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COMMON FIXED POINT FOR WEAK COMPATIBLE MAPPINGS OF TYPE (α) IN MENGER SPACES

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Abstract. In this paper we prove common fixed point theorem for four mappings, under the condition of compatible mappings of type (α) in Menger space, without taking any function continuous. We improve results of Pathak, Kang and Baek [13] and Cho, Murthy and Stojakovic [37].

1. Introduction

The notion of probabilistic metric space was introduced by Menger [14] which is generalization of metric space, and the study of these spaces was expanded rapidly

with the pioneering works of Schweizer and Sklar [5],[6]. The theory of probabilistic metric spaces is of fundamental importance in probabilistic function analysis.

For the detailed discussions of these spaces and their applications, we refer to [1],[3],[10],[11],[12],[19],[21] and [25]. Fixed point theorems in probabilistic metric spaces have been proved by many authors including Bharucha-Reid [2], Boscon [7], Chang [27], Ciric [16], Hadzic [22],[23],[24], Hicks [33], Singh and Pant [28],[29],[30], Stojakovic [17],[18],[19], Tan [30] and many others ([4],[8],[15], [26], [31] and [33]).

Fixed point theorems in Menger spaces have been proved by many authors including Cho, Murthy and Stojakovic [37], Dedeic and Sarapa

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[26], Radu [34],[35],[36], Stojakovic [17],[18],[19] and Pathak, Kang and Baek [13].

Many authors have shown that the papers involving contractive definition that do not require continuity of the function.

Sessa [32] defined weak commutativity and proved common fixed point theorem for weakly commuting mappings. Further Jungck [9] introduced more generalized commutativity, so called compatibility, which is more general than that of weak commutativity. Since than various fixed point theorems for compatible mappings satisfying contractive type conditions and assuming continuity of at least one of the mappings, have been obtained by many authors.

In this paper we improve results of Cho, Murthy and Stojakovic [37] and Pathak, Kang and Baek [13].

2. Preliminaries

Let R denote the set of reals and R^+ the non-negative reals. A mapping $F : R \to R^+$ is called a distribution function if it is non-decreasing left continuous with inf $F = 0$ and sup $F = 1$. We will denote by L the set of all distribution functions.

A probabilistic metric space is a pair (X, F), where X is a non empty set and F is a mapping from X×X to L.

For (u,v) $\in$ X×X, the distribution function F(u,v) is denoted by $F_{u,v}$. The functions $F_{u,v}$ are assumed to satisfy the following conditions:

(P_1) $F_{u,v}(x) = 1$ for every $x > 0$ if and only if $u = v$,

(P_2) $F_{u,v}(0) = 0$ for every $u,v \in X$,

(P_3) $F_{u,v}(x) = F_{v,u}(x)$ for every $u,v \in X$,

(P_4) If $F_{u,v}(x) = 1$ and $F_{v,w}(y) = 1$, then $F_{u,w}(x + y) = 1$ for every $u,v,w \in X$.

In a metric space (X,d),the metric d induces a mapping $F : X\times X \to L$ such that $F(u,v)(x) = F_{u,v}(x) = H(x - d(u,v))$ for every $u,v \in X$ and $x \in R$, where H is a distribution function defined by

$$H(x) = \begin{cases} 0, & x \leq 0 \\ 1, & x > 0 \end{cases}$$

A function t : [0,1] $\times$ [0,1] $\to$ [0,1] is called T-norm if it satisfies the following conditions:

(t_1) t(a,1) = a for every a $\in$ [0,1] and t(0,0) = 0,

(t_2) t(a,b) = t(b,a) for every a,b $\in$ [0,1],

(t_3) if a $\leq$ c and b $\leq$ d, then t(a,b) $\leq$ t(c,d),

(t_4) t(t(a,b),c) = t(a,t(b,c)) for every a,b,c $\in$ [0,1].

A Menger space is a triple (X,F,t), where (X,F) is a PM-space and t is a T-norm with the following condition:

(P_5) $F_{u,w}(x + y) \geq t(F_{u,v}(x), F_{v,w}(y))$ for every u,v,w $\in$ X and x,y $\in$ R^+.

