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INTUITIONISTIC FUZZY SETS IN GAMMA-SEMIGROUPS

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ABSTRACT. We consider the intuitionistic fuzzification of the concept of several Γ-ideals in a Γ-semigroup S, and investigate some properties of such Γ-ideals.

1. Introduction

The notion of a fuzzy set in a set was introduced by L. A. Zadeh [10], and since then this concept has been applied to various algebraic structures. K. T. Atanassov [1] defined the notion of an intuitionistic fuzzy set, as a concept more general than a fuzzy set (see also [2]). Using fuzzy ideals, N. Kuroki [5] discussed characterizations of semigroups (see also [6]). K. H. Kim and Y. B. Jun [3] considered the intuitionistic fuzzification of the notion of several ideals in a semigroup, and investigated some properties of such ideals (see also [4]). M. K. Sen and N. K. Saha [9] defined the concept of a Γ-semigroup, and established a relation between regular Γ-semigroup and Γ-group (see also [7], [8]). In this paper, we introduce the notion of an intuitionistic fuzzy Γ-ideal of a Γ-semigroup, and we investigate some properties connected with intuitionistic fuzzy Γ-ideals in a Γ-semigroup.

2. Preliminaries

Let $S = \{x, y, z, \ldots\}$ and $\Gamma = \{\alpha, \beta, \gamma, \ldots\}$ be two non-empty sets. Then S is called a Γ-*semigroup* if it satisfies

- $x\gamma y \in S$,
- $(x\beta y)\gamma z = x\beta(y\gamma z)$

for all $x, y, z \in S$ and $\beta, \gamma \in \Gamma$. A non-empty subset U of a Γ-semigroup S is said to be a Γ-*subsemigroup* of S if $U\Gamma U \subseteq U$. A *left* (*right*) Γ-*ideal* of a Γ-semigroup S is a non-empty subset U of S such that $S\Gamma U \subseteq U$ ($U\Gamma S \subseteq U$). If U is both a left and a right Γ-ideal of a Γ-semigroup S, then we say that U is

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a Γ-*ideal* of S. A Γ-subsemigroup U of a Γ-semigroup S is called an *interior* Γ-*ideal* of S if $S\Gamma U\Gamma S \subseteq U$. A Γ-*bi-ideal* of a Γ-semigroup S is a Γ-subsemigroup U of S such that $U\Gamma S\Gamma U \subseteq U$. Let $L[x]$ denote the principal left Γ-ideal of a Γ-semigroup S generated by x in S, that is, $L[x] = \{x\} \cup S\Gamma x$. A Γ-semigroup S is said to be *regular* if, for each $x \in S$, there exist $s \in S$ and $\beta, \gamma \in \Gamma$ such that $x = x\beta s\gamma x$. A Γ-semigroup S is called *left-zero* (*right-zero*) if $x\gamma y = x$ ($x\gamma y = y$) for all $x, y \in S$ and $\gamma \in \Gamma$. A Γ-semigroup S is said to be *left* (*right*) *simple* if S has no proper left (right) Γ-ideals. If a Γ-semigroup S has no proper Γ-ideals, then we say that S is *simple*. An element e in a Γ-semigroup S is called an *idempotent* if $e\gamma e = e$ for some $\gamma \in \Gamma$. Let E_S denote the set of all idempotents in a Γ-semigroup S.

By a *fuzzy set* μ in a non-empty set X we mean a function $\mu : X \longrightarrow [0,1]$ and the complement of μ, denoted by $\bar{\mu}$, is the fuzzy set in X given by $\bar{\mu}(x) = 1 - \mu(x)$ for all $x \in X$. An *intuitionistic fuzzy set* (briefly IFS) A in a non-empty set X is an object having the form

$$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle \mid x \in X\},$$

where the functions $\mu_A : X \longrightarrow [0,1]$ and $\nu_A : X \longrightarrow [0,1]$ define the degree of membership and the degree of non-membership of the element $x \in X$ to the set A, which is a subset of X, respectively, and

$$0 \leq \mu_A(x) + \nu_A(x) \leq 1$$

for all $x \in X$. An intuitionistic fuzzy set $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle \mid x \in X\}$ in X can be identified to an ordered pair (μ_A, ν_A) in $I^X \times I^X$ where $I = [0,1]$. For the sake of simplicity, we shall use the symbol $A = (\mu_A, \nu_A)$ for the IFS $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle \mid x \in X\}$. Let χ_U denote the characteristic function of a non-empty subset U of a Γ-semigroup S.

