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Wavelet을 이용한 광역보정위성항법을 위한 전리층 모델링

Ionospheric Modeling using Wavelet for WADGPS

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요 약

전리층 지연은 보정위성항법시스템(DGPS), 위성항법시스템(GPS)을 이용한 시각동기화 및 광역보정위성항법시스템(WADGPS)의 주요한 오차원인이다. 이러한 전리층 지연은 위성 신호가 통과하는 전리층의 환경에 따라 달라지므로 일반적인 보정위성항법시스템의 기준국이 보정할 수 있는 사용자와의 거리는 약 100km로 제한된다. 따라서 광역보정위성항법의 경우 여러 기준국의 측정치를 이용하여 보정구간 전리층 전체를 모델링하여 보정정보를 단일 주파수 수신기 사용자들에게 보내주게 된다. 이를 위해 이미 기존의 격자 알고리즘이 구현되어 있으나 기존의 격자 알고리즘에서는 전리층에 자기폭풍현상이 일어났을 경우에 대한 대처와 정확도가 고려되지 않고 있다. 자기폭풍이 일어나면 수직전리층 값이 공간적으로 noisy한 분포를 나타내게 되기 때문에 격자 알고리즘으로의 경우 모델링의 정확도가 낮아지게 된다. 또한 정확도를 높이기 위한 다른 함수 기반 전리층 모델의 경우 자기 폭풍이 일어났을 때 보정정보 값의 연속성이 보장되지 않는다. 본 논문에서 제시하는 wavelet을 이용한 알고리즘은 보정정보의 개수가 같을 때 기존의 격자 알고리즘보다 더 높은 정확도를 보이며, 특히 자기폭풍이 왔을 때도 비교모델인 spherical harmonics 기반 알고리즘에 비해서도 정확도가 향상됨을 볼 수 있다. 또한 다른 함수기반 알고리즘의 경우 정확도는 높지만 전송해야하는 보정정보 값이 자기폭풍시에 불연속이 되는데 반해 본 알고리즘은 연속성이 보장된다. 따라서 본 알고리즘을 이용하면 자기폭풍시에도 적용가능함으로서 기존의 알고리즘들의 문제를 개선할 수 있다.

Abstract

Ionospheric time delay is one of the main error source for single-frequency DGPS applications, including time transfer and Wide Area Differential GPS (WADGPS). Grid-based algorithm was already developed for WADGPS but that algorithm is not applicable to geomagnetic storm condition in accuracy and management. In geomagnetic storm condition, the spatial distribution of vertical ionospheric delay is noisy and therefore the accuracy of modeling become low in grid-based algorithm. For better accuracy, function based algorithm can be used but the continuity of correction message is not guranteed.In this paper, we propose the ionospheric model using wavelet based algorithm. This algorithm shows better accuracy with the same number of correction message than the existing spherical harmonics algorithm and guarantees the continuity of correction messages when the number of message is expanded for geomagnetic storm condition.

Key words : Ionospheric Model, Ionospheric Storm, Wavelet

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I. Introduction

Ionospheric correction in the Wide Area Differential GPS concept can be separated into three components: an estimation problem, a transmission problem, and a prediction problem[1]. The first step in the differential correction process is to construct a model of the ionosphere. Then given a set of biased and noisy measurements of ionospheric delay, the second step is to fit the model to the available measurements and generate a confidence bound on the residual error. Taken together these two steps constitute the estimation problem.

Once complete, the solution to the estimation problem at the current point in time may be encoded and sent to remote users. While not the focus of this paper, the WAAS solution to the transmission problem is both powerful and elegant. Using a state space model the WAAS separates states by their necessary update rates, the vector corrections have been condensed into a single 250bps message stream. This highly efficient correction stream is suitable for transmission on practically any communication channel, most significantly geosynchronous satellite broadcast where the coverage region closely matches the service volume.

The correction information decoded from the WAAS message stream is applied by reconstructing the state estimates (ionosphere, clock, ephemeris) and projecting them onto the user's observation geometry. For the ionospheric term this is the prediction problem. In the case of the ionospheric corrections it amounts to predicting the ionospheric delay along a line-of-sight (LOS) and generating a confidence bound on the residual error between the prediction and the true delay.

