

대수부등식들만을 이용한 갈라거 랜덤 코딩 바운드의 유도

Derivations of Gallager Random Coding Bound by Simple Manipulations of Algebraic Inequalities

유도식*, 임종태**

Do-Sik Yoo*, Jong-Tae Lim**

요 약

이 논문에서 우리는 이산무기억통신로 (Discrete Memoryless Channel)에 대한 갈라거 랜덤 코딩 바운드를 개념적으로 이해하기 어려운 랜덤코딩 방법론을 따르지 않고 순전히 대수부등식들만을 이용함으로써 유도해 낸다. 갈라거 랜덤코딩바운드는 결정영역(Decision Region)을 알수 없는 경우에도 적용될 뿐만아니라 채널코딩정리까지 유도할 수 있는 매우 강력한 바운드로서 정보 및 부호이론 연구에 있어서 매우 중요한 도구이다. 그동안 개념적으로 이해하기 어려웠던 갈라거 랜덤코딩바운드를 대수적으로 차근차근 유도해 봄으로써 다양한 문제에 쉽게 적용할 수 있는 이론적 바탕을 마련해 보고자 한다.

Abstract

In this letter, we derive the Gallager random coding bound for discrete memoryless channels purely by simple manipulations of algebraic inequalities rather than invoking conceptually difficult random coding arguments. Gallager random coding bound is a very useful tool in information and coding theory due to its applicability to situations in which it is difficult to determine the decision regions and due to the fact that it can be used to derive the channel coding theorem. The readers will find it relatively easy to apply to many practical problems of interest the step-by-step algebraic derivations of the Gallager random coding bound with appropriate modifications.

Key words : Gallager random coding bound, Simple manipulations of algebraic inequalities

I. Introduction

In this letter, we derive the Gallager random coding bound for discrete memoryless channels by a sequence of algebraic manipulations of simple inequalities rather

than exploiting the random coding arguments [1].

Random coding bound analysis has been one of the most important analysis tools in information and coding theory [2]. First of all, it provides the cut off rate [3] which has long played the role of practical limits on

* 홍익대학교 전기전자공학부 (School of Electronic & Electrical Engineering of Hongik University)

** 한국항공대학교 항공전자 및 정보통신공학과 (School of Electronics, Telecommunication & Computer Engineering of Korea Aerospace University)

· 제1저자 (First Author) : 유도식

· 교신저자 (Corresponding Author) : 임종태

· 접수일자 : 2007년 10월 4일

achievable transmission rate of a given communication system design. Secondly, it has served as a primary information theoretic tool for finite packet length performance analysis. While channel capacity can be used instead of cut-off rate for the performance limit, it fails to provide useful information about the performance of a coded system with very limited packet length. However, random coding bound can still provide insightful information about the performance of finite packet length systems. This is particularly useful when we are dealing with fading channels. Thirdly, random coding bound and cut-off rate provide useful performance metric in system design [4].

However, many people find it difficult to understand the concept of random coding arguments. This is particularly true when Gallager bound rather than union bound is used. In this letter, we derive the random coding bound not by the random coding arguments but by step-by-step applications of algebraic inequalities. The procedures in this letter will lead the reader to understand the rigorous steps involved in the derivations of the random coding bound and hence will eventually make them better understand the random coding arguments itself so that they can apply the methods to practical system designs of their interest without difficulty.

II. System Model

In this letter, we consider a q -ary (M, N) block code over a discrete memoryless channel with input alphabet $A = \{1, \dots, q\}$ and output alphabet $B = \{1, \dots, r\}$. Here, M denotes the number of codewords in the code and N the number of symbols in a codeword. We denote by T_{ij} the transition probability from the input alphabet i to the output alphabet j . Hence, if we denote by X and Y the input and output symbols of the channel, then

$$P[Y=j|X=i] = T_{ij} \quad (1)$$

for $i = 1, \dots, q$, and $j = 1, \dots, r$.

We assume that the M codewords are selected equally likely and the receiver is assumed to use the maximum likelihood decoding rule to minimize the codeword error probability. Since each codeword consists of N q -ary symbols, there are q^{NM} possible choices of codes. Let us denote these q^{NM} codes by $C_1, \dots, C_{q^{NM}}$. For example, C_1 shall denote the code in which all M codewords are given by $(1, \dots, 1)$, namely, the codeword made solely by the symbol 1 and similarly $C_{q^{NM}}$ the code in which all M codewords are (q, \dots, q) . Obviously, the system will perform very poorly when either C_1 or $C_{q^{NM}}$ is chosen for its channel coding scheme. Consequently, the system performance will naturally be dependant upon which code is chosen.

