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정수 비트 할당을 위한 최대 탐욕 및 최소 탐욕 알고리즘에 관한 연구

The Most and Least Greedy Algorithms for Integer Bit Allocation

임종태*, 유도식**

Jong-Tae Lim*, Do-Sik Yoo**

요 약

변환부호화기(Transform coders)를 설계함에 있어서 비트 할당(Bit allocation)은 중요한 설계 요인 중의 하나이다. 본 논문에서는 고해상(high-resolution) 이론에 의한 수식들을 바탕으로 각 계수 양자화기들의 비트율을 정수값으로 할당해주는 최적의 알고리즘인 최대 탐욕 알고리즘과 최소 탐욕 알고리즘을 제안하였다. 특히, 제안된 최대 탐욕 알고리즘과 최소 탐욕 알고리즘에서 쌍대성(duality) 성질을 확인할 수 있었다.

Abstract

In designing transform coders bit allocation is one of the important issues. In this paper we propose two optimal algorithms for integer bit allocation in transform coding. Based on high-resolution formulas for bit allocation, the most and least greedy algorithms are developed to optimally adjust non-integer bit rates of coefficient quantizers to integer values. In particular, a duality property is observed between the two greedy algorithms.

Key words : Bit Allocation, Transform Coding, Quantization, Integer Constraint, Duality

Ⅰ. Introduction

One of the fundamental problems in designing transform coders is rate allocation: distributing a given average bit rate among the coefficient quantizers to minimizing the overall distortion. Many numerical algorithms and high-resolution formulas for optimal bit allocation are developed for noiseless and noisy channels [1]-[4].

Since the bit allocation determined by the high-resolution formulas normally gives non-integer values of rates, the allocated bits need to be adjusted to integer values for the applications where each quantizer of transform coders is confined to have an integer rate. For this adjustment, a suboptimal iterative round-up adjustment method is proposed by Wintz and Kurtenbach [5], and a non-iterative round-up algorithm

* 한국항공대학교 항공전자 및 정보통신공학과(School of Electronics, Telecommunication & Computer Engineering of Korea Aerospace University)
** 홍익대학교 전기전자공학부(School of Electronic & Electrical Engineering of Hongik University)
· 제1저자 (First Author) : 임종태
· 교신저자 (Corresponding Author) : 유도식
· 접수일자 : 2007년 10월 22일

is developed by Guo and Meng [6].

In this paper, we consider two algorithms called the most and least greedy algorithms for the optimal bit allocation and study their dual relationship. In particular, we provide dual propositions for bit allocation based on the well-known optimal bit allocation formula. These propositions can easily be applied to integer-bit rate adjustment problems and lead to dual optimal algorithms, namely, the most and least greedy algorithms.

Ⅱ. Transform Coding and Rate Allocation

Consider an N dimensional transform code with rate allocation R1, R2, •••, RN as shown in Fig. 1.

The overall distortion is the average of each quantizer's distortion:

$$D(R_1, R_2, \cdots, R_N) = \frac{1}{N}\sum_{i=1}^{N} D_i(R_i), \quad (1)$$

where Di(Ri) is the distortion of the ith quantizer with rate Ri. Given average bit rate R high-resolution analysis gives the optimal bit allocation minimizing the distortion as

$$R_i = R + \frac{1}{2}\log\frac{\sigma_i^2}{\left(\prod_{i=1}^{N}\sigma_i^2\right)^{1/N}}, \quad (2)$$

where σ_i^2 is the variance of the ith coefficient. With the optimal bit allocation each quantizer's distortion becomes

$$D_i(R_i) = \frac{1}{12}\beta\sigma_i^2 2^{-2R_i} = \frac{1}{12}\beta\left(\prod_{i=1}^{N}\sigma_i^2\right)^{1/N} 2^{-2R} \triangleq D^*(R), \quad (3)$$

where si is the variance of the ith coefficient and b is Panter-Dite factor defined as $\left(\int_{-\infty}^{\infty} p^{1/3}(x)dx\right)^3$, and where p(x) is the probability density function of the input source.

In the following, we denote the ceiling and floor function by $\lceil R_i \rceil$ and $\lfloor R_i \rfloor$, respectively. Let ri be $R_i - \lfloor R_i \rfloor$ and si be $\lceil R_i \rceil - R_i$. We assume that there are Q quantizers whose rates are already integer values in the optimal allocation obtained by (2), i.e., $r_j = 0 = s_j$ and $\lceil R_j \rceil = R_j = \lfloor R_j \rfloor$ for the quantizers.