It is the nature of the ionosphere that allows the differential correction concept to work. Consider the ionospheric delay along one LOS as a random process. We are only able to predict the delay on a user's observations because it is, at a minimum, correlated with the delay along other LOSs. The focus of this work is to characterize the structure of that correlation. More explicitly, we seek to characterize the correlation structure of the ionosphere under the model invoked in the estimation problem

II. Theory

2-1 Conventional 2D-modeling

We have used Klobuchar's assumptions in 2D ionospheric time-delay model (Klobuchar, 1987)[2]. Figure 1 shows these assumptions;

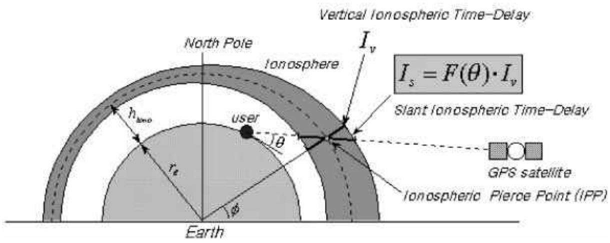


그림 1. 2차원 전리층 모델링 가정
Fig 1. 2D Ionospheric time-delay modelling assumptions.

The ionosphere is assumed to be concentrated at the Ionospheric Pierce Point (IPP) and its average height (hiono) is 350 km~450km from the ground. This is the key concept of the 2D ionospheric model[3]. The real delay of the GPS signal is a slant ionospheric time delay, but this is not appropriate for the 2D model because it varies according to satellite elevation angle; hence, the vertical ionospheric time delay should be used. Vertical and slant ionospheric time delays are related by an obliquity factor: Is =F x Iv , which is only a function of the satellite elevation angle: F = F(θ) (Qiu et al., 1994).

As ionosphere activity is dominated by local time and geomagnetic latitude, the ionospheric time-delay model should be expressed in the coordinate of local time (λ) and geomagnetic latitude (Φ) of the IPP. These can be calculated from GPS time, geographical latitude and longitude.

In the implementation, the ionospheric vertical delay is modeled and expanded by k-th order spherical harmonics[4], i.e.

$$I_v = \sum_{n=0}^{k}\sum_{m=0}^{n}\{C_{nm}\cos(m\lambda) + S_{nm}\sin(m\lambda)\}P_{nm}(\sin(\phi)) \qquad (1)$$

where P_{nm} is the legendre function.

For determining ionospheric model, we must solve coefficients (C_{nm} , S_{nm})

2-2 Ionospheric observation

The time delay of GPS radiowave propagating from transmitter to receiver through ionosphere is given by

$$\Delta t = \frac{1}{c}\int_{R(r)}^{SV(r)}(1-n)dl(r) \qquad (2)$$

where c is the speed of light, SV is the transmitter location, R is the receiver location, n is the index of refraction, and r is a four dimensional position vector. The effect of the ionosphere is captured in the index of refraction, n, which is a function of both radiowave frequency and position along the phase path. The full expression for the complex index of refraction in a plasma such as the ionosphere is given by the Appleton-Hartree equation.

$$n^2 = \frac{X}{1 - iZ - \dfrac{Y_T^2}{2(1-X-iZ)} \pm \left[\dfrac{Y_T^4}{4(1-X-iZ)^2} + Y_L^2\right]^{\frac{1}{2}}}$$

where

$$X = \frac{N(r)e^2}{\varepsilon_0 m f^2} = \frac{f_N^2}{f^2} : \text{thermal motion of the electrons}$$

$$Y_L = \frac{eB_L}{mf} = \frac{f_H\cos\theta}{f} : \text{longitudinal component of the Lorentz force}$$

$$Y_T = \frac{eB_T}{mf} = \frac{f_H\sin\theta}{f} : \text{transverse component of the Lorentz force} \qquad (3)$$

$$Z = \frac{v}{f} : \text{the ratio of collision frequency to radio frequency}$$

N(r) is the local electric density of the plasma, e is the charge on an electron, ε_0 is the permittivity of free space, m is the mass of an electron, B_L and B_T terms are the longitudinal and transverse components of the geomagnetic field, f_H is the gyro (cyclotron) frequency and θ is the angle between the geomagnetic field vector and wave vector. Typically the local plasma frequency in the ionosphere is around 10 MHz, gyro frequency is around 1 MHz, and the collision frequency is around 10 kHZ. So, the L-band approximation to the Appleton-Hartree equation is

