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FUZZY NEARLY C-COMPACTNESS IN GENERALIZED FUZZY TOPOLOGY

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ABSTRACT. In this paper the concept of fuzzy nearly C-compactness is introduced in Generalized fuzzy topological spaces. Several characterizations and some interesting properties of these spaces in Generalized fuzzy topological spaces are discussed. The properties of fuzzy almost continuous and fuzzy almost open functions in Generalized fuzzy topological spaces are also studied.

1. Introduction

C. L. Chang [2] introduced and developed the concept of fuzzy toplogical spaces based on the concept of fuzzy sets introduced by Zadeh [18]. Since then, various important notions in the classical topology such as compactness have been extended to fuzzy topological spaces. The concept of nearly C-compactness in general topology was introduced in [17]. The same was introduced in fuzzy topological spaces in [16]. The concept of Generalized topology was introduced and studied in [4] and this concept has been introduced and studied in fuzzy topological spaces in [15].

The purpose of this paper is to introduce and study the concept of fuzzy nearly C-compactness in Generalized fuzzy topological spaces introduced and studied in [15]. Several characterizations and some interesting properties of these spaces in Generalized fuzzy topological spaces are discussed. Section 2 deals with preliminaries. Section 3

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deals with the concept of fuzzy nearly C-compactness in Generalized fuzzy topological spaces and some of its characterizations. In Section 4, the concepts of $(\mathcal{G}, \mathcal{G}')$-fuzzy almost continuous and $(\mathcal{G}, \mathcal{G}')$-fuzzy almost open functions are introduced and studied. Section 5 deals with the concepts of fuzzy nearly C-compactness in Generalized fuzzy Bitopological spaces.

2. Preliminaries

Let X be a non-empty set. Let $\gamma : I^X \to I^X$ be a function. γ is said to be monotonic if $\lambda_1 \leq \lambda_2 \Rightarrow \gamma(\lambda_1) \leq \gamma(\lambda_2)$ for all $\lambda_1, \lambda_2 \in I^X$. $\Gamma_F(X)$ denotes the collection of all such monotone functions. A fuzzy subset $\lambda \in I^X$ is said to be γ-fuzzy open if $\lambda \leq \gamma(\lambda)\,; \gamma \in \Gamma_F(X)$.

Let $\mathcal{G}$ denote a collection of fuzzy subsets of X. Then $\mathcal{G}$ is called a generalized fuzzy topology on X (briefly GFT) if

(i) $\overline{0}$, the zero fuzzy set is in $\mathcal{G}$
(ii) If $\lambda_\alpha \in \mathcal{G}$ for $\alpha \in \Delta$, then $\bigvee_{\alpha\in\Delta} \lambda_\alpha \in \mathcal{G}$.

Let $\mathcal{G}$ be any generalized fuzzy topology. Then there is a monotonic function $\gamma : I^X \to I^X$ such that $\mathcal{G}$ is the collection of all γ- fuzzy open sets, we can suppose that γ satisfies

(i) $\gamma\left(\overline{0}\right) = \overline{0}$ where $\overline{0}$ is the zero fuzzy set is in $\mathcal{G}$
(ii) $\gamma(\lambda) < \lambda$
(iii) $\gamma\gamma(\lambda) = \gamma(\lambda)$ for $\lambda \in I^X$.

We define γ-fuzzy interior of λ as $i_\gamma^F(\lambda) = \vee\{\mu/\mu \leq \lambda, \mu \text{ is } \gamma - \text{fuzzy open}\}$ and γ-fuzzy closure of λ as $c_\gamma^F(\lambda) = \wedge\{\mu/\lambda \leq \mu, \mu \text{ is } \gamma\text{-fuzzy closed}\}$.

λ is γ- fuzzy regular open if and only if $i_\gamma^F C_\gamma^F(\lambda) = \lambda$.

λ is γ- fuzzy regular open if and only if $C_\gamma^F i_\gamma^F(\lambda) = \lambda$.

Let X be any non-empty set. Let $\gamma : I^X \to I^X$ be any monotone function. Then we know the set of all γ-fuzzy open sets form generalized fuzzy topology say $\mathcal{G}$.

3. Fuzzy nearly C-compactness in Generalized fuzzy topology

In [16] the concept of fuzzy nearly C-compact space in fuzzy topological spaces was defined as follows:

DEFINITION 3.1. Let (X,T) be a fuzzy topological space. (X,T) is said to be *fuzzy nearly C-compact* if for any ordinary subset A of X, $A \neq X$ such that $\mathcal{X}_A$ (The characteristic function of $A \subset X$) is a proper fuzzy regular closed set and for each fuzzy open cover of $\{\lambda_\alpha/\alpha \in \Delta\}$ of $\mathcal{X}_A$ there exists a finite subfamily $\lambda_{\alpha_1}, \lambda_{\alpha_2}, \cdots, \lambda_{\alpha_n}$ such that $\mathcal{X}_A \leq \bigvee_{i=1}^{n} Cl(\lambda_{\alpha_i})$.

Motivated by the above concept we are now ready to make the following:

DEFINITION 3.2. X is said to be *γ-fuzzy nearly C-compact* if for any subset A of X, $A \neq X$ such that χ_A is proper γ-fuzzy regular closed and for each γ- fuzzy open $\{\lambda_\alpha\}_{\alpha\in\Delta}$ of χ_A there exists a finite subfamily $\lambda_{\alpha_1}, \cdots, \lambda_{\alpha_n}$ such that $\chi_A \leq \bigvee_{i=1}^{n} C_\gamma^F(\lambda_{\alpha_i})$.

Result: The following result established in [16] are used in this paper and they are stated here for easy reference.