Let us denote by $P(C)$ the decoding error probability when a particular code C is chosen. Also, we define by $P^{(m)}(C)$ the decoding error probability when a code C is employed and its m^{th} codeword is transmitted. Then, since all the codewords are assumed to be equally likely to be transmitted, it follows that

$$P(C) = \frac{1}{M} \sum_{m=1}^M P^{(m)}(C) \quad (2)$$

III. Mathematical Preliminaries

In this section, we present a series of lemmas needed to derive the random coding bound. Before proceeding, to facilitate the discussion, we define the function $P_r(\alpha_1, \dots, \alpha_{q^{MN}})$ called the random coding bound by

$$P_r(\alpha_1, \dots, \alpha_{q^{MN}}) = \sum_{k=1}^{q^{MN}} \alpha_k P(C_k) \quad (3)$$

for non-negative real numbers $\alpha_1, \dots, \alpha_{q^{MN}}$ satisfying $\alpha_1 + \dots + \alpha_{q^{MN}} = 1$. We note that the random coding

bound is a kind of weighted average of the decoding error probabilities over all possible codes. There are a number of possible ways to interpret the random coding bound. To see the most important one of them, let us consider the following lemma.

Lemma 1 : Let $\alpha_1, \dots, \alpha_{q^{NM}}$ be arbitrary non-negative real numbers such that $\alpha_1 + \dots + \alpha_{q^{NM}} = 1$. Then, there exists at least one code C_n such that

$$P(C_n) \leq P_r(\alpha_1, \dots, \alpha_{q^{NM}}). \quad (4)$$

Proof : Assume on the contrary that

$$P(C_k) > P_r(\alpha_1, \dots, \alpha_{q^{NM}}) \quad (5)$$

holds for all code C_k . But, we note that at least one α_n must be strictly positive so that

$$\alpha_n P(C_n) > \alpha_n P_r(\alpha_1, \dots, \alpha_{q^{NM}}) \quad (6)$$

and hence that

$$\sum_{k=1}^{q^{NM}} \alpha_k P(C_k) > \sum_{k=1}^{q^{NM}} \alpha_k P_r(\alpha_1, \dots, \alpha_{q^{NM}}) \quad (7)$$

This is a contradiction because both the left and right hand sides of the above inequality indicates the same quantity $P_r(\alpha_1, \dots, \alpha_{q^{NM}})$.

Lemma 1 says that there exists at least one code that has decoding error probability lower than the random coding bound. In other words, the random coding bound serves as an upper bound for performance achievable by one of the q^{NM} possible codes. But, we note that the random coding bound itself is a function of non-negative real numbers $\alpha_1, \dots, \alpha_{q^{NM}}$ that can be chosen arbitrary as long as they satisfy the condition $\alpha_1 + \dots + \alpha_{q^{NM}} = 1$. The next lemma provide a class of possible choices of

$\alpha_1, \dots, \alpha_{q^{NM}}$.

Before proceeding to Lemma 2, we define q functions n_1, \dots, n_q defined on A^N so that $n_i(\mathbf{c})$ denotes the number of symbol i contained in the vector $\mathbf{c} \in A^N$. By $n_i(\mathbf{c})$, we desire to indicate the number of symbol i contained in a codeword $\mathbf{c} \in A^N$. Next, we define another set of functions N_1, \dots, N_q defined on the set of codes $\{C_1, \dots, C_{q^{NM}}\}$ so that $N_i(C_k)$ indicates the number of symbol i in all the codewords in C_k . Then, it follows that

$$N_i(C_k) = \sum_{\mathbf{c} \in C_k} n_i(\mathbf{c}) \quad (8)$$

Lemma 2 : Let p_1, \dots, p_q be non-negative real numbers with $p_1 + \dots + p_q = 1$ and define α_k by

$$\alpha_k = p_1^{N_1(C_k)} p_2^{N_2(C_k)} \dots p_q^{N_q(C_k)} \quad (9)$$

for $k = 1, \dots, q^{NM}$. Then, α_k 's are non-negative real numbers and $\alpha_1 + \dots + \alpha_{q^{NM}} = 1$.