$$n \approx 1 - \frac{X}{2} \qquad (4)$$

This is comparatively simple and yet good to better than 1% error. By substituting (4) into (2),

$$\begin{aligned}\Delta t &= \frac{1}{c}\int_{R(r)}^{SV(r)}(1-n)dl(r) = \frac{1}{c}\int_{R(r)}^{SV(r)}\frac{X}{2}dl(r) \\ &= \frac{a}{cf^2}\int_{R(r)}^{SV(r)}N(r)dl(r), \quad a = \frac{e^2}{8\pi^2\varepsilon_0 m}\end{aligned} \qquad (5)$$

If we apply equation (5) in L1,L2 and subtracting each other, we can get following equation.

$$\delta(\Delta t) = \Delta t_{L_2} - \Delta t_{L_1} = \frac{a}{c}\int_{R(r)}^{SV(r)}Ndl(r)\left(\frac{1}{f_{L_2}{}^2} - \frac{1}{f_{L_1}{}^2}\right) \qquad (6)$$

Then. the total electron content (TEC) along the line of sight including IFB can be observed by dual frequency GPS receivers with the instantaneous code delay observation[5]

$$TEC = \frac{f_{L_1}^2 \cdot f_{L_2}^2}{f_{L_1}^2 - f_{L_2}^2} \times \frac{c\cdot\delta(\Delta t)}{a} = \frac{f_{L_1}^2 \cdot f_{L_2}^2}{f_{L_1}^2 - f_{L_2}^2} \times \frac{\rho_2 - \rho_1}{a} - IFB \qquad (7)$$

Typical IFBs can be as large as 15(m) which is unacceptable considering the ionospheric delay ranges from 2 to 30(m). The IFB depends on the antenna, pre-amp, cable, RF filters in the receiver and even the environment (temperature primarily), and the IFB is unique to every receiver installation.

Ⅲ. Wavelet algorithm

With the widespread development of wavelet theory since the ground breaking publication by Daubechies in 1988, wavelets have been applied in a variety of areas, image and data compression, de-noising and filtering in signal processing and inversion of linear systems.

The two properties of wavelets are bounded support and annihilation of moments. The simplest wavelet, called the Haar wavelet, illustrates these two properties. A series of one dimensional Haar wavelets are shown Figure 2.

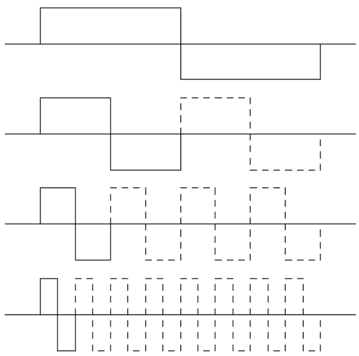


그림 2. Haar 웨이블렛
Fig 2. Haar wavelet

The individual wavelets shown as solid lines are replicated over the extent of the line interval, each wavelet disjoint from its neighbor. Moving from the bottom line to the top increases the scale size, or simply scale, of the wavelet. The support of any individual wavelet is doubled stepping from one scale to the next. An important property of wavelets is that they are orthonormal with respect to other wavelets in the same scale and the wavelets at all other scales.

The simple Haar wavelet[8]

$$\varphi^s(t) = \begin{cases} 2^{-s} & \text{if } -1 <= t < 0 \\ -2^{-s} & \text{if } 0 <= t < 1 \\ 0 & \text{otherwise} \end{cases} \tag{8}$$

has only a single vanishing moment

$$\int \varphi(l) f(l) dl = 0 \quad \text{if and only if } f(l) = c \ \ \forall l \tag{9}$$

Given a one-dimensional wavelet basis such as the Haar wavelet, we can construct a two-dimensional basis. Taking a tensor product of three bases along the ordinate directions, latitude, longitude, and altitude yields a complete basis.

$$\begin{aligned} \Psi(\theta,\phi) &= \{\varphi(\theta) \otimes \varphi(\phi)\} \\ &= \left\{ \prod_{skl}^{JKL} \varphi_k^s(\theta) \varphi_l^s(\phi) \right\} \end{aligned} \tag{10}$$