3. Intuitionistic fuzzy Γ-ideals

In what follows, let S denote a Γ-semigroup unless otherwise specified.

Definition 3.1. For an IFS $A = (\mu_A, \nu_A)$ in S, consider the following axioms:

(ΓS_1) $\mu_A(x\gamma y) \geq \min\{\mu_A(x), \mu_A(y)\}$,
(ΓS_2) $\nu_A(x\gamma y) \leq \max\{\nu_A(x), \nu_A(y)\}$

for all $x, y \in S$ and $\gamma \in \Gamma$. Then $A = (\mu_A, \nu_A)$ is called a *first* (*resp. second*) *intuitionistic fuzzy* Γ-*subsemigroup* (briefly $IF\Gamma S_1$ (resp. $IF\Gamma S_2$)) of S if it satisfies (ΓS_1) (resp. (ΓS_2)). Also, $A = (\mu_A, \nu_A)$ is said to be an *intuitionistic fuzzy* Γ-*subsemigroup* (briefly $IF\Gamma S$) of S if it is both a first and a second intuitionistic fuzzy Γ-subsemigroup.

Theorem 3.2. *If U is a Γ-subsemigroup of S, then $\tilde{U} = (\chi_U, \bar{\chi}_U)$ is an $IF\Gamma S$ of S.*

Proof. Let $x, y \in S$ and $\gamma \in \Gamma$. From the hypothesis, $x\gamma y \in U$ if $x, y \in U$. In this case

$$\chi_U(x\gamma y) = 1 \geq \min\{\chi_U(x), \chi_U(y)\}$$

and

$$\begin{aligned}\bar{\chi}_U(x\gamma y) &= 1 - \chi_U(x\gamma y) \\ &\leq 1 - \min\{\chi_U(x), \chi_U(y)\} \\ &= \max\{1 - \chi_U(x), 1 - \chi_U(y)\} \\ &= \max\{\bar{\chi}_U(x), \bar{\chi}_U(y)\}.\end{aligned}$$

If $x \notin U$ or $y \notin U$, then $\chi_U(x) = 0$ or $\chi_U(y) = 0$. Thus

$$\chi_U(x\gamma y) \geq 0 = \min\{\chi_U(x), \chi_U(y)\}$$

and

$$\begin{aligned}\max\{\bar{\chi}_U(x), \bar{\chi}_U(y)\} &= \max\{1 - \chi_U(x), 1 - \chi_U(y)\} \\ &= 1 - \min\{\chi_U(x), \chi_U(y)\} \\ &= 1 \geq \bar{\chi}_U(x\gamma y).\end{aligned}$$

This completes the proof. □

Theorem 3.3. *Let U be a non-empty subset of S. If $\tilde{U} = (\chi_U, \bar{\chi}_U)$ is an $IF\Gamma S_1$ or $IF\Gamma S_2$ of S, then U is a Γ-subsemigroup of S.*

Proof. Suppose that $\tilde{U} = (\chi_U, \bar{\chi}_U)$ is an $IF\Gamma S_1$ of S and $x \in U\Gamma U$. In this case, $x = u\gamma v$ for some $u, v \in U$ and $\gamma \in \Gamma$. It follows from (ΓS_1) that

$$\chi_U(x) = \chi_U(u\gamma v) \geq \min\{\chi_U(u), \chi_U(v)\} = 1.$$

Hence $\chi_U(x) = 1$, i.e. $x \in U$. Thus U is a Γ-subsemigroup of S.

