization is based on the assumption that the geometry of structure is defined into the shape in its boundary and that an optimal design can be found by varying the shape of an existing initial design. Thus, this formulation cannot remove existing boundaries or add new boundaries to the design. The solutions obtained from the same topology as the initial design are far from optimal because other competing topologies cannot be explored. For these reasons, in order to be able to come up with good initial designs, topology optimization is becoming increasingly important.

The paper presents the integrated optimization procedure to generate solutions to weight reduction structure design and the effectiveness in the sizing, shape and topology designs of continuum structures for least weight and maximum stiffness. This design procedure can efficiently be applied to the typical components in cases where the appropriate treatment of structural details arise in connection with inner panels or where the inner and outer panels are adhesively bonded to form a weight reduction structure.

2. Optimization theory

2.1 Sizing and shape optimization

These methods allow to determine the physical dimensions such as thickness, height, width and the optimum shape of variable contour edges which define the geometry of the surface. The optimization algorithm belongs to the family of methods generally referred to as "gradient-based", since, in addition to function values, they use function gradients to assist in the numerical search for an optimum. The first step in a numerical search procedure is determining the direction to search.

In general, we at least need to know the gradient of our objective function and perhaps some of the constraint functions as well. In the sizing optimization, we are usually concerned with a vector of design variables, Δx, which are thickness, height, width. The gradient of the function can be written as

$$\nabla F(\bar{x}) = \begin{Bmatrix} \dfrac{\partial F}{\partial x_1} \\ \vdots \\ \dfrac{\partial F}{\partial x_n} \end{Bmatrix} = \begin{Bmatrix} \dfrac{F(x+\Delta x_1)-F(x)}{\Delta x_1} \\ \vdots \\ \dfrac{F(x+\Delta x_n)-F(x)}{\Delta x_n} \end{Bmatrix} \tag{1}$$

where each partial derivative is a single correspondent of the n dimensional vector.

Physically, in the direction of increasing objective function, we will actually move in a direction opposite to that of the gradient. The steepest descent algorithm searches in the direction defined by the negative of the objective function gradient, or

$$S = -\nabla F \tag{2}$$

For a search direction S and a vector of design variables x, the new design at the conclusion of our search in this direction can be written as

$$x^{i+1} = x^i + a\,S^{i+1} \tag{3}$$

In the shape optimization, the design domain is determined by movements of control points along the directions of the vectors required. Thus, shape design variables represent translation of the so-called control points along previously selected directions. These points describe the geometry of the boundary curves, that is, the shape of the overall model. However, because of the hierarchical construction of points, curves and surfaces of the model and the variations within an upper and lower limit, it is difficulty in treating the overall domain of structure. Thus, it is necessary that the designed domains out of the overall structure have to be chosen.

2.2 Topology optimization

The fundamental theory of topology optimization is to distribute the material property of element density for the structural rigidity. In the given domain, each element can be distributed with the following material property relationship. Once the parameter is chosen, the Young's modulus of cell can be directly represented by Eq. (5). When $n > 1$, the ratio of relative density is forced to 0 or 1, as given by

$$\rho_i = \kappa_i \, \rho_0 \tag{4}$$

$$E_i = (\kappa_i)^n \, E_0 \tag{5}$$

where ρ element density, E Young's modulus, E_i element elastic modulus, E_0 reference elastic modulus, ρ_i density of element i, ρ_0 reference density of element i, κ_i relative density ratio of element i, n density index.

From the relationship of stress-strain ($\{\sigma\}=[D]\{\varepsilon\}$), the elastic constant, $[D]$, can be given as the relative density ratio. The elasticity constant of a plane stress problem for isotropic material is

$$D(\rho_i) = \frac{E_0 \, \kappa_i^n}{1-\nu^2} \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & (1-\nu)/2 \end{bmatrix} \tag{6}$$

where $[D(\ )]$elastic constant matrix

Fig. 1 shows the microstructure of element cell. In two-dimensional case, the microstructure is formed inside an empty rectangle in a unit cell, where a, b and θ are regarded as the design variables. In order to develop a complete void, both a and b must be 1, whereas for solid material a and b must be 0. In three-dimensional case, the microstructure is formed inside an empty rectangle box in a unit cell, where a, b, c, ϕ, φ and θ are regarded as the design variables. In order to develop a complete void, a, b and c must all be 1, whereas for a solid material a, b and c must be 0. The variables ϕ, φ and θ represent the three-dimensional rotations of unit cell.