Proof : We can regard that a code C_k can be regarded as a string of NM symbols from A with each N consecutive symbols indicating a codeword. Then, the set $\{C_1, \dots, C_{q^{NM}}\}$ of all codes denotes the set of all strings of symbols from A of length NM . Consequently, it follows that

$$\begin{aligned} & \sum_{k=1}^{q^{NM}} p_1^{N_1(C_k)} \dots p_q^{N_q(C_k)} \quad (10) \\ &= \sum_{\mathbf{c}^{(1)} \in A^N} \dots \sum_{\mathbf{c}^{(M)} \in A^N} p_1^{N_1(\{\mathbf{c}^{(1)}, \dots, \mathbf{c}^{(M)}\})} \dots p_q^{N_q(\{\mathbf{c}^{(1)}, \dots, \mathbf{c}^{(M)}\})} \\ &= \sum_{\mathbf{c}^{(1)} \in A^N} \dots \sum_{\mathbf{c}^{(M)} \in A^N} p_1^{\sum_{i=1}^M n_1(\mathbf{c}^{(i)})} \dots p_q^{\sum_{i=1}^M n_q(\mathbf{c}^{(i)})} \\ &= \left[\sum_{\mathbf{c}^{(1)} \in A^N} \prod_{i=1}^q p_i^{n_i(\mathbf{c}^{(1)})} \right] \dots \left[\sum_{\mathbf{c}^{(M)} \in A^N} \prod_{i=1}^q p_i^{n_i(\mathbf{c}^{(M)})} \right] \end{aligned}$$

which should be equal to 1 since

$$\begin{aligned}
\sum_{\mathbf{c} \in A^N} \prod_{i=1}^q p_i^{n_i(\mathbf{c})} &= \sum_{c_1 \in A} \cdots \sum_{c_N \in A} \prod_{i=1}^q p_i^{n_i((c_1, \dots, c_N))} \quad (11) \\
&= \sum_{c_1=1}^q \cdots \sum_{c_N=1}^q p_{c_1} \cdots p_{c_N} \\
&= \left(\sum_{c_1=1}^q p_{c_1} \right) \cdots \left(\sum_{c_N=1}^q p_{c_N} \right) \\
&= 1
\end{aligned}$$

We shall use the α_k 's defined in Lemma 2 in deriving Gallager random coding bound. We shall first obtain a Gallager bound on $P^{(m)}(C_k)$. For that purpose, we define a notation $P(\mathbf{y}|\mathbf{c})$ to denote the probability of receiving \mathbf{y} when the codeword \mathbf{c} is transmitted. Since we are dealing with a discrete memoryless channel with transition probability T_{ij} , it follows that

$$P(\mathbf{y}=(j_1, \dots, j_N)|\mathbf{c}=(i_1, \dots, i_N)) = T_{i_1 j_1} \cdots T_{i_N j_N}. \quad (12)$$

Lemma 3 : Let $\mathbf{c}_k^{(m)}$ denote the m^{th} codeword of the code C_k . Then, for any $\beta > 0$, we have

$$\begin{aligned}
P^{(m)}(C_k) &\leq \sum_{\mathbf{y} \in B^N} [P(\mathbf{y}|\mathbf{c}_k^{(m)})]^{\frac{1}{1+\beta}} \quad (13) \\
&\quad \times \left\{ \sum_{l \neq m} [P(\mathbf{y}|\mathbf{c}_k^{(l)})]^{\frac{1}{1+\beta}} \right\}^\beta
\end{aligned}$$

Proof : Since the decoder is assumed to use the maximum likelihood decision rule, the decision region $D^{(m)}(C_k)$ is given by

$$D_k^{(m)} = \{ \mathbf{y} \in B^N : P(\mathbf{y}|\mathbf{c}_k^{(m)}) \geq \max_l P(\mathbf{y}|\mathbf{c}_k^{(l)}) \}.$$

Now, let β be an arbitrarily chosen positive real number. Then, since for any $\mathbf{y} \notin D^{(m)}(C_k)$,

$$\left\{ \sum_{l \neq m} \left[\frac{P(\mathbf{y}|\mathbf{c}_k^{(l)})}{P(\mathbf{y}|\mathbf{c}_k^{(m)})} \right]^{\frac{1}{1+\beta}} \right\}^\beta \geq 1 \quad (14)$$