Now, assume that $\tilde{U} = (\chi_U, \bar{\chi}_U)$ is an $IF\Gamma S_2$ of S and $x' \in U\Gamma U$. Then $x' = u'\gamma' v'$ for some $u', v' \in U$ and $\gamma' \in \Gamma$. Using (ΓS_2), we get that

$$\begin{aligned}\bar{\chi}_U(x') &= \bar{\chi}_U(u'\gamma' v') \\ &\leq \max\{\bar{\chi}_U(u'), \bar{\chi}_U(v')\} \\ &= \max\{1 - \chi_U(u'), 1 - \chi_U(v')\} = 0\end{aligned}$$

and so $\bar{\chi}_U(x') = 1 - \chi_U(x') = 0$. Therefore $\chi_U(x') = 1$, i.e. $x' \in U$. This completes the proof. □

Definition 3.4. For an IFS $A = (\mu_A, \nu_A)$ in S, consider the following axioms:
$(L\Gamma I_1)$ $\mu_A(x\gamma y) \geq \mu_A(y)$,
$(L\Gamma I_2)$ $\nu_A(x\gamma y) \leq \nu_A(y)$
for all $x, y \in S$ and $\gamma \in \Gamma$. Then $A = (\mu_A, \nu_A)$ is called a *first* (*resp. second*) *intuitionistic fuzzy left* Γ*-ideal* (briefly $IFL\Gamma I_1$ (resp. $IFL\Gamma I_2$)) of S if it satisfies $(L\Gamma I_1)$ (resp. $(L\Gamma I_2)$). Also, $A = (\mu_A, \nu_A)$ is said to be an *intuitionistic fuzzy left* Γ*-ideal* (briefly $IFL\Gamma I$) of S if it is both a first and a second intuitionistic fuzzy left Γ-ideal.

Definition 3.5. For an IFS $A = (\mu_A, \nu_A)$ in S, consider the following axioms:
$(R\Gamma I_1)$ $\mu_A(x\gamma y) \geq \mu_A(x)$,
$(R\Gamma I_2)$ $\nu_A(x\gamma y) \leq \nu_A(x)$
for all $x, y \in S$ and $\gamma \in \Gamma$. Then $A = (\mu_A, \nu_A)$ is called a *first (resp. second) intuitionistic fuzzy right Γ-ideal* (briefly $IFR\Gamma I_1$ (resp. $IFR\Gamma I_2$)) of S if it satisfies $(R\Gamma I_1)$ (resp. $(R\Gamma I_2)$). Also, $A = (\mu_A, \nu_A)$ is said to be an *intuitionistic fuzzy right Γ-ideal* (briefly $IFR\Gamma I$) of S if it is both a first and a second intuitionistic fuzzy right Γ-ideal.

Definition 3.6. Let $A = (\mu_A, \nu_A)$ be an IFS in S. Then $A = (\mu_A, \nu_A)$ is called an *intuitionistic fuzzy Γ-ideal* (briefly $IF\Gamma I$) of S if it is both an intuitionistic fuzzy left and an intuitionistic fuzzy right Γ-ideal.

Proposition 3.7. *Let U be a left-zero Γ-subsemigroup of S. If $A = (\mu_A, \nu_A)$ is an $IFL\Gamma I$ of S, then the restriction of A to U is constant, that is, $A(x) = A(y)$ for all $x, y \in U$.*

Proof. Let $x, y \in U$. Since U is left-zero, $x\gamma y = x$ and $y\gamma x = y$ for all $\gamma \in \Gamma$. In this case, from the hypothesis, we have that

$$\mu_A(x) = \mu_A(x\gamma y) \geq \mu_A(y),$$
$$\mu_A(y) = \mu_A(y\gamma x) \geq \mu_A(x)$$

and

$$\nu_A(x) = \nu_A(x\gamma y) \leq \nu_A(y),$$
$$\nu_A(y) = \nu_A(y\gamma x) \leq \nu_A(x).$$

Thus we obtain $\mu_A(x) = \mu_A(y)$ and $\nu_A(x) = \nu_A(y)$ for all $x, y \in U$. Hence $A(x) = A(y)$ for all $x, y \in U$. □

Lemma 3.8. *If U is a left Γ-ideal of S, then $\tilde{U} = (\chi_U, \bar{\chi}_U)$ is an $IFL\Gamma I$ of S.*