PROPOSITION 3.3. *Let $(X,\mathcal{G})$ and $(X',\mathcal{G}')$ be any two generalized fuzzy topologies. Let $f:(X,\mathcal{G}) \to (X',\mathcal{G}')$ be a $(\mathcal{G},\mathcal{G}')$- fuzzy almost continuous mapping and $(\mathcal{G},\mathcal{G}')$- fuzzy almost open mapping. Then*

(a) *the inverse image $f^{-1}(\lambda)$ of each γ'-fuzzy regular open set λ of X' is a γ-fuzzy regular open set in X;*

(b) *the inverse image $f^{-1}(\mu)$ of each γ'-fuzzy regular closed set μ of X' is a γ-fuzzy regular closed set in X.*

Proof. (a) Let λbe an arbitrary γ-fuzzy regular open set in X. Then since f is $(\mathcal{G},\mathcal{G}')$- fuzzy almost continuous mapping, $f^{-1}(\lambda)$ is $(\mathcal{G},\mathcal{G}')$- fuzzy almost open mapping and hence we obtain that $f^{-1}(\lambda) \leq i_\gamma^F C_\gamma^F$ $f^{-1}(\lambda)$ is fuzzy regular closed $f^{-1}\left(C_\gamma^F(\lambda)\right)$ is fuzzy closed and hence $i_\gamma^F C_\gamma^F f^{-1}(\lambda) \leq C_\gamma^F f^{-1}\left(C_\gamma^F(\lambda)\right) = f^{-1}\left(C_\gamma^F(\lambda)\right)$, since $C_\gamma^F(\lambda)$ is fuzzy regular closed . As f is $(\mathcal{G}, \mathcal{G}')$- fuzzy almost open and $i_\gamma^F\ f^{-1}(\lambda)$ is γ-fuzzy regular open set in $X, f\left(i_\gamma^F C_\gamma^F f^{-1}(\lambda)\right) \leq f f^{-1}\left(C_\gamma^F(\lambda)\right) \leq C_\gamma^F(\lambda)$ and thus $f\left(i_\gamma^F C_\gamma^F f^{-1}(\lambda)\right) = i_\gamma^F f\left[i_\gamma^F C_\gamma^F f^{-1}(\lambda)\right] = i_\gamma^F C_\gamma^F(\lambda) = \lambda$. Hence $i_\gamma^F C_\gamma^F f^{-1}(\lambda) \leq f^{-1}(\lambda)$ and since $f^{-1}(\lambda)$ is γ-fuzzy open set in X we have $f^{-1}(\lambda) \leq i_\gamma^F C_\gamma^F f^{-1}(\lambda)$. Thus $f^{-1}(\lambda)$ is a γ-fuzzy regular open set in X. This proves (a).

(b) follows easily from (a). □

PROPOSITION 3.4. *The following assertions are equivalent:*

1. *X is γ-fuzzy nearly C-compact.*
2. *For each subset $A \subset X$ such that χ_A is proper γ-fuzzy regular closed and for each γ-fuzzy regular open cover $\mathfrak{U} = \{\lambda_\alpha\}_{\alpha\in\Delta}$ of χ_A there exists a finite subfamily $\lambda_{\alpha_1}, \lambda_{\alpha_2}, \cdots, \lambda_{\alpha_n}$ of $\mathfrak{U}$ such that $\chi_A \leq \bigvee_{i=1}^{n} C_\gamma^F(\lambda_{\alpha_i})$.*
3. *For each subset $A \subset X$ such that χ_A is proper γ-fuzzy regular closed set and for each family $\mathfrak{F} = \{\mu_\alpha\}_{\alpha\in\Delta}$ of nonzero γ-fuzzy regular closed set such that $\left(\bigwedge_{\alpha\in\Delta} \mu_\alpha\right) \bigwedge \chi_A = 0$, there exists a finite subfamily $\mu_{\alpha_1}, \mu_{\alpha_2}, \cdots, \mu_{\alpha_n}$ of $\mathfrak{F}$ such that $\left(\bigwedge_{i=1}^{n} i_\gamma^F(\mu_{\alpha_i})\right) \bigwedge \chi_A = 0$.*
4. *For each subset $A \subset X$ such that χ_A is proper γ-fuzzy regular closed set and for each family $\mathfrak{F} = \{\mu_\alpha\}_{\alpha\in\Delta}$ of γ-fuzzy regular closed sets such that for each finite subfamily $\mu_{\alpha_1}, \mu_{\alpha_2}, \cdots, \mu_{\alpha_n}$ of $\mathfrak{F}$ we have $\left(\bigwedge_{i=1}^{n} i_\gamma^F(\mu_{\alpha_i})\right) \bigwedge \chi_A \neq 0$ then $\left[\bigwedge_{\alpha\in\Delta} \mu_\alpha\right] \bigwedge \chi_A \neq 0$.*

Proof. (1)$\Longrightarrow$(2) follows from the definition.

(2)$\Longrightarrow$(1) Suppose (2) holds. Let A be any subset of X such that χ_A is proper γ- fuzzy regular closed set. Let $\{\lambda_\alpha\}_{\alpha\in\Delta} = \mathfrak{U}$ be a γ-fuzzy open cover of χ_A . Then $\left\{C_\gamma^F i_\gamma^F \lambda_a\right\}_{\alpha\in\Delta}$ will be a γ- fuzzy regular open cover of χ_A. Then by (2) there exists a finite subfamily $\left\{C_\gamma^F i_\gamma^F \lambda_{a_i}\right\}_{i=1}^{n}$ such that

$$\chi_A \leq \vee C_\gamma^F \left\{C_\gamma^F i_\gamma^F \lambda_{a_i}\right\} \leq \bigvee_{i=1}^{n} C_\gamma^F i_\gamma^F(\lambda_{a_i}) \leq \bigvee_{i=1}^{n} C_\gamma^F(\lambda_{a_i}).$$

This proves (2)$\Longrightarrow$(1).