and since $D^{(m)}(C_k) \subseteq B^N$, it follows

$$\begin{aligned}
P^{(m)}(C_k) &= \sum_{\mathbf{y} \notin D_k^{(m)}} P(\mathbf{y}|\mathbf{c}_k^{(m)}) \quad (15) \\
&\leq \sum_{\mathbf{y} \notin D_k^{(m)}} P(\mathbf{y}|\mathbf{c}_k^{(m)}) \left\{ \sum_{l \neq m} \left[\frac{P(\mathbf{y}|\mathbf{c}_k^{(l)})}{P(\mathbf{y}|\mathbf{c}_k^{(m)})} \right]^{\frac{1}{1+\beta}} \right\}^\beta \\
&\leq \sum_{\mathbf{y} \in B^N} P(\mathbf{y}|\mathbf{c}_k^{(m)}) \left\{ \sum_{l \neq m} \left[\frac{P(\mathbf{y}|\mathbf{c}_k^{(l)})}{P(\mathbf{y}|\mathbf{c}_k^{(m)})} \right]^{\frac{1}{1+\beta}} \right\}^\beta \\
&= \sum_{\mathbf{y} \in B^N} [P(\mathbf{y}|\mathbf{c}_k^{(m)})]^{\frac{1}{1+\beta}} \left\{ \sum_{l \neq m} [P(\mathbf{y}|\mathbf{c}_k^{(l)})]^{\frac{1}{1+\beta}} \right\}^\beta
\end{aligned}$$

IV. Random Coding Bound

In this section, using the lemmas presented in the previous section, we derive the random coding bound purely by algebraic manipulations of simple inequalities. The result is presented in the following theorem.

Theorem 1 (Random Coding Bound):

Let p_1, \dots, p_q be arbitrarily chosen non-negative real numbers such that $\sum_{i=1}^q p_i = 1$. Then, for any $\beta \in (0, 1)$, there exists at least one code C_n such that

$$P(C_n) \leq (M-1)^\beta \left[\sum_{j=1}^r \left\{ \sum_{i=1}^q p_i T_{ij}^{\frac{1}{1+\beta}} \right\}^{1+\beta} \right]^{NM} \quad (16)$$

Proof : First, we define $\alpha_1, \dots, \alpha_{pNM}$ as in Lemma 2. Also we define $P_r^{(m)}(\alpha_1, \dots, \alpha_{pNM})$ by

$$P_r^{(m)}(\alpha_1, \dots, \alpha_{pNM}) = \sum_{k=1}^{qNM} \alpha_k P^{(m)}(C_k) \quad (17)$$

so that

$$P_r(\alpha_1, \dots, \alpha_{qNM}) = \frac{1}{M} \sum_{m=1}^M P_r^{(m)}(\alpha_1, \dots, \alpha_{qNM}) \quad (18)$$

Then,

$$\begin{aligned}
 & P_r^{(m)}(\alpha_1, \dots, \alpha_{qNM}) \\
 &= \sum_{\mathbf{c}^{(1)} \in A^N} \dots \sum_{\mathbf{c}^{(M)} \in A^N} p_1^{N_1(\{\mathbf{c}^{(1)}, \dots, \mathbf{c}^{(M)}\})} \dots p_q^{N_q(\{\mathbf{c}^{(1)}, \dots, \mathbf{c}^{(M)}\})} \\
 &\quad \times P^{(m)}(\{\mathbf{c}^{(1)}, \dots, \mathbf{c}^{(M)}\}) \\
 &= \sum_{\mathbf{c}^{(1)} \in A^N} \dots \sum_{\mathbf{c}^{(M)} \in A^N} p_1^{\sum_{l=1}^M n_1(\mathbf{c}^{(l)})} \dots p_q^{\sum_{l=1}^M n_q(\mathbf{c}^{(l)})} \\
 &\quad \times P^{(m)}(\{\mathbf{c}^{(1)}, \dots, \mathbf{c}^{(M)}\})
 \end{aligned} \tag{19}$$

Now, let β be arbitrarily chosen real number in $(0, 1)$. Then, from Lemma 3, we obtain