Proof. Let $x, y \in S$ and $\gamma \in \Gamma$. Since U is a left Γ-ideal of S, $x\gamma y \in U$ if $y \in U$. It follows that

$$\chi_U(x\gamma y) = 1 = \chi_U(y)$$

and

$$\bar{\chi}_U(x\gamma y) = 1 - \chi_U(x\gamma y) = 0 = 1 - \chi_U(y) = \bar{\chi}_U(y).$$

If $y \notin U$, then $\chi_U(y) = 0$. In this case

$$\chi_U(x\gamma y) \geq 0 = \chi_U(y)$$

and

$$\bar{\chi}_U(y) = 1 - \chi_U(y) = 1 \geq \bar{\chi}_U(x\gamma y).$$

Consequently, $\tilde{U} = (\chi_U, \bar{\chi}_U)$ is an $IFL\Gamma I$ of S. □

Theorem 3.9. *Let $A = (\mu_A, \nu_A)$ be an $IFL\Gamma I$ of S. If E_S is a left-zero Γ-subsemigroup of S, then $A(e) = A(e')$ for all $e, e' \in E_S$.*

Proof. Let $e, e' \in E_S$. From the hypothesis, $e\gamma e' = e$ and $e'\gamma e = e'$ for all $\gamma \in \Gamma$. Thus, since $A = (\mu_A, \nu_A)$ is an $IFL\Gamma I$ of S, we get that

$$\mu_A(e) = \mu_A(e\gamma e') \geq \mu_A(e'),$$
$$\mu_A(e') = \mu_A(e'\gamma e) \geq \mu_A(e)$$

and

$$\nu_A(e) = \nu_A(e\gamma e') \leq \nu_A(e'),$$
$$\nu_A(e') = \nu_A(e'\gamma e) \leq \nu_A(e).$$

Hence we have $\mu_A(e) = \mu_A(e')$ and $\nu_A(e) = \nu_A(e')$ for all $e, e' \in E_S$. This completes the proof. □

Theorem 3.10. *Let S be regular. If, for every non-empty subset U of S, $\tilde{U} = (\chi_U, \bar{\chi}_U)$ is an $IFL\Gamma I_1$ (or $IFL\Gamma I_2$) of S and $\tilde{U}(e) = \tilde{U}(e')$ for all $e, e' \in E_S$, then E_S is a left-zero Γ-subsemigroup of S.*

Proof. Since S is regular, E_S is non-empty. Let $e = e\gamma e$, $e' = e'\gamma' e' \in E_S$ where $\gamma, \gamma' \in \Gamma$. Because of S is regular, $L[e] = S\Gamma e$. Since $L[e]$ is a left Γ-ideal of S, we obtain $\tilde{L}[e] = (\chi_{L[e]}, \bar{\chi}_{L[e]})$ is an $IFL\Gamma I_1$ (or $IFL\Gamma I_2$) of S by Lemma 3.8. In this case, from the hypothesis, we get that

$$\chi_{L[e]}(e') = \chi_{L[e]}(e) = 1 \text{ (or } \bar{\chi}_{L[e]}(e') = \bar{\chi}_{L[e]}(e) = 0).$$

Hence $e' \in L[e] = S\Gamma e$. Thus

$$e' = x\beta e = x\beta(e\gamma e) = (x\beta e)\gamma e = e'\gamma e$$

for some $x \in S$ and $\beta \in \Gamma$. Consequently, E_S is a left-zero Γ-semigroup. □

Definition 3.11. For an IFS $A = (\mu_A, \nu_A)$ in S, consider the following axioms:

($I\Gamma I_1$) $\mu_A(x\beta s\gamma y) \geq \mu_A(s)$,
($I\Gamma I_2$) $\nu_A(x\beta s\gamma y) \leq \nu_A(s)$

for all $s, x, y \in S$ and $\beta, \gamma \in \Gamma$. Then $A = (\mu_A, \nu_A)$ is called a *first* (*resp. second*) *intuitionistic fuzzy interior* Γ*-ideal* (briefly $IFI\Gamma I_1$ (resp. $IFI\Gamma I_2$)) of S if it is an $IF\Gamma S_1$ (resp. $IF\Gamma S_2$) satisfying ($I\Gamma I_1$) (resp. ($I\Gamma I_2$)). Also, $A = (\mu_A, \nu_A)$ is said to be an *intuitionistic fuzzy interior* Γ*-ideal* (briefly $IFI\Gamma I$) of S if it is both a first and a second intuitionistic fuzzy interior Γ-ideal.