The concept of neighborhoods in PM-spaces was introduced by Schweizer and Sklar [5]. If u $\in$ X, $\epsilon > 0$ and $\lambda \in (0,1)$, then an (ϵ,λ)-neighborhood of u, denoted by $U_u(\epsilon,\lambda)$, is defined by

$U_u(\epsilon,\lambda) = \{ v \in X : F_{u,v}(\epsilon) > 1- \lambda \}$.

If (X,F,t) is a Menger space with the continuous T-norm t, then the family

$\{ U_u(\epsilon,\lambda) : u \in X, \epsilon > 0, \lambda \in (0,1) \}$

of neighbourhoods induces a Hausdorff topology on X and if sup t(a,a) = 1, a < 1, it is metrizable.

An important T-norm is the T-norm t(a,b) = min{a,b} for all a,b $\in$ [0,1] and this is the unique T-norm such that t(a,a) $\geq$ a for every a $\in$ [0,1]. Indeed, if it satisfies the condition, we have

$$\begin{aligned} \min\{a,b\} &\leq \mathrm{t}(\min\{a,b\},\min\{a,b\}) \leq \mathrm{t}(a,b) \\ &\leq \mathrm{t}(\min\{a,b\},1) = \min\{a,b\}. \end{aligned}$$

Therefore, t = min.

In the sequel, we need the following definitions due to Radu [35].

Definition 2.1. Let (X,F,t) be a Menger space with continuous T-norm t. A sequence $\{p_n\}$ in X is said to be convergent to a point p $\in$ X if for every $\epsilon > 0$ and $\lambda > 0$, there exists an integer N = N(ϵ,λ) such that $p_n \in U_p(\epsilon,\lambda)$ for all n $\geq$ N, or equivalently, $F_{p,p_n}(\epsilon) > 1-\lambda$ for all n $\geq$ N. We write $p_n \to p$ as $n \to \infty$ or $\lim_{n\to\infty} p_n = p$.

Definition 2.2. Let (X,F,t) be a Menger space with continuous T-norm t. Then F is a lower semi-continuous function of points in X, that is, for any fixed x $\in$ R^+, if $q_n \to$ q and $p_n \to$ p as n $\to \infty$, then $\lim_{n\to\infty}$ inf F_{p_n, q_n} (x) = $F_{p,q}$ (x).

Definition 2.3. Let (X,F,t) be a Menger space with continuous T-norm t. A sequence $\{p_n\}$ of points in X is said to be Cauchy sequence if for every $\epsilon > 0$ and $\lambda > 0$, there exists an integer N = N(ϵ,λ) > 0 such that F_{p_n,p_m} (ϵ) > 1- λ for all m,n $\geq$ N.

Definition 2.4. A Menger space (X,F,t) with the continuous T-norm t is said to be complete if every Cauchy sequence in X converges to a point in X.

Lemma 2.1. [6],[29] Let $\{x_n\}$ be a sequence in a Menger space (X,F,t), where t is a continuous T-norm and t(x,x) $\geq$ x for all x $\in$ [0,1]. If there exists a constant k $\in$ (0,1) such that $F_{x_n,x_{n+1}}(kx) \geq F_{x_{n-1},x_n}$ (x), for all x > 0 and n $\in$ N, then $\{x_n\}$ is a Cauchy sequence in X.

The following theorems establish the relations between metric spaces and Menger spaces. It is well known that every metric space (X,d) is a Menger space (X,F, min), where the mapping $F_{x,y}$ is defined by $F_{x,y}$ (ϵ) = H(ϵ - d(x,y)). The space (X,F, min) is called the induced Menger space.

Theorem 2.1. Let t be a T-norm defined by t(a,b) = min {a,b}. Then the induced Menger space (X,F, t) is complete if a metric space (X,d) is complete.

Definition 2.5. [13] Let (X, F,t) be a Menger space such that the T-Norm t is continuous and S, T be mappings form X into itself. S and T are said to be weak compatible of type (α) if

$$\lim_{n\to\infty} F_{STx_n,TTx_n}(x) \geq F_{TSx_n,TTx_n}(x)$$

and

$$\lim_{n\to\infty} F_{TSx_n,SSx_n}(x) \geq F_{STx_n,SSx_n}(x)$$

for all x > 0, whenever $\{x_n\}$ is a sequence in X such that

$\lim_{n\to\infty} Sx_n = \lim_{n\to\infty} Tx_n = z$ for some $z \in X$.