(2)$\Longrightarrow$(3) Let $A \subset X$ be such that χ_A is proper γ- fuzzy regular closed. Let $\mathfrak{F} = \{\mu_\alpha\}_{\alpha\in\Delta}$ be a family of non-zero γ- fuzzy regular closed sets of X such that $(\wedge\mu_\alpha)_{\alpha\in\Delta} \wedge \chi_A = 0$ for each proper γ- fuzzy regular closed set χ_A of X. Then $\mathfrak{U} = \{1-\mu_\alpha\}_{\alpha\in\Delta}$ is a γ- fuzzy regular open cover of the γ- fuzzy regular closed set χ_A and therefore by assumption (2) there exists a finite subfamily$\{\lambda_{a_i} = 1-\mu_{a_i} | i = 1, 2, \cdots, n\}$ of $\mathfrak{U}$ such that $\chi_A \leq \bigvee_{i=1}^{n} C_\gamma^F(\lambda_{a_i})$.

Now for each α_i we have

$$i_\gamma^F(\mu_{a_i}) = i_\gamma^F(1-\lambda_{a_i}) = 1 - C_\gamma^F(1-(1-\lambda_{a_i})) = 1 - C_\gamma^F(\mu_{a_i}).$$

Therefore

$$\bigwedge_{i=1}^{n} i_\gamma^F(\mu_{a_i}) = 1 - \bigvee_{i=1}^{n}\left\{C_\gamma^F(\lambda_{a_i})\right\} \leq 1 - \chi_A$$

and so

$$\left(\bigwedge_{i=1}^{n} i_\gamma^F(\mu_{a_i})\right) \bigwedge \chi_A \leq (1-\chi_A) \bigwedge \chi_A = \chi_{X-A} \bigwedge \chi_A = 0.$$

This proves (2) $\Longrightarrow$(3).

(3) $\Longrightarrow$(2) Let $\mathfrak{U} = \{\lambda_\alpha\}_{\alpha\in\Delta}$ be γ- fuzzy regular open cover of the proper γ- fuzzy regular closed set χ_A of X where $A \subset X$. Now $\chi_A \leq \bigvee_{\alpha\in\Delta} \lambda_\alpha$ implies that $(1-\chi_A) \geq \left(1 - \bigvee_{\alpha\in\Delta}\lambda_\alpha\right)$ and

$$\begin{aligned}\bigwedge_{\alpha\in\Delta}(1-\lambda_\alpha)\bigwedge\chi_A &= \left(1-\bigvee_{\alpha\in\Delta}\lambda_\alpha\right)\bigwedge\chi_A \\ &\leq (1-\chi_A)\bigwedge\chi_A \\ &= \chi_{X-A}\bigwedge\chi_A = 0.\end{aligned}$$

So $\{1-\lambda_\alpha\}_{\alpha\in\Delta}$ is a family of γ- fuzzy regular closed sets such that $\bigwedge_{\alpha\in\Delta}(1-\lambda_\alpha)\bigwedge\chi_A = 0$ and so by (3) there exists a finite subfamily $\{1-\lambda_{\alpha_1}, \cdots 1-\lambda_{\alpha_n}\}$ such that $\left[\bigwedge_{i=1}^{n} i_\gamma^F(1-\lambda_{\alpha_i})\right]\bigwedge\chi_A = 0$. Now it follows that $\chi_A \leq \bigvee_{i=1}^{n} \left[1 - i_\gamma^F (1-\lambda_{\alpha_i})\right]$. But for each α_i we have

$$i_\gamma^F (1-\lambda_{\alpha_i}) = 1 - C_\gamma^F (1-(1-\lambda_{\alpha_i})) = 1 - C_\gamma^F (\lambda_{\alpha_i}).$$

Therefore we conclude that

$$\chi_A \leq \bigvee_{i=1}^{n} C_\gamma^F (\lambda_{\alpha_i}).$$

This proves (3) $\Longrightarrow$(2).

(3) $\Longrightarrow$(4) Let $A \subset X$ be such that χ_A is a proper γ- fuzzy regular closed set and suppose $\{\mu_\alpha\}_{\alpha\in\Delta}$ is a family of γ- fuzzy regular closed set such that for every finite family $\mu_{a_1}, \cdots, \mu_{a_n}$ of $\{\mu_\alpha\}_{\alpha\in\Delta}$,

$$\bigwedge_{i=1}^{n} i_\gamma^F (\mu_{\alpha_i}) \wedge \chi_A \neq 0.$$

We want to show that $\bigwedge_{\alpha\in\Delta} \mu_\alpha \bigwedge \chi_A \neq 0$. If we suppose $\bigwedge_{\alpha\in\Delta} \mu_\alpha \bigwedge \chi_A = 0$, then by assumption (3) there exists a finite subfamily $\mu_{a_1}, \cdots, \mu_{a_n}$ such that $\bigwedge_{i=1}^{n} i_\gamma^F (\mu_{\alpha_i}) \bigwedge \chi_A = 0$, which is a contradiction. Hence $\bigwedge_{\alpha\in\Delta} \mu_\alpha \bigwedge \chi_A \neq 0$.

(4) $\Longrightarrow$(3) Let $A \subset X$ be such that χ_A is a proper γ- fuzzy regular closed set. Let $\{\mu_\alpha\}_{\alpha\in\Delta}$ be a family of γ- fuzzy regular closed sets of X such that $\bigwedge_{\alpha\in\Delta} \mu_\alpha \bigwedge \chi_A = 0$. We have to show that there exists a finite integer (say) n_0 such that $\bigwedge_{i=1}^{n_0} i_\gamma^F (\mu_{\alpha_i}) \bigwedge \chi_A = 0$. Suppose now that for every finite integer n_0 we have $\bigwedge_{i=1}^{n_0} i_\gamma^F (\mu_{\alpha_i}) \bigwedge \chi_A \neq 0$. Then by assumption (4) we have $\bigwedge_{\alpha\in\Delta} i_\gamma^F \mu_\alpha \bigwedge \chi_A \neq 0$.Therefore $0 \neq \bigwedge_{\alpha\in\Delta} i_\gamma^F (\mu_\alpha) \bigwedge \chi_A \leq \bigwedge_{\alpha\in\Delta} (\mu_\alpha) \bigwedge \chi_A$, which is a contradiction. Hence there exists a finite integer n_0 such that $\bigwedge_{i=1}^{n_0} i_\gamma^F (\mu_{\alpha_i}) \bigwedge \chi_A = 0$. This proves (4) $\Longrightarrow$(3). □