For example, to make analogy to the idea of a cellular body consisting of unit cells with rectangular holes, κ_i may be written as,

$$\kappa_i = 1 - a_i \, b_i \tag{7}$$

where a_i and b_i are the void dimensions of element i as shown in the left of Fig. 1.

The matrix of elasticity constant of a plane stress problem for isotropic material can be written as,

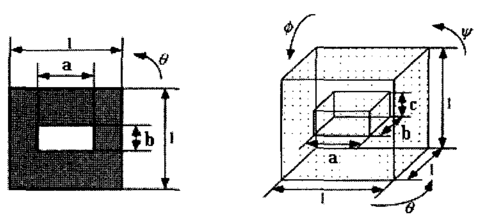
$$D(\rho_i) = \frac{E_0 \, (1-a_i \, b_i)^n}{1-\nu^2} \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & (1-\nu)/2 \end{bmatrix} \tag{8}$$

For the solid element, the elasticity constant can be given by,

$$D(\rho_i) = \frac{E_0(1-\nu)(1-a_i b_i c_i)^n}{(1+\nu)(1-2\nu)} * \begin{bmatrix} 1 & \frac{\nu}{1-\nu} & \frac{\nu}{1-\nu} & 0 & 0 & 0 \\ \frac{\nu}{1-\nu} & 1 & \frac{\nu}{1-\nu} & 0 & 0 & 0 \\ \frac{\nu}{1-\nu} & \frac{\nu}{1-\nu} & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1-2\nu}{2(1-\nu)} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1-2\nu}{2(1-\nu)} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1-2\nu}{2(1-\nu)} \end{bmatrix} \tag{9}$$

where a_i, b_i and c_i are the void dimensions of element i as shown in the right of Fig. 1.

In order to design the lightweight structure with high structural rigidity, the objective must be defined as mean compliance and the constraint as mass.



(a) 2 D design domain (b) 3 D design domain

Fig. 1 Design domain of microstructure

$$Minimize \int_{\Gamma} F_i \, x_i \, d\Gamma$$

$$Subject\ to \int_{\Omega} \rho_i(\kappa)\, d\Omega \le V_0 \text{ , at } 0 \le \kappa_i \le 1 \tag{10}$$

where F_i force vector on element i, x_i displacement vector of element i, Γ design space, Ω design domain, V_0 given volume

The variation of structural rigidity with respect to material element density can be calculated using the following relationships.

For the static problem,

$$[K]\{u\} = \{F\}$$

$$[K]\frac{\partial\{u\}}{\partial x} = -\frac{\partial[K]}{\partial x}\{u\} = \{\tilde{f}\} \tag{11}$$

where $[K]$ stiffness matrix, $\{F\}$ force vector, $\{\tilde{f}\}$ is the pseudo-load.

for the eigenvalue problem,

$$[K]\{\phi\} - \lambda[M]\{\phi\} = 0$$

$$\frac{\partial \lambda_i}{\partial x} = \frac{\{\phi_i\}^T \left(\frac{\partial[K]}{\partial x} - \lambda_i \frac{\partial[M]}{\partial x} \right) \{\phi_i\}}{\{\phi_i\}^T [M] \{\phi_i\}} \tag{12}$$

$$= \{\phi_i\}^T \frac{\partial[K]}{\partial x}\{\phi_i\} - \lambda_i \{\phi_i\}^T \frac{\partial[M]}{\partial x}\{\phi_i\}$$

$$= \frac{E'(x)}{E(x)}\{\phi_i\}^T [K]\{\phi_i\} - \frac{1}{x}\lambda_i \{\phi_i\}^T [M]\{\phi_i\}$$

where $\{x\}$ design variable vector, $[M]$ mass matrix, $\{\phi_i\}^T[M]\{\phi_i\} = 1$

The entries of stiffness matrix $[K]$ can be written by Eq. (7).

$$[K] = \int_V [B]^T [D(V)][B]\, dV \tag{13}$$

where $[B]$ spatial derivative matrix of displacement variables.