Remark 3.12. It is clear that every $IF\Gamma I$ of S is an $IFI\Gamma I$ of S.

Theorem 3.13. *If S is regular, then every $IFI\Gamma I$ of S is an $IF\Gamma I$ of S.*

Proof. Let $A = (\mu_A, \nu_A)$ be an $IFI\Gamma I$ of S and $x, y \in S$. In this case, because of S is regular, there exist $s, s' \in S$ and $\beta, \beta', \gamma, \gamma' \in \Gamma$ such that $x = x\beta s\gamma x$

and $y = y\beta' s'\gamma' y$. Thus

$$\begin{aligned}\mu_A(x\alpha' y) &= \mu_A(x\alpha'(y\beta' s'\gamma' y)) \\ &= \mu_A(x\alpha' y\beta'(s'\gamma' y)) \\ &\geq \mu_A(y)\end{aligned}$$

and

$$\begin{aligned}\nu_A(x\alpha' y) &= \nu_A(x\alpha'(y\beta' s'\gamma' y)) \\ &= \nu_A(x\alpha' y\beta'(s'\gamma' y)) \\ &\leq \nu_A(y)\end{aligned}$$

for all $\alpha' \in \Gamma$. It follows that $A = (\mu_A, \nu_A)$ is an $IFL\Gamma I$ of S. Similarly, we can show that $A = (\mu_A, \nu_A)$ is an $IFR\Gamma I$ of S. This completes the proof. □

Theorem 3.14. *If U is an interior Γ-ideal of S, then $\tilde{U} = (\chi_U, \bar{\chi}_U)$ is an $IFI\Gamma I$ of S.*

Proof. Since U is a Γ-subsemigroup of S, we have that $\tilde{U} = (\chi_U, \bar{\chi}_U)$ is an $IF\Gamma S$ of S by Theorem 3.2. Let $s, x, y \in S$ and $\beta, \gamma \in \Gamma$. From the hypothesis, $x\beta s\gamma y \in U$ if $s \in U$. In this case

$$\chi_U(x\beta s\gamma y) = 1 = \chi_U(s)$$

and

$$\bar{\chi}_U(x\beta s\gamma y) = 1 - \chi_U(x\beta s\gamma y) = 0 = 1 - \chi_U(s) = \bar{\chi}_U(s).$$

If $s \notin U$, then $\chi_U(s) = 0$. Thus

$$\chi_U(x\beta s\gamma y) \geq 0 = \chi_U(s)$$

and

$$\bar{\chi}_U(s) = 1 - \chi_U(s) = 1 \geq \bar{\chi}_U(x\beta s\gamma y).$$

Consequently, $\tilde{U} = (\chi_U, \bar{\chi}_U)$ is an $IFI\Gamma I$ of S. □

Theorem 3.15. *Let S be regular and U a non-empty subset of S. If $\tilde{U} = (\chi_U, \bar{\chi}_U)$ is an $IFI\Gamma I_1$ or $IFI\Gamma I_2$ of S, then U is an interior Γ-ideal of S.*

Proof. It is clear that U is a Γ-subsemigroup of S by Theorem 3.3. Suppose that $\tilde{U} = (\chi_U, \bar{\chi}_U)$ is an $IFI\Gamma I_1$ of S and $x \in S\Gamma U\Gamma S$. In this case, $x = s\beta u\gamma t$ for some $s, t \in S$, $u \in U$ and $\beta, \gamma \in \Gamma$. It follows from $(I\Gamma I_1)$ that

$$\chi_U(x) = \chi_U(s\beta u\gamma t) \geq \chi_U(u) = 1.$$

Hence $\chi_U(x) = 1$, i.e. $x \in U$. Thus U is an interior Γ-ideal of S.