Proposition 2.1: - Let (X, F,t) be a Menger space such that the T-norm t is continuous and $t(x,x) \geq t$ for all $x \in [0,1]$ and $S,T : X \to X$ be mappings. If S and T are weak compatible of type (α) and Sz = Tz for some $z \in X$, then SSz = STz = TSz = TTz.

Proof : Suppose that $\{x_n\}$ is a sequence in X defined by $x_n = z$, n = 1, 2, for some $z \in X$ and Sz = Tz. Then we have Sx_n, Tx_n = Sz as $n \to \infty$. Since S and T are weak compatible of type (α), for every $\epsilon > 0$,

$$\begin{aligned} F_{STz,TTz}(\epsilon) &= \lim_{n\to\infty} F_{STx_n,TTx_n}(\epsilon) \\ &\geq F_{TSx_n,TTx_n}(\epsilon) = F_{TSz,TTz}(\epsilon) = 1 \end{aligned}$$

Hence, we have STz = TTz, Therefore, we have STz = SSz = TTz = TSz since Tz = Sz. This completes the proof.

3. Common fixed point theorem

Theorem 3.1. Let (X, F,t) be a complete Menger space with t (x,y) = min{x,y} for all $x,y \in [0,1]$ and A,B,S and T be mappings from X into itself such that,

(3.1) $A(X) \subset T(X)$ and $B(X) \subset S(X)$,

(3.2) the pairs {A,S} and {B,T} are weak compatible of type (α),

(3.3) $$\begin{aligned}[F_{Au,Bv}(kx)]^2 = \min\{&[F_{Su,Tv}(x)]^2, F_{Su,Au}(x).F_{Tv,Bv}(x), \\ &F_{Su,Tv}(x).F_{Su,Au}(x), F_{Su,Tv}(x).F_{Tv,Bv}(x), \\ &F_{Su,Tv}(x).F_{Su,Bv}(2x), F_{Su,Tv}(x).F_{Tv,Au}(x), \\ &F_{Su,Bv}(2x).F_{Tv,Au}(x), F_{Su,Au}(x).F_{Tv,Au}(x), \\ &F_{Su,Bv}(2x).F_{Tv,Bv}(x)\},\end{aligned}$$

for all $u, v \in X$ and $x \geq 0$, where $k \in (0,1)$.

Then A,B,S and T have a unique common fixed point in X.

Proof : by (3.1), since $A(X) \subset T(X)$, so for any arbitrary $x_0 \in X$, there exists a point $x_1 \in X$ such that $Ax_0 = Tx_1$. Since $B(X) \subset S(X)$, for this point x_1 we can choose a point $x_2 \in X$ such that $Bx_1 = Sx_2$ and so on. Inductively, we can define a sequence $\{y_n\}$ in X such that,

(3.4) $y_{2n} = Tx_{2n+1} = Ax_{2n}$ and

$y_{2n+1} = Sx_{2n+2} = Bx_{2n+1}$ for n = 0,1,2 . . .,

Now we shall prove $F_{y_{2n},y_{2n+1}}(kx) \geq F_{y_{2n-1},y_{2n}}(x)$ for all x > 0, where k ∈ (0,1). Suppose that $F_{y_{2n},y_{2n+1}}(kx) < F_{y_{2n-1},y_{2n}}(x)$. Then by using (3.3) and $F_{y_{2n},y_{2n+1}}(kx) \leq F_{y_{2n},y_{2n+1}}(x)$, we have