PROPOSITION 3.5. *For any GF Topological space $(X, \mathcal{G})$ the following assertions are equivalent*
1. X is γ-fuzzy nearly C-compact
2. If $A \subset X$ is such that χ_A is a proper γ-fuzzy regular closed and $\mathfrak{F} = \{\lambda_\alpha\}_{\alpha\in\Delta}$ is a family of γ-fuzzy regular closed sets of X such that $\chi_A \leq \left(1 - \bigwedge_{\alpha\in\Delta} \lambda_\alpha\right)$, then there exists a finite number of elements of $\mathfrak{F}$ say $\lambda_{\alpha_1}, \lambda_{\alpha_2}, \cdots, \lambda_{\alpha_n}$ such that $\chi_A < 1 - \bigwedge_{i=1}^{n} i_\gamma^F(\lambda_{\alpha_i})$.

Proof. (1) $\Longrightarrow$ (2). Suppose X is γ-fuzzy nearly C-compact. Let $A \subset X$ be such that χ_A is a proper γ-fuzzy regular closed set. Let $\mathfrak{F}$ be a family of γ-fuzzy regular closed sets of X such that $\chi_A < \left(1 - \bigwedge_{\alpha\in\Delta} \lambda_\alpha\right) = \bigvee_{\alpha\in\Delta} (1 - \lambda_\alpha)$. Clearly $\mathfrak{U} = (1 - \lambda_\alpha)_{\alpha\in\Delta}$ is a γ-fuzzy regular open cover of χ_A. Hence by assumption (1) there exists a finite number of elements (say) $\lambda_{\alpha_1}, \lambda_{\alpha_2}, \cdots, \lambda_{\alpha_n}$ such that $\chi_A \leq \bigvee_{i=1}^{n} c_\gamma^F(\lambda_{\alpha_i})$. Therefore $\bigwedge_{i=1}^{n} i_\gamma^F(\lambda_{\alpha_i}) = 1 - \bigvee_{i=1}^{n} c_\gamma^F(\lambda_{\alpha_i}) \leq 1 - \chi_A$. That is $\chi_A \leq 1 - \bigwedge_{i=1}^{n} i_\gamma^F(\lambda_{\alpha_i})$. This proves (1) $\Longrightarrow$ (2).

(2) $\Longrightarrow$ (1). Let $A \subset X$ be such that χ_A is a proper γ-fuzzy regular closed set in X. Let $\mathfrak{F}$ be a family of γ-fuzzy regular open sets of X such that $\chi_A < \bigvee_{\lambda\in\mathfrak{F}} \lambda$. Put $\mathfrak{U} = \{1 - \lambda\}_{\lambda\in\mathfrak{F}}$. Then $\mathfrak{U}$ is clearly a family of γ-fuzzy regular closed set of X such that $\chi_A < \bigvee_{\lambda\in\mathfrak{F}} \lambda = \bigvee_{\lambda\in\mathfrak{F}} [1 - (1 - \lambda)] = 1 - \bigwedge_{\lambda\in\mathfrak{F}} (1 - \lambda)$. Hence by assumption (2) there exists a finite number of elements say $1 - \lambda_1, 1 - \lambda_2, \cdots, 1 - \lambda_n$ such that $\chi_A \leq \bigwedge_{i=1}^{n} i_\gamma^F(1 - \lambda_i) = \bigvee_{i=1}^{n} 1 - i_\gamma^F(1 - \lambda_i) \leq \bigvee_{i=1}^{n} c_\gamma^F(\lambda_i)$. This proves (2) $\Longrightarrow$ (1). □

4. Properties of generalized fuzzy almost continuous and generalized fuzzy almost open functions

DEFINITION 4.1. Consider two GFT's $\mathcal{G}$ and $\mathcal{G}'$ on X and X' respectively, we say that f is $(\mathcal{G}, \mathcal{G}')$-*fuzzy continuous* if $\lambda' \in \mathcal{G}' \implies f^{-1}(\lambda') \in \mathcal{G}$.

DEFINITION 4.2. Let $(X, \mathcal{G})$ and $(X^{'}, \mathcal{G}^{'})$ be any two generalized fuzzy topological spaces. A mapping $f : (X, \mathcal{G}) \to (X^{'}, \mathcal{G}^{'})$ is said to be $(\mathcal{G}, \mathcal{G}')$-*fuzzy almost continuous* if the inverse image of every γ'-fuzzy regular open(closed) set is γ-fuzzy open(closed).

DEFINITION 4.3. Let $(X, \mathcal{G})$ and $(X^{'}, \mathcal{G}^{'})$ be any two generalized fuzzy topological spaces. A mapping $f : (X, \mathcal{G}) \to (X^{'}, \mathcal{G}^{'})$ is said to be $(\mathcal{G}, \mathcal{G}')$-*fuzzy almost open (closed)* if the image of every γ-fuzzy regular open(closed) set is γ'-fuzzy open(closed).

