# **Converter Coupled Finite Element Analysis** of Single Phase Switched Reluctance Motor

# Youn-Hyun Kim\*

**Abstract** – This paper deals with the characteristic analysis of the single phase switched reluctance motor based on the converter coupled FE analysis. The converter comprises a rectifier, LC filter and switching power device. A computational technique for the converter coupled FE analysis is introduced and its validity is proven by the experiment.

Keywords: Converter Coupled Analysis, Finite Element Method, Power Factor Correction, Single Phase Switched Reluctance Motor

## 1. Introduction

THE application of Single Phase Switched reluctance motors (SPSRM) for home appliances such as vacuum cleaners is on the increase because they have simple construction, high speed, and large torque. Above all, they are able to lower the cost of drive converters owing to the reduction of the number of power devices [1-2]. The drive converter of SPSRM adopts a capacitor with a low capacitance in dc link due to the cost and size of the system, and an inductor for the power factor correction. A low capacitance causes a large dc link voltage ripple and the amplitude of ripple voltage depends on LC parameter, source voltage and drive state of the SPSRM [3]. It brings about an unexpected result to analyze the driving performance of the SPSRM under the constant dc link voltage, because an applied voltage becomes different according to dc link voltage ripple. Also, LC parameters affect the power factor and Harmonics of source power, which are the chief causes in power consumption and EMI because of the elongated power cable. Accordingly, the optimal LC parameter should be determined in the operation of the SPSRM and converter. Even power factor and efficiency should be calculated by source power rather than motor terminal. Therefore, the FE analysis coupled driving converter with the LC filter is required for the accurate evaluation of SPSRM drive performance and the reflection on the design.

This paper introduces a computational technique for the converter coupled FE analysis, and it shows its results and proves the validity of the analysis. The characteristics of SPSRM and its converter based on the converter coupled

FE analysis are also presented.

### 2. Converter Coupled FE Analysis

#### 2.1 Analysis Model

Fig. 1. shows the cross sectional of the basic analysis model, single phase 6/6 SRM. The motor is configured to have 3 pole pairs, and the path of the magnetic flux is made to be as short as possible in order to reduce the magnetic resistance by parallel connection of each of the windings. This paper will be focused on the application of the motor for a vacuum cleaner. The output power of a vacuum cleaner is 540[W] and the efficiency is 60[%] while the rated output of the motor is 900[W], respectively. Since a capacitor with a low capacitance is placed at the DC link of the converter to lower the cost, the voltage across the DC link can be assumed to be a fully rectified waveform of a conventional AC 220[V] sinusoidal

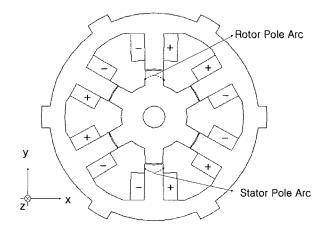


Fig. 1. Cross section of the basic SPSRM model

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Table 1. Sp	pecifications (	Of The Basic	Analysis Model

Index	Quantity	Dimension
Rated Output Power	900	W
Rated Voltage	190	V
Speed .	35,000	rpm
Phase	1	
Stator Pole	6	
Rotor Pole	6	
Rotor Diameter	35	mm
Stator Yoke	6	mm
Rotor Yoke	6	mm
Stator Diameter	80	mm
Air Gap	0.3	mm
Stack Length	33	mm
Coil Part Ratio	10	%

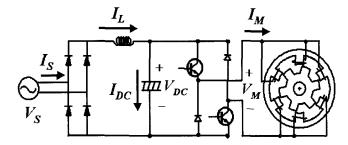


Fig. 2. SPSRM and its converter

source, instead of a smoothly regulated voltage. Thus, the value of the input voltage to the motor is 190[V], which is the average value of the fully rectified voltage of AC 220[V]. The converter to lower the cost and the rated speed is 35,000[rpm], and other specifications are shown in Table 1.

#### 2.1.1 Finite Element Formulations

Fig. 2. presents the SPSRM and its converter. The driving converter comprises diode rectifier, power factor correction LC filter, and switching power devices. For the converter coupled FE analysis, circuit equations of the converter should be formulated and combined with the magnetic field of the SPSRM.

The two-dimensional governing equation for the SPSRM can be expressed as follows:

$$\frac{\partial}{\partial x} \left[ \frac{1}{\mu} \frac{\partial A_z}{\partial x} \right] + \frac{\partial}{\partial y} \left[ \frac{1}{\mu} \frac{\partial A_z}{\partial y} \right] + J_0 = 0 \tag{1}$$

where  $\mu$  is the permeability,  $A_z$  is z component of the magnetic vector potential, and  $J_o$  is the input current density.