$$\begin{aligned}
 & P_r^{(m)}(\alpha_1, \dots, \alpha_{qNM}) \\
 &\leq \sum_{\mathbf{c}^{(1)} \in A^N} \dots \sum_{\mathbf{c}^{(M)} \in A^N} p_1^{\sum_{l=1}^M n_1(\mathbf{c}^{(l)})} \dots p_q^{\sum_{l=1}^M n_q(\mathbf{c}^{(l)})} \\
 &\quad \sum_{\mathbf{y} \in B^N} [P(\mathbf{y}|\mathbf{c}^{(m)})]^{\frac{1}{1+\beta}} \left\{ \sum_{l \neq m} [P(\mathbf{y}|\mathbf{c}^{(l)})]^{\frac{1}{1+\beta}} \right\}^\beta \\
 &= \sum_{\mathbf{y} \in B^N} \left\{ \sum_{\mathbf{c}^{(m)} \in A^N} \left(\prod_{i=1}^q p_i^{n_i(\mathbf{c}^{(m)})} \right) [P(\mathbf{y}|\mathbf{c}^{(m)})]^{\frac{1}{1+\beta}} \right\} \\
 &\quad \times \prod_{l \neq m} \left(\sum_{\mathbf{c}^{(l)} \in A^N} \prod_{j=1}^q p_j^{n_j(\mathbf{c}^{(l)})} \right) \left\{ \sum_{l \neq m} [P(\mathbf{y}|\mathbf{c}^{(l)})]^{\frac{1}{1+\beta}} \right\}^\beta
 \end{aligned} \tag{20}$$

Next, we apply the Jensen inequality according to which

$$t_1 x_1^\delta + \dots + t_r x_r^\delta \leq (t_1 x_1 + \dots + t_r x_r)^\delta \tag{21}$$

holds for any non-negative real numbers t_1, \dots, t_r and x_1, \dots, x_r such that $t_1 + \dots + t_r = 1$ when $0 < \delta < 1$.

Now, applying the above inequality, we obtain

$$\begin{aligned}
 & \left\{ \prod_{l \neq m} \left(\sum_{\mathbf{c}^{(l)} \in A^N} \prod_{j=1}^q p_j^{n_j(\mathbf{c}^{(l)})} \right) \right\} \left\{ \sum_{n \neq m} [P(\mathbf{y}|\mathbf{c}^{(n)})]^{\frac{1}{1+\beta}} \right\}^\beta \\
 &\leq \left[\prod_{l \neq m} \left(\sum_{\mathbf{c}^{(l)} \in A^N} \prod_{j=1}^q p_j^{n_j(\mathbf{c}^{(l)})} \right) \sum_{n \neq m} [P(\mathbf{y}|\mathbf{c}^{(n)})]^{\frac{1}{1+\beta}} \right]^\beta \\
 &= \left[\sum_{n \neq m} \prod_{l \neq m} \left(\sum_{\mathbf{c}^{(l)} \in A^N} \prod_{j=1}^q p_j^{n_j(\mathbf{c}^{(l)})} \right) [P(\mathbf{y}|\mathbf{c}^{(n)})]^{\frac{1}{1+\beta}} \right]^\beta
 \end{aligned} \tag{22}$$

Hence, it follows

$$\begin{aligned}
 & P_r^{(m)}(\alpha_1, \dots, \alpha_{qNM}) \\
 &\leq (M-1)^\beta \sum_{\mathbf{y} \in B^N} \left\{ \sum_{\mathbf{c} \in A^N} \left(\prod_{i=1}^q p_i^{n_i(\mathbf{c})} \right) [P(\mathbf{y}|\mathbf{c})]^{\frac{1}{1+\beta}} \right\}^{\beta+1}
 \end{aligned} \tag{23}$$

Here, if we write $\mathbf{c} = (c_1, \dots, c_N)$ and $\mathbf{y} = (y_1, \dots, y_N)$, then

$$p_1^{n_1(\mathbf{c})} \dots p_q^{n_q(\mathbf{c})} = p_{c_1} \dots p_{c_N} \tag{24}$$

and

$$P(\mathbf{y}|\mathbf{c}) = T_{c_1 y_1} \dots T_{c_N y_N} \tag{25}$$

Consequently, by rewriting the above inequality, we obtain

$$\begin{aligned}
 & P_r^{(m)}(\alpha_1, \dots, \alpha_{qNM}) \\
 &\leq (M-1)^\beta \sum_{\mathbf{y} \in B^N} \left\{ \sum_{\mathbf{c} \in A^N} \prod_{i=1}^N \left(p_{c_i} [T_{c_i y_i}]^{\frac{1}{1+\beta}} \right) \right\}^{\beta+1}
 \end{aligned} \tag{26}$$