PROPOSITION 4.4. *Let $f : (X, \mathcal{G}) \to (X', \mathcal{G}')$ be a $(\mathcal{G}, \mathcal{G}')$-fuzzy almost open (almost closed) map of a GFT space $(X, \mathcal{G})$ onto a GFT space $(X', \mathcal{G}')$ and if $h : (X', \mathcal{G}') \to (X'', \mathcal{G}'')$. If $h \circ f$ is a $(\mathcal{G}, \mathcal{G}'')$-fuzzy almost continuous and $(\mathcal{G}, \mathcal{G}'')$- fuzzy almost open map ,then h is $(\mathcal{G}', \mathcal{G}'')-$ fuzzy almost continuous.*

Proof. First let us assume f is $(\mathcal{G}, \mathcal{G}')-$ fuzzy almost closed. Let λ be γ-fuzzy regular closed subset of X''. Then

$$(h \circ f)^{-1}(\lambda) = f^{-1}\left(h^{-1}(\lambda)\right)$$

and by Proposition 3.3 $(h \circ f)^{-1}(\lambda)$ is a γ-fuzzy regular closed set in X. Since f is $(\mathcal{G}, \mathcal{G}^{'})$-fuzzy almost closed and surjective,

$$f\left(f^{-1}\left(h^{-1}(\lambda)\right)\right) = h^{-1}(\lambda)$$

is γ-fuzzy regular closed inX'. Thus we have shown that h is $(\mathcal{G}', \mathcal{G}'')$-fuzzy almost continuous. The proof is similar when f is $(\mathcal{G}, \mathcal{G}^{'})$- fuzzy almost open. □

PROPOSITION 4.5. *Assume that $f : (X, \mathcal{G}) \to (X', \mathcal{G}')$ is $(\mathcal{G}, \mathcal{G}')$-fuzzy almost continuous and let $h : X' \to X''$ be a $(\mathcal{G}^{'}, \mathcal{G}'')$- fuzzy almost continuous and $(\mathcal{G}^{'}, \mathcal{G}'')$- fuzzy almost open map. Then $h \circ f$ is $(\mathcal{G}, \mathcal{G}')$- fuzzy almost continuous.*

Proof. Let λ be a γ-fuzzy regular open in X''. Then by Proposition 3.3 $h^{-1}(\lambda)$ is γ-fuzzy regular open in X' and $(h \circ f)^{-1}(\lambda)$ $= f^{-1}[h^{-1}(\lambda)]$ is γ-fuzzy open in X. This proves $h \circ f$ is $(\mathcal{G}, \mathcal{G}'')$-fuzzy almost continuous. □

PROPOSITION 4.6. *Let $f : (X, \mathcal{G}) \to (X', \mathcal{G}')$ and $h : X' \to X''$ and suppose that $h \circ f$ is a $(\mathcal{G}, \mathcal{G}'')$-fuzzy almost open [$(\mathcal{G}, \mathcal{G}'')$-fuzzy almost closed] map. If f is $(\mathcal{G}, \mathcal{G}'')$-fuzzy almost continuous and $(\mathcal{G}, \mathcal{G}')$-fuzzy almost open surjection, then h is a $(\mathcal{G}', \mathcal{G}'')$-fuzzy almost open (closed) map.*

Proof. Let λ be any γ-fuzzy regular open set in X'. Then by Proposition 3.3, $f^{-1}(\lambda)$ is γ-fuzzy regular open in X. Since $h \circ f$ is $(\mathcal{G}, \mathcal{G}'')$-fuzzy almost open $h \circ f[f^{-1}(\lambda)] = h[ff^{-1}(\lambda)] = h(\lambda)$ is γ-fuzzy open in X''. This proves that h is a $(\mathcal{G}, \mathcal{G}')$- fuzzy almost open. The proof is similar when f is $(\mathcal{G}, \mathcal{G}')$- fuzzy almost closed. □

LEMMA 4.7. *Let $f : (X, \mathcal{G}) \to (X', \mathcal{G}')$ be any $(\mathcal{G}, \mathcal{G}')$- fuzzy almost open map. Given any $\lambda \in I^{X'}$ and γ-fuzzy regular closed set μ containing $f^{-1}(\lambda)$, there exists a fuzzy closed set $\theta \geq \lambda$ such that $f^{-1}(\lambda) \leq \mu$.*

Proof. Let $\lambda \in I^X$ and let μ be any γ-fuzzy rgular closed set such that $f^{-1}(\lambda) \leq \mu$. Since f is $(\mathcal{G}, \mathcal{G}^{'})$- fuzzy almost open and $1 - \mu$ is γ-fuzzy regular open set, it follows that $\theta = 1_{X'} - f(1_X - \mu)$ is a γ-fuzzy closed set in X' and $\theta = 1_{X'} - f(1_X - \mu) \geq 1_{X'} - f[1_X - f^{-1}(\lambda)] \geq \lambda$. Now $1_X - \mu \leq f^{-1}f(1_X - \mu)$ which implies that $1_X - (1_X - \mu) \geq 1_X - f^{-1}f[1 - \mu]$. That is, $\mu \geq 1_X - f^{-1}f[1 - \mu]$. We conclude that $f^{-1}(\theta) = f^{-1}[1_{X'} - f^{-1}(1_X - \mu)] = 1_X - f^{-1}[f(1_X - \mu)] \leq \mu$. □

LEMMA 4.8. *Let $f : (X, \mathcal{G}) \to (X^{'}, \mathcal{G}^{'})$ be a $(\mathcal{G}, \mathcal{G}^{'})$- fuzzy almost continuous and $(\mathcal{G}, \mathcal{G}^{'})$- fuzzy almost open map and let λ be any γ-fuzzy closed subset of Y. Then*

1. $c_\gamma^F\left(f^{-1}\left(i_\gamma^F(\lambda)\right)\right) = f^{-1}\left(c_\gamma^F\left(i_\gamma^F(\lambda)\right)\right)$;
2. *If λ is a γ-fuzzy regular closed subset, then* $c_\gamma^F\left(f^{-1}\left(i_\gamma^F(\lambda)\right)\right) = f^{-1}(\lambda)$.