2.3 Integrated optimization procedure

The integrated optimization approach combines the optimum design techniques for maximum stiffness design of structures. In the optimization procedure, the objective function to minimize is the total elastic strain energy with a constraint on the total available volume.

$$Minimize \quad U(x_1, x_2, \ldots, x_n)$$

$$Subject\ to \quad V(x_1, x_2, \ldots, x_n) \le V_0$$

$$x_i^{\min} \le x_i \le x_i^{\max}, \quad i = 1, 2, \ldots, n \tag{14}$$

In the loop of topology optimization, material densities and orientations are solved in two separate steps for reaching the optimum. First is to define the material layout in the design domain. Second is to define the local layout in the global topological layout, which is the main topology maintaining the structural rigidity.

Since the stiffness may change dramatically when local curvature is modified, if this separate approach is used, the shape and material distribution can be geometrically optimized. And then, the sizing and shape optimization are used as the detailed optimization design.

The sizing optimization is concerned with the physical dimensions and the shape optimization is concerned with the robust local profile on the design domain. Both the detailed optimizations are inter-complemented, since the changes of local geometry on the domain can improve the stiffness relative to the increase of physical dimensions.

Through this method, the subsequent changes of geometry and material distribution in the sublevel can help

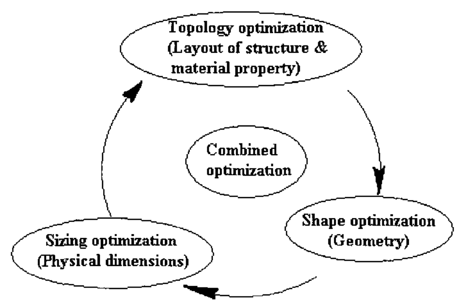


Fig. 2 Combined sizing, shape and topology optimization

to find the optimum convergence, without the influence on each other and the change of global stiffness.

3. Results and discussions of applications

Based on the proposed approach, an example is presented to demonstrate the capability and effectiveness of this implemented combined optimization method. This integrated procedure can be applied to the weight-reduced structure such as control arm, hood, door, tailgate and roof. For topology, sizing and shape optimizations, the commercial finite element code are used.

3.1 Aluminum control arm

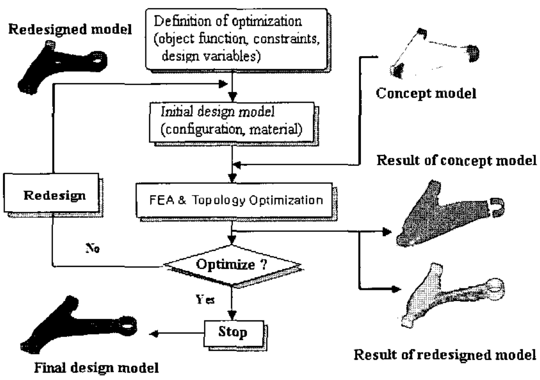
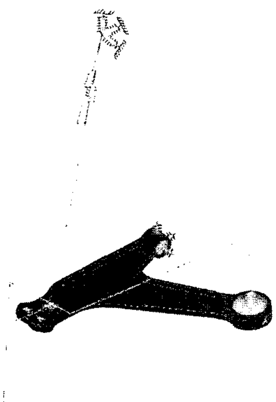
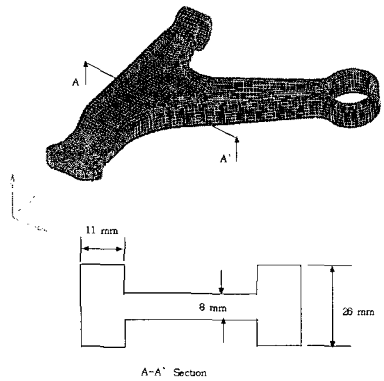


Fig. 3 Design process of aluminum control arm



(a) Finite element model



(b) Shape profile for aluminum control arm

Fig. 4 Design model descriptions

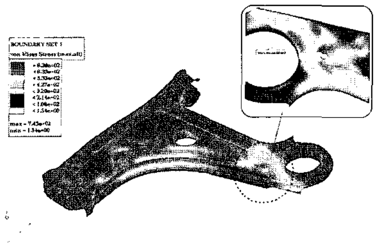


Fig. 5 Stress contour of steel control arm(Pothole brake)