From the defining equation introduced back in previous chapter,

$$TEC = \int_{R(r)}^{SV(r)} N(r) dl(r) \tag{11}$$

A user can reconstruct a local observation matrix from her known line of sight vectors and the ionospheric basis functions.

$$A_{ij} = \begin{cases} r 4^s (-1)^{s+k+l} \Delta l \csc(\theta_m) \\ 0 \end{cases} \tag{12}$$

Once the user's observation matrix is built, the differential ionospheric correction for that line of sight is reconstructed by dotting it with the broadcast correction.

$$TEC^j = A_{user} x \tag{13}$$

IV. Simulation

4-1 Simulation Circumstance

Actually, we can not know the true inter-frequency bias of receiver. So we compute IFBs before simulation. Simulation area is all over the USA and 39 stations. Stations and area are shown in figure 3. Each quiet day and stormy day are selected. The ionosphere consists of density distribution as latitude, longitude, height in USA.

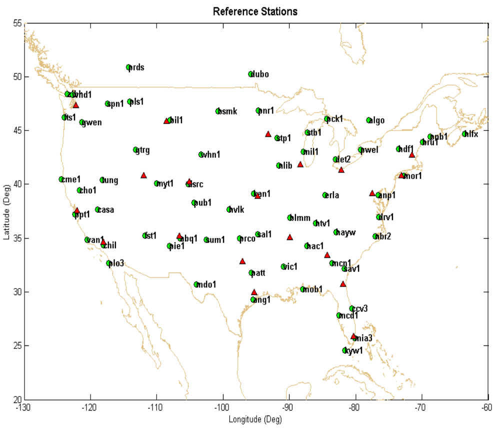


그림 3. 시뮬레이션에 사용된 기준국 분포
Fig 3. Selected stations

4-2 Simulation result

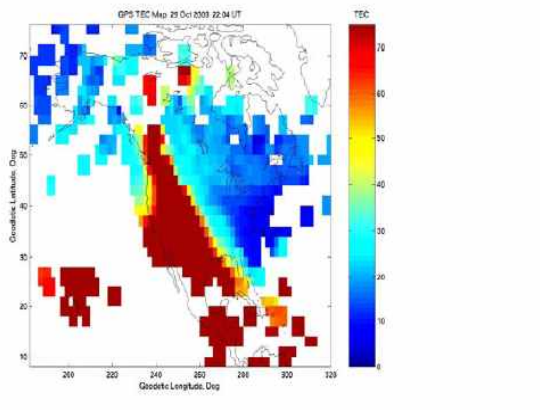
In a quiet day, we compare the results of spherical harmonics and wavelet. Table 1 shows the result

표 1. 2003년 10월 28일 비교결과 (Quiet day)
Table 1. 2003.301day.173280 (Quiet day)

No. of param	Spherical harmonics		HAAR	
	RMS(m)	Max(m)	RMS(m)	Max(m)
16(3rd)	0.4921	2.2809	0.4960	2.2863
25(4th)	0.4889	2.1522	0.4883	2.117
36(5th)	0.4810	2.0861	0.4719	2.2075
49(6th)	0.4624	2.0609	0.4642	2.0232
64(7th)	0.4561	1.9089	0.4560	1.9306

From the results, errors are similar in same number of parameters at a quiet day, so we can conclude that wavelet algorithm has no problem compared with conventional function-based algorithm.

Figure 4 shows the TEC distributions in strong stormy day. In stormy day, we have very steep gradient area like red dot line in figure 4. Conventional algorithms of single scale function have problems with steep gradient but wavelet of multi scale function can fit TEC distributions of stormy day with better accuracy.



GPS TEC map derived from 400+ reference stations, 22:04 UT October 29, 2003.