Proof. (1) Let λ be any γ-fuzzy closed set of X'. Assume $i_\gamma^F(\lambda) \neq 0$(since there is nothing to prove if $i_\gamma^F(\lambda) = 0$). Hence $i_\gamma^F(\lambda)$ is γ-fuzzy regular open. Since f is $(\mathcal{G}, \mathcal{G}^{'})$-fuzzy almost continuous, $f^{-1}\left(i_{\gamma}^{F}(\lambda)\right)$ is γ-open in X. Also $c_{\gamma}^{F}\left[f^{-1}\left(i_{\gamma}^{F}(\lambda)\right)\right]$ is a γ-fuzzy regular closed set in X containing $f^{-1}\left(i_{\gamma}^{F}(\lambda)\right)$. Therefore by Lemma 4.7, there exists a γ- fuzzy closed set $\theta \geq i_{\gamma}^{F}(\lambda)$ such that $f^{-1}\left(i_{\gamma}^{F}(\lambda)\right) \leq f^{-1}(\theta) \leq c_{\gamma}^{F}\left(f^{-1}\left(i_{\gamma}^{F}(\lambda)\right)\right)$. This implies that $f^{-1}\left(c_{\gamma}^{F}\left(i_{\gamma}^{F}(\lambda)\right)\right) \leq c_{\gamma}^{F} f^{-1}\left(i_{\gamma}^{F}(\lambda)\right)$. Now since f is $(\mathcal{G}, \mathcal{G}^{'})$-fuzzy almost continuous, $f^{-1}\left[c_{\gamma}^{F}\left(i_{\gamma}^{F}(\lambda)\right)\right]$ is γ-fuzzy closed in X and so $f^{-1}\left(i_{\gamma}^{F}(\lambda)\right) \leq f^{-1}\left[c_{\gamma}^{F}\left(i_{\gamma}^{F}(\lambda)\right)\right]$. Therefore $c_{\gamma}^{F}\left[f^{-1}\left(i_{\gamma}^{F}(\lambda)\right)\right] \leq c_{\gamma}^{F}\left[f^{-1}\left(c_{\gamma}^{F}\left(i_{\gamma}^{F}(\lambda)\right)\right)\right] = f^{-1}\left[c_{\gamma}^{F}\left(i_{\gamma}^{F}(\lambda)\right)\right] \leq c_{\gamma}^{F}\left[f^{-1}\left(i_{\gamma}^{F}(\lambda)\right)\right]$. This proves (1).

(2) If λ is γ-fuzzy regular closed subset, then $c_{\gamma}^{F}\left[\left(i_{\gamma}^{F}(\lambda)\right)\right] = \lambda$.Using in (1), we get $c_{\gamma}^{F}\left[f^{-1}\left(i_{\gamma}^{F}(\lambda)\right)\right] = f^{-1}(\lambda)$. This proves (2). □

PROPOSITION 4.9. *Let $f : (X, \mathcal{G}) \to (X^{'}, \mathcal{G}^{'})$ be a $(\mathcal{G}, \mathcal{G}^{'})$-fuzzy almost continuous and $(\mathcal{G}, \mathcal{G}^{'})$-fuzzy almost open map. If μ is γ-fuzzy open set in $X^{'}$, then*

1. $i_{\gamma}^{F}\left[c_{\gamma}^{F}\left[i_{\gamma}^{F}\left(c_{\gamma}^{F}(\lambda)\right)\right]\right] = i_{\gamma}^{F}\left[c_{\gamma}^{F} f^{-1}\left(c_{\gamma}^{F}(\mu)\right)\right] = i_{\gamma}^{F}\left[c_{\gamma}^{F}(\mu)\right];$
2. $f^{-1}\left[i_{\gamma}^{F}\left(c_{\gamma}^{F}(\lambda)\right)\right] = i_{\gamma}^{F}\left[f^{-1} c_{\gamma}^{F}(\lambda)\right]$
3. *If μ is γ-fuzzy regular open set in Y, then* $i_{\gamma}^{F}\left[f^{-1}\left[f^{-1}\left(c_{\gamma}^{F}(\mu)\right)\right]\right] = f^{-1}(\mu)$.

Proof. (1) Let μ be a γ-fuzzy open set in X'. Since $c_{\gamma}^{F}(\mu)$ is a fuzzy regular set in X', $f^{-1}\left(c_{\gamma}^{F}(\mu)\right)$ is γ-fuzzy closed in X. In fact, by Lemma 4.8, we have $c_{\gamma}^{F}\left[f^{-1}\left(i_{\gamma}^{F}\left(c_{\gamma}^{F}(\mu)\right)\right)\right] = f^{-1}\left(c_{\gamma}^{F}(\mu)\right)$.Therefore we have $i_{\gamma}^{F} c_{\gamma}^{F}\left[f^{-1}\left(i_{\gamma}^{F} c_{\gamma}^{F}(\mu)\right)\right] = i_{\gamma}^{F} c_{\gamma}^{F}\left[f^{-1} c_{\gamma}^{F}(\mu)\right] = i_{\gamma}^{F}\left[f^{-1} c_{\gamma}^{F}(\mu)\right]$. This proves (1).

(2) By Proposition 3.3 we have that $f^{-1}\left(i_{\gamma}^{F}\left(c_{\gamma}^{F}(\mu)\right)\right)$ is γ-fuzzy regular open set in X. Thus

$$i_{\gamma}^{F} c_{\gamma}^{F}\left[c_{\gamma}^{F}(\mu)\right] = f^{-1}\left[i_{\gamma}^{F} c_{\gamma}^{F}(\mu)\right] = i_{\gamma}^{F}\left[f^{-1} c_{\gamma}^{F}(\mu)\right].$$

This proves (2).