The voltage source circuit equation of the SPSRM is expressed as in (2), and the relation of the input voltage of SPSRM and dc link voltage is equal to (3).

$$R_m I_M + L_m \frac{dI_M}{dt} + E_M - V_M = 0$$
(2)

$$V_M - S_S V_{DC} = 0 (3)$$

where  $V_M$  is the input voltage of SPSRM,  $I_M$  is the current of the SPSRM,  $R_m$  is the phase resistance,  $L_m$  is the leakage inductance of the end coil,  $E_M$  is the electromotive force induced in the coil,  $V_{DC}$  is the dc link voltage and  $S_S$  is the

switching function, which indicates the state of power devices, and is defined as follows:

$$S_s \in \{1, -1\}, \quad 1: on, \quad -1: off$$
 (4)

That is,  $V_{DC}$  or  $-V_{DC}$  is applied to the motor according to the switching state of the power device.

By substituting (3) for (2), the circuit equation of the motor may be rewritten as (5).

$$R_m I_M + L_m \frac{dI_M}{dt} + E_M - S_S V_{DC} = 0$$
 (5)

DC link voltage  $V_{DC}$  and current  $I_{DC}$  flowing into the capacitor C is written as in (6), the connection of current  $I_L$  in inductor L,  $I_{DC}$ , and  $I_M$  is given as (7).

$$\frac{dV_{DC}}{dt} - \frac{1}{C}I_{DC} = 0 \tag{6}$$

$$S_S I_M + I_{DC} - I_L = 0 (7)$$

Equation (6) is also replaced with (8) by substitution (7) for (6).

$$\frac{dV_{DC}}{dt} + \frac{1}{C} (S_S I_M - I_L) = 0$$
(8)

The circuit equation concerned in the inductor L is similar to (9). From the characteristic of the diode rectifier,  $I_L$  flows only from source to capacitor and  $I_L$  flowing from capacitor to source is blocked forcibly in the computation.

$$L\frac{dI_L}{dt} + V_{DC} - abs(V_S) = 0 (9)$$

Input source current  $I_s$  is calculated by (10).

$$abs(I_S) = I_L \tag{10}$$

After applying the Gelerkin method to (1) and combining with (2)\_(9), the overall system matrix can be obtained by using the backward difference method to deal with the time derivative term in the circuit equation as in (11).

$$\begin{bmatrix}
[S] & Q & 0 & 0 \\
a & b & -S_S & 0 \\
0 & c & d & e \\
0 & 0 & 1 & f
\end{bmatrix}
\begin{cases}
[A] \\
I_M \\
V_{DC} \\
I_L
\end{cases}^t$$

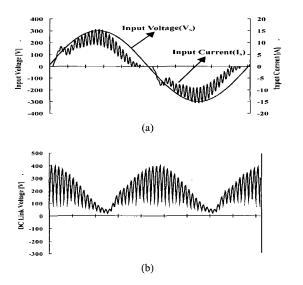
$$= \begin{bmatrix}
[0] & 0 & 0 & 0 \\
a & g & 0 & 0 \\
0 & 0 & d & 0 \\
0 & 0 & 0 & f
\end{bmatrix}
\begin{Bmatrix}
[A] \\
I_M \\
V_{DC} \\
I_L
\end{Bmatrix}^t$$

$$+ \begin{Bmatrix}
[0] \\
0 \\
0 \\
abs(V_S)
\end{Bmatrix}$$
(11)

where 
$$a = {F \choose \Delta t}$$
,  $b = R_m + {L_m \choose \Delta t}$ ,  $c = {S_S \choose C}$ ,  $d = {1 \choose \Delta t}$ ,  $e = {-1 \choose C}$ ,  $f = {L \choose \Delta t}$ ,  $g = {L_m \choose \Delta t}$ 

### 3. Analysis Results and Verification by Experiment

Fig. 3 shows the results of the converter coupled FE analysis. The terms of drive are speed 35,000[RPM], source voltage 60[Hz],  $220[V_{RMS}]$ , DC link C and L  $13[\mu F]$ , 4[mH] respectively. Turn on and off are 5° and 25° respectively from the unaligned position. Fig. 3(a) indicates the source voltage and current waveform. The current includes the ripple of power device switching frequency, and its harmonics and power factor of source power are much improved over just the converter and capacitor.