그림 4. storm day의 수직전리층지연 분포
Fig 4. TEC distribution in storm day

In a stormy day, we compare the results of spherical harmonics and wavelet. Figure 5 and table 2 show the result

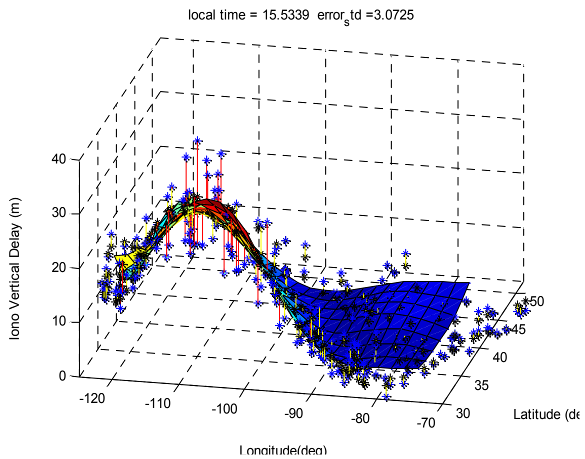


그림5. 측정치와 모델의 차이
(경도축,위도축,수직전리층지연값축)
Fig 5. The difference between measurement and model (longitude vs lattitude vs iono delay)

In figure 5, real measurements are distributed over red line but conventional model cannot catch up measurements because of steep gradient. Wavelet algorithm can supplement this limitation with multi scale fitting.

표 2. 2003년 10월 29일 비교결과 (storm day)
Table 2. 2003.302day.338400 (Strong Storm)

No. of param	Spherical harmonics		HAAR	
	RMS(m)	Max(m)	RMS(m)	Max(m)
16(3rd)	3.6849	13.688	3.3164	13.004
25(4th)	3.2102	12.411	2.7608	10.846
36(5th)	3.0725	11.689	2.4907	10.157
49(6th)	2.7996	11.627	2.2957	10.096
64(7th)	2.6779	11.574	2.1477	10.039

From the results, wavelet algorithm shows better accuracy than conventional function-based algorithm about 15~20% in same number of parameters at a stormy day, because of wavelet properties, the continuity of correction messages can be guaranteed when the number of message is expanded for geomagnetic storm condition.

Ⅴ. Conclusion

Ionospheric time delay is one of the main error source for single-frequency DGPS applications, including time transfer and Wide Area Differential GPS (WADGPS). Grid-based algorithm was already developed for WADGPS but that algorithm is not applicable to geomagnetic storm condition in accuracy and management. In geomagnetic storm condition, the spatial distribution of vertical ionospheric delay is noisy and therefore the accuracy of modeling become low in grid-based algorithm. For better accuracy, function based algorithm can be used but the continuity of correction message is not guranteed.

In this paper, we propose the ionospheric model using wavelet based algorithm. This algorithm shows better accuracy with the same number of correction message than the existing algorithm and guarantees the continuity of correction messages when the number of message is expanded for geomagnetic storm condition. We can apply this algorithm to storm condition for better accuracy and continuity.

References

[1] Chao, Y., Tsai, Y., Walter, T., Kee, C., Enge, P. and Parkinson, B. (1995). The ionospheric model improvement of the Stanford WAAS network. *Proceedings of National Technical Meeting, Institute of Navigation, Anaheim*, pp. 531-538.

[2] Klobuchar, J. A. (1987). Ionospheric time-delay algorithms for single-frequency GPS users. *IEEE Transactions on Aerospace and Electronic systems,* Vol. AES-23, No. 3, pp. 325-331.

[3] Coco, D., Coker, C., Dahlke S. and Clynch, J. (1991). Variability of GPS satellites differential group delay biases. *IEEE Transactions on Aerospace and Electronic systems*, Vol. 27, No. 6, pp. 931-938.

[4] Hobson, E. (1931). The Theory of Spherical and Elliptical Harmonics, *Cambridge University Press*, pp. 777-778..

[5] Kee, C. (1995). Wide Area Differential GPS. Global Positioning System Theory and Applications Volume II, edited by Parkinson, B. W. and Spilker, J. J., Jr., Vol. 164, Progress in Astronautics and Aeronautics, AIAA, pp. 81-115.

[6] Kee, C., Walter, T., Enge, P. and Parkinson, P. (1996). Quality control algorithms on wide-area reference station for WAAS. Proceedings of 52nd ION Annual Meeting, Cambridge, Massachusetts, pp. 487-495.

[7] Kee, C. and Yun, D. (1997). Development of local ionospheric time delay model for single-frequency DGPS users. *Proceedings of ION GPS-97, Kansas City,*

Missouri, pp. 1113-1122

[8] Andrew J. Hansen, Todd Walter, and Per Enge. Ionospheric correction using tomography. P*roceedings of the Institute of Navigation GPS-97,* pages 249-57, September 1997

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