Now, assume that $\tilde{U} = (\chi_U, \bar{\chi}_U)$ is an $IFI\Gamma I_2$ of S and $x' \in S\Gamma U\Gamma S$. Then $x' = s'\beta' u'\gamma' t'$ for some $s', t' \in S$, $u' \in U$ and $\beta', \gamma' \in \Gamma$. Using $(I\Gamma I_2)$, we obtain

$$\bar{\chi}_U(x') = \bar{\chi}_U(s'\beta' u'\gamma' t') \leq \bar{\chi}_U(u') = 1 - \chi_U(u') = 0$$

and so $\bar{\chi}_U(x') = 1 - \chi_U(x') = 0$. Therefore $\chi_U(x') = 1$, i.e. $x' \in U$. This completes the proof. □

Definition 3.16. S is called *first (resp. second) intuitionistic fuzzy left simple* if every $IFL\Gamma I_1$ (resp. $IFL\Gamma I_2$) of S is constant. Also, S is said to be *intuitionistic fuzzy left simple* if it is both first and second intuitionistic fuzzy left simple, i.e. every $IFL\Gamma I$ of S is constant.

Theorem 3.17. *If S is left simple, then S is intuitionistic fuzzy left simple.*

Proof. Let $A = (\mu_A, \nu_A)$ be an $IFL\Gamma I$ of S and $x, x' \in S$. In this case, because of S is left simple, there exist $s, s' \in S$ and $\gamma, \gamma' \in \Gamma$ such that $x = s\gamma x'$ and $x' = s'\gamma' x$. Thus, since $A = (\mu_A, \nu_A)$ is an $IFL\Gamma I$ of S, we get that

$$\mu_A(x) = \mu_A(s\gamma x') \geq \mu_A(x'),$$
$$\mu_A(x') = \mu_A(s'\gamma' x) \geq \mu_A(x)$$

and

$$\nu_A(x) = \nu_A(s\gamma x') \leq \nu_A(x'),$$
$$\nu_A(x') = \nu_A(s'\gamma' x) \leq \nu_A(x).$$

Hence we have $\mu_A(x) = \mu_A(x')$ and $\nu_A(x) = \nu_A(x')$ for all $x, x' \in S$, that is, $A(x) = A(x')$ for all $x, x' \in S$. Consequently, S is intuitionistic fuzzy left simple. □

Theorem 3.18. *If S is first or second intuitionistic fuzzy left simple, then S is left simple.*

Proof. Let U be a left Γ-ideal of S. Suppose that S is first (or second) intuitionistic fuzzy left simple. Because of $\tilde{U} = (\chi_U, \bar{\chi}_U)$ is an $IFL\Gamma I$ of S by Lemma 3.8, $\tilde{U} = (\chi_U, \bar{\chi}_U)$ is an $IFL\Gamma I_1$ (and $IFL\Gamma I_2$) of S. From the hypothesis, χ_U (and $\bar{\chi}_U$) is constant. Since U is non-empty, it follows that $\chi_U = \mathbf{1}$ (or $\bar{\chi}_U = \mathbf{0}$), where $\mathbf{1}$ and $\mathbf{0}$ are fuzzy sets in S defined by $\mathbf{1}(x) = 1$ and $\mathbf{0}(x) = 0$ for all $x \in S$, respectively. Thus $x \in U$ for all $x \in S$. This completes the proof. □

Theorem 3.19. *If S is simple, then every $IFI\Gamma I$ of S is constant.*

Proof. Let $A = (\mu_A, \nu_A)$ be an $IFI\Gamma I$ of S and $x, x' \in S$. In this case, because of S is simple, there exist $s, s', t, t' \in S$ and $\beta, \beta', \gamma, \gamma' \in \Gamma$ such that $x = s\beta x'\gamma t$ and $x' = s'\beta' x\gamma' t'$. Thus, since $A = (\mu_A, \nu_A)$ is an $IFI\Gamma I$ of S, we obtain that

$$\mu_A(x) = \mu_A(s\beta x'\gamma t) \geq \mu_A(x'),$$
$$\mu_A(x') = \mu_A(s'\beta' x\gamma' t') \geq \mu_A(x)$$

and

$$\nu_A(x) = \nu_A(s\beta x'\gamma t) \leq \nu_A(x'),$$
$$\nu_A(x') = \nu_A(s'\beta' x\gamma' t') \leq \nu_A(x).$$