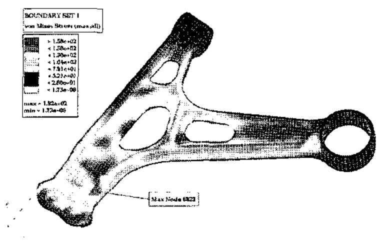


Fig. 6 Stress contour of aluminum control arm(Pothole brake)

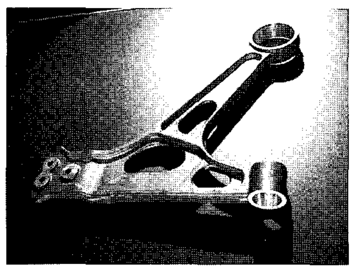


Fig. 7 Manufactured aluminum control arm

Table 1 Comparison of max. stress(unit; MPa)

Load cases	Steel	Aluminum
Pothole brake	745	182
Pothole corner	640	104
Ultimate vertical	386	62.5
Reverse brake	103	68.5
Lateral kerb strike	336	185
Oblique kerb strike	476	260

3.2 Inner panel of hood

The hood design is done for the topology shape optimization of stiffener, which is the inner panel. In this case, the hood outer panel is chosen as the initial design domain as the panel model shown in Fig. 8. Both bending and twisting loads are considered for the topological distribution of elements. The object of the shape optimization is to reduce the maximum deflections without increasing its weight. The configuration optimization problem is to find the width and height of the channel of each inner panel.

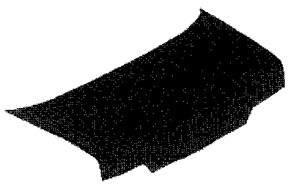


Fig. 8 Topology pattern 1 of reinforcement hood

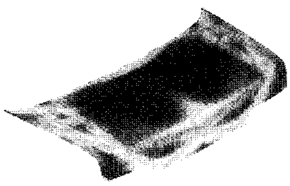


Fig. 9 Topology pattern 2 of reinforcement hood

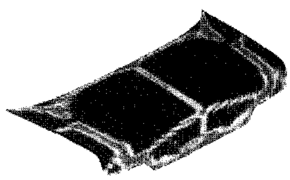


Fig. 10 Topology pattern 3 of reinforcement hood

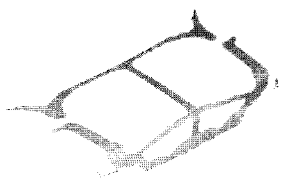


Fig. 11 Topology pattern 4 of reinforcement hood

$$\textit{Minimize the } \max\textit{imum deflections}$$

$$\textit{Subject to weight} \leq \textit{original weight}$$

for sizing optimization

$$\textit{thickness}$$

for shape optimization

$$h^L \leq \textit{change of height}(h) \leq h^U$$

$$w^L \leq \textit{change of width}(w) \leq w^U$$

$$a^L{}_j \leq \textit{configuration vector}(a_j) \leq a^U{}_j \,, j = 1,2,\ldots,n$$

where h and w are the dimensions of reinforced rib in the inner panel. a_j is the move vector of reinforced bead.

The design variables of shape optimization are the width and the height of each channel of inner panel. The number of design variables is 14.

4. Conclusions

This paper presents the optimization design methodology in order to secure the structural rigidities and lightweight of weight reduction structure.

(1) The initially structural topology is determined by topology optimization, the detailed profiles are designed by the shape optimization, and the detailed dimensions such as panel's thickness and mounting location are studied by sizing optimization.

(2) This method seems to provide an efficient tool to predict the maximum stiffness design of weight-reduction structures and serves as an excellent alternative to simultaneously optimize not only the geometry but

also the material distribution, in the early stage of development.

(3) As the result, by applying topological optimization method, the ratio of weight-reduced decreased 20% and the stiffness of structure increased 30%. These ratios can be differed by the choice of sensitivity of design variables.

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