$$
\begin{aligned}
[F_{y_{2n},\, y_{2n+1}}(kx)]^2 &= [F_{Ax_{2n},\, Bx_{2n+1}}(kx)]^2 \\
&\geq \min \{ [F_{Sx_{2n},\, Tx_{2n+1}}(x)]^2, F_{Sx_{2n},\, Ax_{2n}}(x) \,.\, F_{Tx_{2n+1},\, Bx_{2n+1}}(x) F_{Sx_{2n},Tx_{2n+1}}(x).\, F_{Sx_{2n},Ax_{2n}}(x), F_{Sx_{2n},Tx_{2n+1}}(x). F_{Tx_{2n+1},\, Bx_{2n+1}}(x), \\
&\quad F_{Sx_{2n},Tx_{2n+1}}(x). F_{Sx_{2n},Bx_{2n+1}}(2x), F_{Sx_{2n},\, Tx_{2n+1}}(x). F_{Sx_{2n+1},Ax_{2n}}(x), F_{Sx_{2n},\, Bx_{2n+1}}(2x). F_{Tx_{2n+1},\, Ax_{2n}}(x), \\
&\quad F_{Sx_{2n},\, Ax_{2n}}(x). F_{Tx_{2n+1},\, Ax_{2n}}(x), F_{Sx_{2n},\, Bx_{2n+1}}(2x) \,.\, F_{Tx_{2n+1},\, Bx_{2n+1}}(x) \}
\end{aligned}
$$

$$
\begin{aligned}
&= \min \{ [F_{y_{2n-1},\, y_{2n}}(x)]^2, F_{y_{2n-1},\, y_{2n}}(x) \,.\, F_{y_{2n},\, y_{2n+1}}(x), F_{y_{2n-1},\, y_{2n}}(x) \,.\, F_{y_{2n-1},\, y_{2n}}(x), F_{y_{2n-1},\, y_{2n}}(x) \,.\, F_{y_{2n},\, y_{2n+1}}(x), \\
&\quad F_{y_{2n-1},\, y_{2n}}(x) \,.\, F_{y_{2n-1},\, y_{2n+1}}(2x), F_{y_{2n-1},\, y_{2n}}(x) \,.\, F_{y_{2n},\, y_{2n}}(x), F_{y_{2n-1},\, y_{2n+1}}(2x) \,.\, F_{y_{2n},\, y_{2n}}(x), \\
&\quad F_{y_{2n-1},\, y_{2n}}(x) \,.\, F_{y_{2n},\, y_{2n}}(x), F_{y_{2n-1},\, y_{2n+1}}(2x) \,.\, F_{y_{2n},\, y_{2n+1}}(x)\}
\end{aligned}
$$

$$
\begin{aligned}
&\geq \min \{ [F_{y_{2n-1},y_{2n}}(x)]^2, F_{y_{2n-1},y_{2n}}(x) \,.\, F_{y_{2n},y_{2n+1}}(x) [F_{y_{2n-1},y_{2n}}(x)]^2, F_{y_{2n-1},y_{2n}}(x) \,.\, F_{y_{2n},y_{2n+1}}(x) F_{y_{2n-1},\, y_{2n}}(x). \\
&\quad t(F_{y_{2n-1},y_{2n}}(x), F_{y_{2n},y_{2n+1}}(x)), F_{y_{2n-1},y_{2n}}(x), t(F_{y_{2n-1},y_{2n}}(x), F_{y_{2n},y_{2n+1}}(x)), F_{y_{2n-1},\, y_{2n}}(x), \\
&\quad t(F_{y_{2n-1},\, y_{2n}}(x), F_{y_{2n},\, y_{2n+1}}(x)).\, F_{y_{2n},y_{2n+1}}(x) \} \\
&> \min \{ [F_{y_{2n},\, y_{2n+1}}(kx)]^2, [F_{y_{2n},\, y_{2n+1}}(kx)]^2, [F_{y_{2n},\, y_{2n+1}}(kx)]^2, [F_{y_{2n},\, y_{2n+1}}(kx)]^2, [F_{y_{2n},\, y_{2n+1}}(kx)]^2 \\
&\quad F_{y_{2n},\, y_{2n+1}}(kx), F_{y_{2n},\, y_{2n+1}}(kx), F_{y_{2n},\, y_{2n+1}}(kx), [F_{y_{2n},\, y_{2n+1}}(kx)]^2\} \\
&= [F_{y_{2n},\, y_{2n+1}}(kx)]^2
\end{aligned}
$$

which is a contradiction. Thus, we have

$F_{y_{2n},y_{2n+1}}(kx) \geq F_{y_{2n-1},y_{2n}}(x)$.