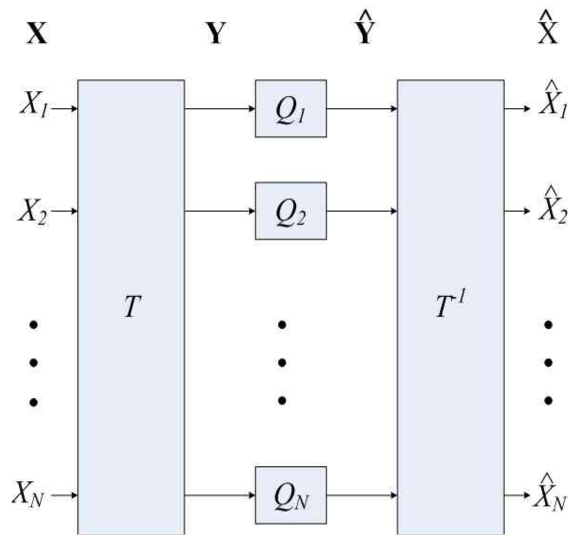

그림 1. 변환 부호화기의 블록 다이어그램
Fig. 1. Block Diagram of a Transform Coding

Proposition 1: Given integer bit allocation $\lfloor R_1 \rfloor, \lfloor R_2 \rfloor, \cdots, \lfloor R_N \rfloor$, where R1, R2, •••, RN are obtained by (2) and assumed to be arranged to satisfy the following order

$$D_1(\lfloor R_1 \rfloor) \ge D_2(\lfloor R_2 \rfloor) \ge \cdots \ge D_N(\lfloor R_N \rfloor), \quad (4)$$

if one more bit is available to allocate, the overall distortion of the transform code is minimized when the available bit is allocated to the first quantizer. (The first quantizer has the biggest distortion and thus it is the most greedy quantizer that is eager to take the available bit to minimize the total distortion.) Moreover, the distortion of the first quantizer becomes the smallest after the one bit reallocation.

Proof: From (3) we have

$$D_i\left(\lfloor R_i \rfloor\right) = \frac{1}{12}\beta_1 \sigma_i^2 2^{-2\lfloor R_i \rfloor} = 2^{2r_i} D^*(R) \quad (5)$$

and

$$D_i\left(\lfloor R_i \rfloor + 1\right) = \frac{1}{12}\beta_1 \sigma_i^2 2^{-2(\lfloor R_i \rfloor + 1)} = 2^{2(r_i - 1)} D^*(R) \quad (6)$$

Combing (5) into (4), we have $2^{2r_1} D^*(R) \geq 2^{2r_2} D^*(R) \geq \cdots \geq 2^{2r_N} D^*(R)$ and thus

$$r_1 \geq r_2 \geq \cdots \geq r_N . \quad (7)$$

If the one bit is allocated to the jth quantizer, the resulting distortion of the transform coder is

$$D = \frac{1}{N}\sum_{i=1}^{N} D_i\left(\lfloor R_i \rfloor\right) - \left[D_j\left(\lfloor R_j \rfloor\right) - D_j\left(\lfloor R_j \rfloor + 1\right)\right] \quad (8)$$

To minimize the distortion, the distortion decrement, $D_j\left(\lfloor R_j \rfloor\right) - D_j\left(\lfloor R_j \rfloor + 1\right)$ should be maximized. Since the term becomes

$$D_j\left(\lfloor R_j \rfloor\right) - D_j\left(\lfloor R_j \rfloor + 1\right) = 2^{2r_j} D^*(R) - 2^{2(r_j - 1)} D^*(R) = \frac{3}{4} 2^{2r_j} D^*(R) \quad (9)$$

it is maximized for j = 1, i.e., the available bit is allocated to the first quantizer with rate $\lfloor R_1 \rfloor$. And from (5) and (6), we have $D_1\left(\lfloor R_1 \rfloor + 1\right) \leq D_i\left(\lfloor R_i \rfloor\right)$, for i = 1, •••, N, thus the distortion of the first quantizer becomes the smallest with the one bit reallocation.

Similarly, we have the following proposition as a dual of Proposition 1.