(3) Since μ is a γ-fuzzy regular open set, $\mu = i_{\gamma}^{F} c_{\gamma}^{F}(\mu)$.Hence it follows that $f^{-1}\left(i_{\gamma}^{F} c_{\gamma}^{F}(\mu)\right) = f^{-1}(\mu) = i_{\gamma}^{F}\left[f^{-1}\left(c_{\gamma}^{F}(\mu)\right)\right]$.This proves (3). □

Proposition 4.10. *The image of a γ-fuzzy nearly C-compact space under a $(\mathcal{G}, \mathcal{G}^{'})$ -fuzzy almost continuous and$(\mathcal{G}, \mathcal{G}^{'})$ - fuzzy almost open mapping is γ-fuzzy nearly C-compact.*

Proof. Let $f : (X, \mathcal{G}) \rightarrow (X^{'}, \mathcal{G}^{'})$ be $(\mathcal{G}, \mathcal{G}^{'})$- fuzzy almost continuous and fuzzy almost open mapping from a γ-fuzzy nearly C-compact space X onto X'. We have to show that is X' is also γ-fuzzy nearly C-compact, Let $A \subset X'$ be any subset of X' such that χ_A is γ-fuzzy regular closed in X'. Let $\mathcal{U} = \{\lambda_i\}_{i\in\Delta}$ be a γ-fuzzy regular open cover of χ_A in X'.Since f is $(\mathcal{G}, \mathcal{G}^{'})$- fuzzy almost continuous and $(\mathcal{G}, \mathcal{G}^{'})$-fuzzy almost open, by Proposition 3.3, $f^{-1}\ (\chi_A)$ is a γ-fuzzy regular closed subset of X and $\{f^{-1}(\lambda_i)\}$ is a γ-fuzzy regular open cover of $f^{-1}\ (\chi_A)$ in X. Since X is γ-fuzzy nearly C-compact, there exists a finite subfamily $\{f^{-1}(\lambda_i)\,/ i = 1, 2, \cdots, n\}$ such that $f^{-1}\ (\chi_A) \leq \bigvee\limits_{i=1}^{n} c_\gamma^F \{f^{-1}(\lambda_i)\} \leq \bigvee\limits_{i=1}^{n} \left\{f^{-1}\left(c_\gamma^F(\lambda_i)\right)\right\}$.That is $\chi_A \leq \bigvee\limits_{i=1}^{n} \left\{c_\gamma^F(\lambda_i)\right\}$. This proves that Y is γ-fuzzy nearly C-compact. □

5. Fuzzy Nearly C-compactness in Generalized Fuzzy Bitopological Spaces

The concept of fuzzy bitopological spaces was introduced in [11] and subsequently further studied by various authors [3, 12]. In [11], the definition of fuzzy bitopological space was given as follows :

Definition 5.1. A fuzzy bitopological space is an ordered triple $(X, \mathcal{G}_1, \mathcal{G}_2)$ where $\mathcal{G}_1$ and $\mathcal{G}_2$ are fuzzy topologies on X.

Based on the above, the generalized fuzzy bitopological spaces is defined as follows :

Definition 5.2. A generalized fuzzy bitopological space is an ordered triple $(X, \mathcal{G}_1, \mathcal{G}_2)$ where $\mathcal{G}_1$ and $\mathcal{G}_2$ are generalized fuzzy topologies in X.

The concept of pairwise fuzzy nearly C-compact space was introduced in [16] as follows.

DEFINITION 5.3. The fuzzy bitopological space $(X, \mathcal{G}_1, \mathcal{G}_2)$ is said to be $(\mathcal{G}_1, \mathcal{G}_2)$- fuzzy nearly C-compact if for every set $A \subset X$ such that χ_A is a proper $\mathcal{G}_1$-fuzzy regular closed set and for every $\mathcal{G}_2$-fuzzy open cover $\mathcal{U}$ of A, there exists a finite sub-collection of $\mathcal{U}$,(say) $\lambda_1, \lambda_2, \cdots, \lambda_n$ such that $\chi_A \leq \text{cl}_{\mathcal{G}_2}(\lambda_i)$.$(X, \mathcal{G}_1, \mathcal{G}_2)$ is said to be pairwise fuzzy nearly C-compact if it is both $(\mathcal{G}_1, \mathcal{G}_2)$- fuzzy nearly C-compact and $(\mathcal{G}_2, \mathcal{G}_1)$ -fuzzy nearly C-compact.

Based on the above defintion we are now ready to make the following.

DEFINITION 5.4. The fuzzy bitopological space $(X, \mathcal{G}_1, \mathcal{G}_2)$ is said to be $(\mathcal{G}_1, \mathcal{G}_2)$-$\gamma$-fuzzy nearly C-compact if for every set $A \subset X$ such that χ_A is a proper $\mathcal{G}_1$-γ-fuzzy open cover $\mathcal{U}$ of χ_A,there exists a finite sub-collection of $\mathcal{U}$,(say) $\lambda_1, \lambda_2, \cdots, \lambda_n$ such that $\chi_A \leq \bigvee_{i=1}^{n} c^F_{\gamma-\mathcal{G}_2}(\lambda_i)$. $(X, \mathcal{G}_1, \mathcal{G}_2)$ is pairwise γ-fuzzy nearly C-compact if it is both $(\mathcal{G}_1, \mathcal{G}_2)$-$\gamma$-fuzzy nearly C-compact and $(\mathcal{G}_2, \mathcal{G}_1)$ -γ-fuzzy nearly C-compact.

DEFINITION 5.5. A mapping $f : (X, \mathcal{G}_1, \mathcal{G}_2) \to (X, \mathcal{F}_1, \mathcal{F}_2)$ is said to be pairwise γ-fuzzy almost continuous (pairwise γ-fuzzy almost open, pairwise γ-fuzzy continuous) if the induced mappings $f : (X, \mathcal{G}_1) \to (X, \mathcal{F}_1)$ and $f : (X, \mathcal{G}_2) \to (X, \mathcal{F}_2)$ are γ- fuzzy almost continuous (γ-fuzzy almost open, γ- fuzzy continuous).