Hence we get $\mu_A(x) = \mu_A(x')$ and $\nu_A(x) = \nu_A(x')$ for all $x, x' \in S$. Consequently, $A = (\mu_A, \nu_A)$ is constant. □

Definition 3.20. For an $IF\Gamma S$ $A = (\mu_A, \nu_A)$ in S, consider the following axioms:

(ΓB_1) $\mu_A(x\beta s\gamma y) \geq \min\{\mu_A(x), \mu_A(y)\}$,
(ΓB_2) $\nu_A(x\beta s\gamma y) \leq \max\{\nu_A(x), \nu_A(y)\}$

for all $s, x, y \in S$ and $\beta, \gamma \in \Gamma$. Then $A = (\mu_A, \nu_A)$ is called an *intuitionistic fuzzy* Γ*-bi-ideal* (briefly $IF\Gamma B$) of S if it satisfies (ΓB_1) and (ΓB_2).

Remark 3.21. It is clear that every $IF\Gamma I$ of S is an $IF\Gamma B$ of S.

Theorem 3.22. *If S is left simple, then every $IF\Gamma B$ of S is an $IFR\Gamma I$ of S.*

Proof. Let $A = (\mu_A, \nu_A)$ be an $IF\Gamma B$ of S and $x, y \in S$. In this case, from the hypothesis, there exist $s \in S$ and $\gamma \in \Gamma$ such that $y = s\gamma x$. Thus, because of $A = (\mu_A, \nu_A)$ is an $IF\Gamma B$ of S, we have that

$$\begin{aligned}\mu_A(x\beta y) &= \mu_A(x\beta s\gamma x)\\ &\geq \min\{\mu_A(x), \mu_A(x)\} = \mu_A(x)\end{aligned}$$

and

$$\begin{aligned}\nu_A(x\beta y) &= \nu_A(x\beta s\gamma x)\\ &\leq \max\{\nu_A(x), \nu_A(x)\} = \nu_A(x)\end{aligned}$$

for all $\beta \in \Gamma$. It follows that $A = (\mu_A, \nu_A)$ is an $IFR\Gamma I$ of S. □

Proposition 3.23. *If U is a Γ-bi-ideal of S, then $\tilde{U} = (\chi_U, \bar{\chi}_U)$ is an $IF\Gamma B$ of S.*

Proof. Since U is a Γ-subsemigroup of S, we obtain that $\tilde{U} = (\chi_U, \bar{\chi}_U)$ is an $IF\Gamma S$ of S by Theorem 3.2. Let $s, x, y \in S$ and $\beta, \gamma \in \Gamma$. From the hypothesis, $x\beta s\gamma y \in U$ if $x, y \in U$. In this case

$$\chi_U(x\beta s\gamma y) = 1 = \min\{\chi_U(x), \chi_U(y)\}$$

and

$$\bar{\chi}_U(x\beta s\gamma y) = 1 - \chi_U(x\beta s\gamma y) = 0 = \max\{\bar{\chi}_U(x), \bar{\chi}_U(y)\}.$$

If $x \notin U$ or $y \notin U$, then $\chi_U(x) = 0$ or $\chi_U(y) = 0$. Thus

$$\chi_U(x\beta s\gamma y) \geq 0 = \min\{\chi_U(x), \chi_U(y)\}$$

and

$$\begin{aligned}\max\{\bar{\chi}_U(x), \bar{\chi}_U(y)\} &= \max\{1 - \chi_U(x), 1 - \chi_U(y)\}\\ &= 1 - \min\{\chi_U(x), \chi_U(y)\}\\ &= 1 \geq \bar{\chi}_U(x\beta s\gamma y).\end{aligned}$$

Consequently, $\tilde{U} = (\chi_U, \bar{\chi}_U)$ is an $IF\Gamma B$ of S. □

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