Next, we note that

$$\begin{aligned}
 & \sum_{\mathbf{y} \in B^N} \left\{ \sum_{\mathbf{c} \in A^N} \prod_{i=1}^N \left(p_{c_i} [T_{c_i y_i}]^{\frac{1}{1+\beta}} \right) \right\}^{\beta+1} \\
 &= \sum_{\mathbf{y} \in B^N} \left\{ \prod_{i=1}^N \left(\sum_{c_i \in A} p_{c_i} [T_{c_i y_i}]^{\frac{1}{1+\beta}} \right) \right\}^{\beta+1} \\
 &= \sum_{\mathbf{y} \in B^N} \prod_{i=1}^N \left\{ \left(\sum_{c_i \in A} p_{c_i} [T_{c_i y_i}]^{\frac{1}{1+\beta}} \right) \right\}^{\beta+1}
 \end{aligned} \tag{27}$$

$$\begin{aligned}
&= \prod_{i=1}^N \left[\sum_{y_i \in B} \left\{ \left(\sum_{c_i \in A} p_{c_i} [T_{c_i y_i}]^{\frac{1}{1+\beta}} \right)^{\beta+1} \right\} \right] \\
&= \left[\sum_{y \in B} \left\{ \left(\sum_{c \in A} p_c [T_{cy}]^{\frac{1}{1+\beta}} \right)^{\beta+1} \right\} \right]^N
\end{aligned}$$

to conclude

$$\begin{aligned}
&P_r^{(m)}(\alpha_1, \dots, \alpha_{qNM}) \\
&\leq (M-1)^\beta \left[\sum_{j=1}^r \left\{ \sum_{i=1}^q p_i [T_{ij}]^{\frac{1}{1+\beta}} \right\}^{1+\beta} \right]^N
\end{aligned} \quad (28)$$

This implies, in turn, that

$$\begin{aligned}
&P_r(\alpha_1, \dots, \alpha_{qNM}) \\
&= \frac{1}{M} \sum_{m=1}^M P_r^{(m)}(\alpha_1, \dots, \alpha_{qNM}) \\
&\leq (M-1)^\beta \left[\sum_{j=1}^r \left\{ \sum_{i=1}^q p_i [T_{ij}]^{\frac{1}{1+\beta}} \right\}^{1+\beta} \right]^N
\end{aligned} \quad (29)$$

Finally, using Lemma 1, we conclude that there exists at least one code C_n such that

$$\begin{aligned}
&P(C_n) \leq P_r(\alpha_1, \dots, \alpha_{qNM}) \\
&\leq (M-1)^\beta \left[\sum_{j=1}^r \left\{ \sum_{i=1}^q p_i [T_{ij}]^{\frac{1}{1+\beta}} \right\}^{1+\beta} \right]^N
\end{aligned} \quad (30)$$

We note that the Gallager random coding bound in Theorem 1 depends on the choices of the probabilities p_1, \dots, p_q and the real number β . Hence, we can further attempt to obtain minimum random coding bound over all possible choices of these values. In fact, it is shown in [1] that the channel coding theorem for discrete memoryless channel can be derived from Theorem 1 by solving this minimization problem.

V. Conclusions

In this letter, we presented an alternative derivation of the Gallager random coding bound based on simple

algebraic manipulations rather than invoking the random coding argument which is conceptually harder to understand. We believe the derivation in this letter will be useful to the readers not only in obtaining various practical performance bounds but also in making themselves better understand the random coding bound arguments itself so that they can easily apply the methods to practical problems of their own interest.

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유도식 (兪堵植)



1990년 8월 : 서울대학교 전기
공학과(공학사)

1994년 2월 : 서울대학교 물리
학과(이학석사)

2002년 4월 : University of
Michigan, 전기공학과(공학박
사)

2002년 2월 ~ 2003년 10월 미

시간대학교 박사후 연구원

2006년 9월 ~ 현재 홍익대학교 전자전기공학부 조교수

관심분야 : 통신이론, 무선통신 시스템 설계, 정보이론,
부호이론, 네트워크 정보이론 등

임종태 (林鍾泰)



1989년 2월 : 서울대학교 전자공
학과(공학사)

1991년 2월 : 서울대학교 전자공
학과(공학석사)

2001년 8월 : The Univ. of
Michigan at Ann Arbor 전기공
학과(공학박사)

1991년 3월 ~ 2004년 8월 (주) 대우일렉트로닉스 영상
연구소 및 신호처리연구소 책임 연구원

2004년 9월 ~ 현재 한국항공대학교 항공전자 및 정보통
신공학부 조교수

관심분야 : Joint Source-Channel Coding, Digital
Broadcasting Systems, Video Signal Processing