PROPOSITION 5.6. *Every pairwise γ-fuzzy almost continuous and pairwise γ-fuzzy almost open image of a pairwise γ-fuzzy nearly C-compact spare is pairwise γ-fuzzy nearly C-compact.*

Proof. Let $f : (X, \mathcal{G}_1, \mathcal{G}_2) \to (Y, \mathcal{F}_1, \mathcal{F}_2)$ be any pairwise γ-fuzzy almost continuous and pairwise γ-fuzzy almost open onto mapping. Assume $(X, \mathcal{G}_1, \mathcal{G}_2)$ is pairwise γ-fuzzy nearly C-compact. We want to show that $(Y, \mathcal{F}_1, \mathcal{F}_2)$ is pairwise γ-fuzzy nearly C-compact.

Let $A \subset Y$ be such that χ_A is a proper $\mathcal{F}_1$-fuzzy regular closed set and let $\mathcal{U}$ be a $\mathcal{F}_2 - \gamma$- fuzzy open cover of χ_A . Since f is $(\mathcal{G}_1, \mathcal{G}_2)$-fuzzy almost continuous and $(\mathcal{G}, \mathcal{G}')-$ fuzzy almost open, $f^{-1}(\chi_A)$ is $\mathcal{G}_1 - \gamma$ - fuzzy regular closed by Proposition 3.3 and $\{f^{-1}(\mu) / \mu \in \mathcal{U}\}$ is a $\mathcal{G}_2 - \gamma-$ fuzzy open cover of $f^{-1}(\chi_A)$. Since $(X, \mathcal{G}_1, \mathcal{G}_2)$ is pairwise - fuzzy nearly C-compact there exists a finite sub-collection

$$\left\{f^{-1}(\mu_k) / k = 1, 2, \cdots n\right\}$$

such that $f^{-1}(\chi_A) \leq \bigvee_{i=1}^{n} c^F_{\gamma-\mathcal{G}_2} f^{-1}(\mu_k)$. Hence

$$\begin{aligned} \chi_A &= ff^{-1}(\chi_A) \\ &\leq \bigvee_{i=1}^{n} f\left[c^F_{\gamma-\mathcal{G}_2} f^{-1}(\mu_k)\right] \\ &\leq \bigvee_{k=1}^{n} f\left[c^F_{\gamma-\mathcal{F}_2} ff^{-1}(\mu_k)\right] \\ &\leq \bigvee_{k=1}^{n} c^F_{\gamma-\mathcal{F}_2}(\mu_k). \end{aligned}$$

This proves that Y is $(\mathcal{G}_1, \mathcal{G}_2)$- γ- fuzzy nearly C-compact. Similarly we can show that Y is also $(\mathcal{G}_2, \mathcal{G}_1)$-$\gamma$-fuzzy nearly C-compact. Thus we have shown that $(Y, \mathcal{F}_1, \mathcal{F}_2)$ is pairwise γ-fuzzy nearly C-compact. □

Proposition 5.7. *Let $(X, \mathcal{G}_1, \mathcal{G}_2)$ be any pairwise γ-fuzzy nearly C-compact space. Then if $A \subset X$ is such that χ_A is proper $\mathcal{G}_1$-fuzzy regular closed and $\mathcal{F}$ is a family of $\mathcal{G}_1 - \gamma -$ fuzzy closed subsets of X such that $\wedge\{\lambda \wedge \chi_A / \lambda \in \mathcal{F}\} = 0$, there exists a finite number of elements, say $\{\lambda_1, \lambda_2, \cdots, \lambda_n\}$ of $\mathcal{F}$ such that $\bigwedge_{k=1}^{n} \left\{ i^F_{\gamma-\mathcal{G}_j} \lambda_k \wedge \chi_A \right\} = 0, i \neq j, j = 1, 2.$*

Proof. Suppose $(X, \mathcal{G}_1, \mathcal{G}_2)$ is pairwise γ-fuzzy nearly C-compact. Let $A \subset X$ be such that χ_A is proper $\mathcal{G}_i - \gamma$-fuzzy regular closed and $\mathcal{F}$ is a family of $\mathcal{G}_j - \gamma$- fuzzy closed sets of X such that $\wedge\{\lambda \wedge \chi_A / \lambda \in \mathcal{F}\} = 0$. Now $\bigwedge_{\lambda \in \mathcal{F}} \{\lambda \wedge \chi_A\} = 0 \Longrightarrow \bigwedge_{\lambda \in \mathcal{F}} (\lambda) \leq 1 - \chi_A \Longrightarrow \chi_A \leq 1 - \bigwedge_{\lambda \in \mathcal{F}} (\lambda) = \vee\{1 - \lambda / \lambda \in \mathcal{F}\}$. So $\{1 - \lambda / \lambda \in \mathcal{F}\}$ is a $\mathcal{G}_j - \gamma$-fuzzy open cover of χ_A which is $\mathcal{G}_i - \gamma -$fuzzy regular closed and hence by assumption we have a finite collection, say $\{1 - \lambda_1, 1 - \lambda_2, \cdots, 1 - \lambda_n\}$ such that

$$\begin{aligned} \chi_A &\leq \bigvee_{k=1}^{n} c^F_{\gamma-\mathcal{G}_j}(1 - \lambda_k) = \bigvee_{k=1}^{n} \left[1 - i^F_{\gamma-\mathcal{G}_j} \lambda_k\right] \\ &= 1 - \bigwedge_{k=1}^{n} i^F_{\gamma-\mathcal{G}_j}(\lambda_k) \end{aligned}$$

This implies $\bigwedge_{k=1}^{n} i^F_{\gamma-\mathcal{G}_j}\lambda_k \leq 1-\chi_A = \chi_{X-A}$. Therefore

$$\chi_A \wedge \left(\bigwedge_{k=1}^{n} i^F_{\gamma-\mathcal{G}_j}\lambda_k\right) \leq \chi_A \wedge \chi_{X-A} = 0 \Longrightarrow \bigwedge_{k=1}^{n} i^F_{\gamma-\mathcal{G}_j}\left[\lambda_k \wedge \chi_A\right] = 0.$$

□

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