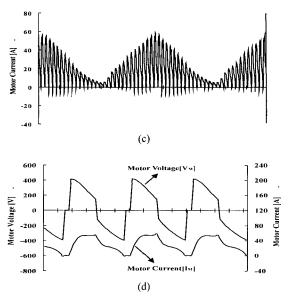
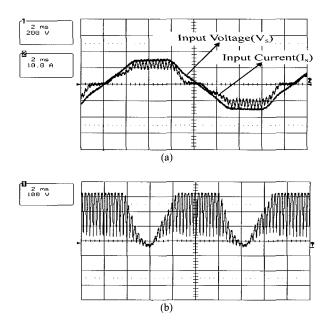


Fig. 3. Analysis result: (a) Source current and voltage, (b) DC link voltage (c) Motor current, (d) Motor current and applied voltage

Fig. 3(b) shows DC link voltage waveform and proves that DC link voltage is not constant. Ripple of power device switching frequency and the two times of source frequency are comprised in the dc link voltage. Each are mainly influenced by L and C, and the extent of the effect is investigated in Chapter IV. Fig. 3(c) reveals the SPSRM current waveform during the period of source. Motor current fluctuates in accordance with the DC link voltage and a long period of torque ripple is also expected. Fig. 3(d) shows the applied voltage and current waveform of the SPSRM during the switching period. Motor voltage doesn't have a constant value owing to the change of DC link voltage.



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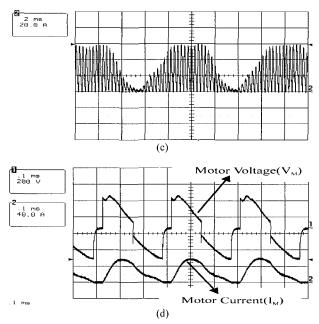


Fig. 4. Experiment result: (a) Source current and voltage, (b) DC link voltage (c) Motor current, (d) Motor current and applied voltage

Fig. 4 presents the experimental results. Excepting that there are some differences between the analysis results and the experimental ones due to the distortion of source voltage, Fig. 4 confirms that both results match well and the proposed analysis scheme is adequate.

# 4. Effectiveness of LC Parameter

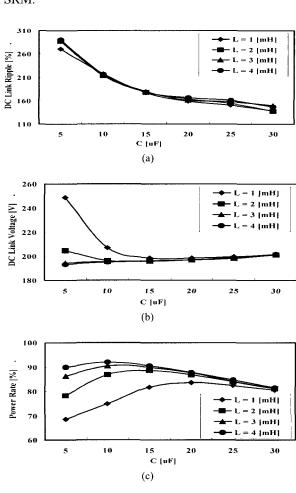
Fig. 5 indicates converter coupled FE analysis results according to the LC parameters. Drive states of the SPSRM are the same as Chapter III. From Fig. 5, the ripple rate of DC link voltage is dependent on C value and its average value increases to the peak of source voltage with the addition of C. Power factor of the source varies seriously according to the LC parameters, is heavily affected by L value especially in small C. THD (Total Harmonics Distortion) of Fig. 5(d) is calculated as in (12).

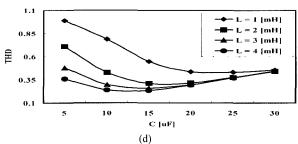
$$THD = \frac{(RMS \ of \ total \ harmonics)}{(RMS \ of \ a \ fundamental)}$$
$$= \frac{\sqrt{(I^2 - I_i^2)}}{I_i}$$
(12)

The switching frequency harmonics diminish by L, but harmonics caused by inrush current rise with the addition of C. Fig. 5(e) shows motor average torque vs. average source current and is largely affected by L value in small C.

Optimal LC parameters for the required performance of application can be selected from the investigation results of

converter coupled FE analysis in drive condition of the SPSRM.





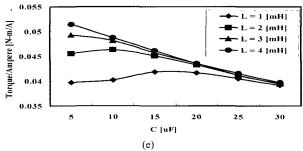


Fig. 5. Analysis result: (a) the ripple rate of DC link voltage, (b) DC link average voltage, (c) Power factor, (d) THD of source current, (e) motor average torque. vs. average source current

#### 5. Conclusion

In this work, the computational scheme for the converter coupled FE analysis was introduced and the validity of the analysis method was verified by the experimental results. The characteristics of the SPSRM and its converter according to LC parameters were also presented through the converter coupled FE analysis. From the analysis results, it is known that not only SPSRM but circuit parameters are important elements of design and its best values can be computed by only the converter coupled FE analysis. Moreover, it is expected that the proposed method will be widely used.

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