Similarly, we have also $F_{y_{2n+1},\ y_{2n+2}}(kx) \geq F_{y_{2n},\ y_{2n+1}}(x)$. Therefore, for every n $\in$ N, $F_{y_n,\ y_{n+1}}(kx) \geq F_{y_{n-1},\ y_n}(x)$. Therefore by lemma 2.1, $\{y_n\}$ is a Cauchy Sequence in X.

Since the Menger space (X, F, t) is complete, therefore sequence $\{y_n\}$ converges to a point z in X and the sub-sequences $\{Ax_{2n}\}$, $\{Bx_{2n+1}\}$, $\{Sx_{2n}\}$ and $\{Tx_{2n+1}\}$ of $\{y_n\}$ also converge to z.

Since B (X) $\subset$ S(X), there exists q $\in$ X such that z = Sq. Then using (3.3) we have

$$[F_{Aq,\ Bx_{2n-1}}(kx)]^2 = \min\{[F_{Sq,\ Tx_{2n-1}}(x)]^2, F_{Sq,\ Aq}(x) \,.\, F_{Tx_{2n-1},Bx_{2n-1}}(x), F_{Sq,Tx_{2n-1}}(x) \,.\, F_{Sq,\ Aq}(x), F_{Sq,\ Tx_{2n-1}}(x) \,.\, F_{Tx_{2n-1},\ Bx_{2n-1}}(x), F_{Sq,\ Tx_{2n-1}}(x) \,.\, F_{Sq,Bx_{2n-1}}(2x), F_{Sq,\ Tx_{2n-1}}(x) \,.\, F_{Tx_{2n-1},Aq}(x), F_{Sq,\ Bx_{2n-1}}(2x) \,.\, F_{Tx_{2n-1},Aq}(x), F_{Sq,Aq}(x) \,.\, F_{Tx_{2n-1},Aq}(x), F_{Sq,\ Bx_{2n-1}}(2x) \,.\, F_{Tx_{2n-1},Bx_{2n-1}}(x)\}.$$

Taking n $\to \infty$, we have

$$[F_{Aq,z}(kx)]^2 \geq \min\{[F_{z,z}(x)]^2, F_{z,Aq}(x). F_{z,z}(x), F_{z,z}(x). F_{z,Aq}(x), F_{z,z}(x). F_{z,z}(x), F_{z,z}(x). F_{z,z}(2x), F_{z,z}(x). F_{z,Aq}(x), F_{z,z}(2x). F_{z,Aq}(x), F_{z,Aq}(x). F_{z,Aq}(x), F_{z,z}(2x). F_{z,z}(x)\}$$
$$= [F_{Aq,z}(x)]^2$$

which is a contradiction, thus we have z = Aq = Sq.

Since A(X) $\subset$ T(X), there exists a point p $\in$ X such that z = Tp. Again using (3.3), we have

$$[F_{Aq,Bp}(kx)]^2 \geq \min\{[F_{Sq,Tp}(x)]^2, F_{Sq,Aq}(x) \,.\, F_{Tp,Bp}(x), F_{Sq,Tp}(x) \,.\, F_{Sq,Aq}(x), F_{Sq,Tp}(x) \,.\, F_{Tp,\ Bp}(x), F_{Sq,\ Tp}(x) \,.\, F_{Sq,\ Bp}(2x), F_{Sq,\ Tp}(x) \,.\, F_{Tp,\ Aq}(x), F_{Sq,\ Bp}(2x) \,.\, F_{Tp,\ Aq}(x), F_{Sq,\ Aq}(x). F_{Tp,\ Aq}(x), F_{Sq,Bp}(2x) \,.\, F_{Tp,\ Bp}(x)\}$$

$$[F_{z,\ Bp}(kx)]^2 \geq \min\{[F_{z,\ z}(x)]^2 F_{z,z}(x). F_{z,\ Bp}(x), F_{z,z}(x). F_{z,z}(x), F_{z,z}(x). F_{z,Bp}(x), F_{z,z}(x). F_{z,\ Bp}(2x), F_{z,z}(x). F_{z,z}(x), F_{z,Bp}(2x) .F_{z,z}(x), F_{z,\ z}(x). F_{z,z}(x), F_{z,Bp}(2x) \,.\, F_{z,Bp}(x)\}$$
$$= [F_{z,Bp}(x)]^2$$

which implies that z = Bp. Therefore z = Bp = Tp = Sq = Aq. Since the pair of maps A and S are weakly campatible of type (α) i.e Az = Sz.