Proposition 2: Given bit allocation $\lceil R_1' \rceil, \lceil R_2' \rceil, \cdots, \lceil R_N' \rceil$, where $R_1', R_2', \cdots, R_N'$ are obtained by (2) and assumed to be arranged to satisfy the following order:

$$D_1\left(\lceil R_1' \rceil\right) \geq D_2\left(\lceil R_2' \rceil\right) \geq \cdots \geq D_N\left(\lceil R_N' \rceil\right), \quad (10)$$

if one bit needs to be de-allocated from one of the quantizers, the overall distortion of the transform code is minimized when the abundant bit is de-allocated from the last quantizers, (The quantizer has the smallest distortion and thus it is the least greedy quantizer in the sense that the distortion increment occurred by reducing one bit rate from the quantizer is the smallest.)Moreover, the distortion of the last quantizer becomes the largest after the one bit de-allocation.

Proof: By the analogy with Proposition 1, we can show that

$$s_1' \leq s_2' \leq \cdots \leq s_N' \quad (11)$$

and the overall distortion increment, $D_j\left(\lceil R_j' \rceil - 1\right) - D_j\left(\lceil R_j' \rceil\right)$ is minimized for j = N, i.e., when the one bit is deprived from the last quantizer that has the smallest distortion before the de-allocation. Also, we have $D_N\left(\lceil R_N' \rceil - 1\right) \geq D_i\left(\lceil R_i' \rceil\right)$, for $i = 1, \cdots, N$, thus the distortion of the last quantizer becomes the largest with the one bit de-allocation.

Ⅲ. Integer Bit Allocation

In this chapter, we present the two dual algorithms called the most and least greedy algorithms for the optimal integer bit allocation.

The most greedy algorithm: After applying the floor function to the optimal rates obtained by high-resolution formula, $L = NR - \sum_{i=1}^{N} \lfloor R_i \rfloor$ remaining bits are reallocated as follows:

Step 1: Set the quantizer's rates as the largest integers less than or equal to the optimal rates obtained by (2).

Step 2: Arrange the rates with the order of (4) or equivalently (7).

Step 3: Add 1 bit each to the first L quantizer's rates.

The flowchart for the most greedy algorithm is shown in Fig. 2.

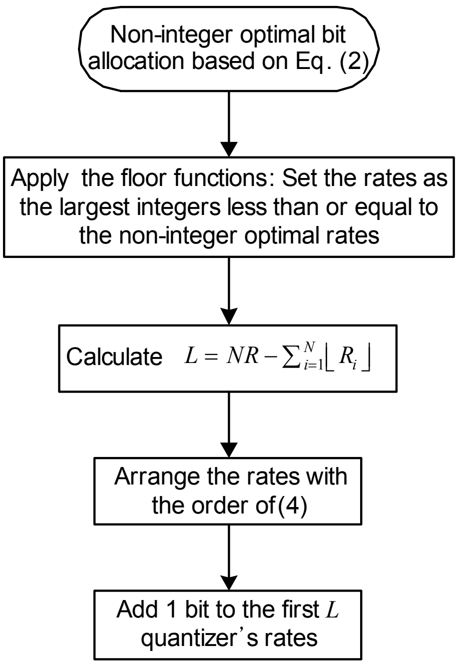

그림 2. 최대 탐욕 알고리즘에 대한 순서도

Fig. 2. Flow Chart for the most greedy algorithm

The optimality of the algorithm can be easily verified by applying Proposition 1 L times. The resulting bit allocation by the most greedy algorithm is

$$\lfloor R_1 \rfloor + 1, \cdots, \lfloor R_L \rfloor + 1, \lfloor R_{L+1} \rfloor, \cdots, \lfloor R_N \rfloor. \quad (12)$$

This algorithm is equivalent to Guo and Meng's round-up algorithm [6], but it is based on more intuitive idea resulting from Proposition 1.

The least greedy algorithm: Similarly, after applying the ceiling function to the optimal rates obtained by high-resolution formula, $M = \sum_{i=1}^{N} \lceil R_i' \rceil - NR$ abundant bits are de-allocated as follows:

Step 1: Set the quantizer's rates as the smallest integers not less than the optimal rates obtained by (2).

Step 2: Arrange the rates with the order of (10) or equivalently (11).

Step 3: Subtract 1 bit each from the last M quantizer's rates.

The flowchart for the least greedy algorithm is shown in Fig. 3. The optimality of the algorithm can be easily verified by applying the dual proposition, Proposition 2, M times. The resulting bit allocation by the least greedy algorithm is

$$\lceil R_1' \rceil, \lceil R_2' \rceil, \cdots, \lceil R_{N-M}' \rceil, \\ \lceil R_{N-M+1}' \rceil - 1, \cdots, \lceil R_N' \rceil - 1. \quad (13)$$

We can easily see a dual relationship between the most greedy algorithm and the least greedy algorithm. Two optimal algorithms necessarily give the same allocation result. This can be shown considering the relationship between the ceiling and floor function. Since $\lfloor R_i \rfloor + 1 = \lceil R_i \rceil$ for non-integer Ri and $\lfloor R_j \rfloor = R_j = \lceil R_j \rceil$ for integer Rj, we have N= L+Q+M. Considering the rates of Q quantizers whose rates are already integer values in the optimal allocation are corresponding to the last Q rates $R_{N-Q+1}, \cdots, R_N$ for the order of (7) and to the first Q rates $R_1', \cdots, R_Q'$ for the order of (11), one can easily show that two allocations (12) and (13) are identical to the following integer-value bit allocation:

$$\lceil R_1 \rceil, \cdots, \lceil R_L \rceil, \lfloor R_{L+1} \rfloor, \cdots, \lfloor R_{L+M} \rfloor, R_{N-Q+1}, \cdots, R_N \qquad (14)$$

for $r_1 \geq r_2 \geq \cdots \geq r_N$, or

$$R_1, \cdots, R_Q, \lceil R_{Q+1} \rceil, \cdots, \lceil R_{Q+L} \rceil, \lfloor R_{N-M+1} \rfloor, \cdots, \lfloor R_N \rfloor \qquad (15)$$

for $s_1 \leq s_2 \leq \cdots \leq s_N$.

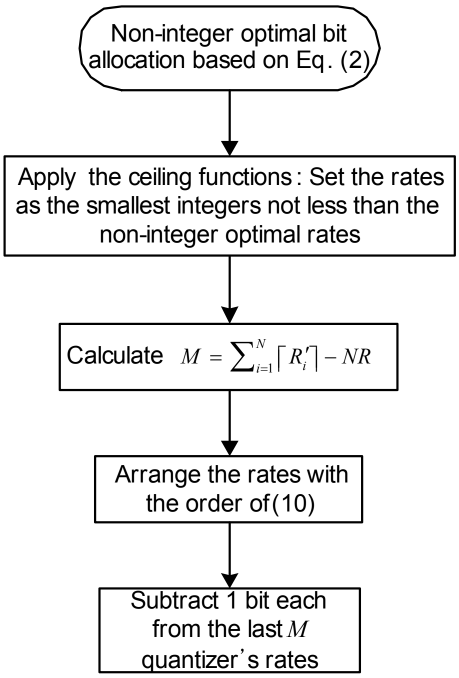

그림 3. 최소 탐욕 알고리즘에 대한 순서도
Fig. 3. Flow Chart for the least greedy algorithm

Ⅳ. Conclusions

In this paper, we investigated the dual properties for the optimal bit allocation based on the well-known high-resolution formular for transform coding. We observed that when one more bits is available in addition to the given bit allocation, it must be allocated to the most greedy quantizer, namely, one that has the greatest distortion among the coefficient quantizers. Also, if one bit needs to be deprived of the given bit allocation, it must be from the least greedy quantizer that has the smallest distortion. Applying these properties to integer-bit allocation problem, we obtained the dual optimal algorithms for adjusting non-integer bit allocation to integer rates.

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임 종 태 (林鍾泰)

1989년 2월 : 서울대학교 전자공학과(공학사)
1991년 2월 : 서울대학교 전자공학과(공학석사)
2001년 8월 : The Univ. of Michigan at Ann Arbor 전기공학과(공학박사)
1991년 3월 ~ 2004년 8월 (주) 대우일렉트로닉스 영상연구소 및 신호처리연구소 책임 연구원
2004년 9월 ~ 현재 한국항공대학교 항공전자 및 정보통신공학부 조교수
관심분야 : Joint Source-Channel Coding, Digital Broadcasting Systems, Video Signal Processing

유 도 식 (兪堵植)

1990년 8월 : 서울대학교 전기공학과(공학사)
1994년 2월 : 서울대학교 물리학과(이학석사)
2002년 4월 :미국 미시간대학교 전기공학과(공학박사)
2002년 2월 ~ 2003년 10월 미시간대학교 박사후 연구원
2006년 9월 ~ 현재 홍익대학교 전자전기공학부 조교수
관심분야 : 통신이론, 무선통신 시스템 설계, 정보이론, 부호이론, 네트워크 정보이론 등