Now we show that z is a common fixed point of A. If Az $\neq$ z, then by (3.3)

$$[F_{Az,z}(kx)]^2 \geq [F_{Az,Bp}(kx)]^2$$
$$\geq \min\{[F_{Sz,Tp}(x)]^2, F_{Sz,Az}(x) \,.\, F_{Tp,Bp}(x),$$
$$F_{Sz,Tp}(x) \,.\, F_{Sz,Az}(x), F_{Sz,Tp}(x) \,.\, F_{Tp,Bp}(x),$$
$$F_{Sz,Tp}(x) \,.\, F_{SzBp}(2x), F_{Sz,Tp}(x) \,.\, F_{Tp,Az}(x),$$
$$F_{Sz,Bp}(2x) \,.\, F_{Tp,Az}(x), F_{Sz,Az}(x) \,.\, F_{Tp,Az}(x),$$
$$F_{Sz,Bp}(2x) \,.\, F_{Tp,Bp}(x)\}$$
$$[F_{Az,z}(kx)]^2 \geq \min\{[F_{Az,z}(x)]^2, Az,Az(x). F_{z,z}(x),$$
$$F_{Az,z}(x). F_{Az,Az}(x),$$
$$F_{Az,z}(x) \,.\, F_{z,z}(x),$$
$$F_{Az,z}(x) \,.\, F_{Az,z}(2x),$$
$$F_{Az,z}(x).F_{z,Az}(x),$$
$$F_{Az,z}(2x). F_{z,Az}(x),$$
$$F_{Az,Az}(x). F_{z,Az}(x),$$
$$F_{Az,z}(2x).F_{z,z}(x)\}$$
$$= [F_{Az,z}(x)]^2$$

which is a contradiction. Thus we have Az = z. Hence z = Az = Sz.

Similarly, pair of maps B and T are weakly compatible of type (α), we have Bz = Tz Again by (3.3), we have

$$[F_{z,Bz}(kx)]^2 \geq \min\{[F_{z,Bz}(x)]^2, F_{z,z}(x) \,.\, F_{Bz,Bz}(x),$$
$$F_{z,Bz}(x). F_{z,z}(x), F_{z,Bz}(x). F_{Bz,Bz}(x),$$
$$F_{z,Bz}(x). F_{z,Bz}(2x), F_{z,Bz}(x). F_{Bz,z}(x),$$
$$F_{z,Bz}(2x) \,.\, F_{Bz,z}(x), F_{z,z}(x) \,.F_{Bz,z}(x),$$
$$F_{z,Bz}(2x) \,.\, F_{Bz,Bz}(x)\}$$
$$= [F_{z,Bz}(x)]^2$$

Therefore, z = Bz = Tz = Az = Sz, and z is a common fixed point of A,B,S and T.

Finally, in order to prove the uniqueness of z, suppose that z and w, (z $\neq$ w), are two common fixed points of A,B,S and T. Then by (3.3), we obtain.

$$
\begin{aligned}
[F_{z,w}(kx)]^2 \geq \min\{ & [F_{z,w}(x)]^2, F_{z,z}(x).\ F_{w,w}(x), \\
& F_{z,w}(x)\ .\ F_{z,z}(x), F_{z,w}(x)\ .\ F_{w,w}(x), \\
& F_{z,w}(x).\ F_{z,w}(2x), F_{z,w}(x)\ .\ F_{w,z}(x), \\
& F_{z,w}(2x)\ .\ F_{w,z}(x), F_{z,z}(x)\ .\ F_{w,z}(x), \\
& F_{z,w}(2x).\ F_{w,w}(x) \} \\
= [F_{z,w}(x)]^2, &
\end{aligned}
$$

which is a contradiction therefore z = w. This completes the proof of the theorem.

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