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Local Influence of the Quasi-likelihood Estimators in Generalized Linear Models

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Abstract

We present a diagnostic method for the quasi-likelihood estimators in generalized linear models. Since these estimators can be usually obtained by iteratively reweighted least squares which are well known to be very sensitive to unusual data, a diagnostic step is indispensable to analysis of data. We extend the local influence approach based on the maximum likelihood function to that on the quasi-likelihood function. Under several perturbation schemes local influence diagnostics are derived. An illustrative example is given and we compare the results provided by local influence and deletion.

Keywords: Cook's distance; diagnostics; generalized linear models; local influence; maximum likelihood estimator; quasi-likelihood estimator.

1. Introduction

Generalized linear models (GLMs; Nelder and Wedderburn, 1972; McCullagh and Nelder, 1989) are a popular method for modeling the relationship between a discrete or continuous response variable and the predictors. However, GLM usually requires the specification of the distribution of the response, which is belonging to the exponential family, and uses the maximum likelihood estimators (MLE). It can be extended to the quasi-likelihood method with only consideration of the link and variance functions of the model. The quasi-likelihood estimator is an alternative to the maximum likelihood estimator when the latter is not available. In the view of estimation and inference the link, variance and independence assumptions are more important than the distributional assumptions. So the quasi-likelihood estimator is very useful in practice.

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In GLM the MLEs are usually obtained using iteratively reweighted least squares (IRLS), a kind of least squares estimates which is known to be sensitive to outliers or influential observations. So are the quasi-likelihood estimates. It requires a diagnostic step for analyzing data based on the quasi-likelihood method in GLM. The diagnostics for GLM used those for Gaussian linear models, which are the Pearson residual, the deviance residual, the hat matrix and the Cook distance (Dobson, 2002, pp. 127–130). Since these diagnostics are single case deletion methods, they cannot detect unusually multiple observations and overcome the errors of masking or swamping effects. Deletion approach is a global influence of observations on the estimates of parameters.

Cook (1986) devised the local influence method as a general diagnostic of investigating locally the influence of observations. The local influence can be obtained by perturbing the statistical model under a given weight. This approach provides influence information about the relative impact of a given perturbation and characterization of most influential observations. Thomas and Cook (1989) suggested diagnostics for influence on the estimated regression coefficients in a GLM. Lesaffre and Verbeke (1998) used local influence in linear models for investigating longitudinal data. However these two approaches are based on the likelihood displacement. To get the local influence of the quasi-likelihood estimator we need a new definition for local influence, not based on the likelihood displacement.

In this work we investigate the influence of observations on the quasi-likelihood estimators in GLMs. In Section 2 we review the quasi-likelihood estimator in GLMs and introduce some notations used in this paper. We define a quasi-likelihood displacement which is an extension of the likelihood displacement and suggest diagnostics of the estimators of parameters using the local influence in Section 3. In Section 4 we derive local influence measures on the quasi-likelihood estimators in GLMs under some meaningful perturbation schemes. In Section 5 an illustrative example is given and we conduct the comparison of the local influence measures and the deletion diagnostics.

2. Quasi-likelihood Estimators

In this section we review the quasi-likelihood estimator in GLMs.

For $i = 1, \dots, n$, let consider the model $Y_i = \mu_i + \epsilon_i$, where $\mu_i = \mathbf{x}_i^T \boldsymbol{\beta}$ and ϵ_i follows $N(0, \sigma^2)$, that is, $Y_i \sim N(\mu_i, \sigma^2)$. This normal linear model can be extended to the GLM by Nelder and Wedderburn (1972) with the consideration of the non-normal response variable and the link function $g(\mu_i) = \mathbf{x}_i^T\boldsymbol{\beta}$. However, the GLM requires the specification of the probability distribution function for the response variable. In case we would know only the information about the type and moments of the response, it is not possible to construct the likelihood function by a GLM. Wedderburn (1974) proposed an estimation method in such situations, which is referred to as the quasi-likelihood method.

Let Y_i have mean μ_i and variance $\phi V(\mu_i)$. And assume that Y_i's are independent. Let define the random variable $U_i = (Y_i - \mu_i)/\phi V(\mu_i)$. Then we have $E(U_i) = 0, \mathrm{Var}(U_i) = 1/\phi V(\mu_i), E(\partial U_i/\partial \mu_i) = -\mathrm{Var}(U_i)$. The integral

$$Q_i(\mu_i, y_i) = \int_{y_i}^{\mu_i} (y_i - t)/\phi V(t)dt,$$

if it exists, should behave like a log-likelihood function for μ_i. It is called the quasi-likelihood for μ_i based on data y_i. Since Y_i's are independent, the quasi-likelihood for the complete data is $Q(\mu, y) = \sum_{i=1}^n Q_i(\mu_i, y_i)$. The quasi-likelihood estimators can be obtained by minimizing the objective function $Q(\mu, y)$, that is, finding the estimates of $\boldsymbol{\beta}$ satisfies the equation

$$\mathbf{0} = \frac{\partial Q}{\partial \boldsymbol{\beta}} = \mathbf{D}^T\mathbf{V}^{-1}(\mathbf{y} - \boldsymbol{\mu})/\phi, \tag{2.1}$$

where $\mathbf{D} = \partial\boldsymbol{\mu}/\partial\boldsymbol{\beta}^T$ and $\mathbf{V} = \mathrm{diag}(V(\mu_1), \ldots, V(\mu_n))$. Beginning with an arbitrary estimate $\hat{\boldsymbol{\beta}}^{(0)}$ sufficiently close to $\boldsymbol{\beta}$, the sequence of parameter estimates generated by the Newton-Raphson method with Fisher scoring is

$$\hat{\boldsymbol{\beta}}^{(m+1)} = \hat{\boldsymbol{\beta}}^{(m)} + (\mathbf{D}^T\mathbf{V}^{-1}\mathbf{D})^{-1}\mathbf{D}^T\mathbf{V}^{-1}(\mathbf{y} - \boldsymbol{\mu}).$$

The conventional estimator of ϕ is a moment estimator based on the residual vector $\mathbf{y} - \hat{\boldsymbol{\mu}}$, namely $\hat{\phi} = \sum_{i=1}^n (y_i - \hat{\mu}_i)^2/((n-p)V(\hat{\mu}_i))$. See McCullagh (1983) and Davis (2002) for the properties and the algorithms of the quasi-likelihood estimates.

This is an estimate of IRLS which is sensitive to outliers or influential observations. So it is indispensable to the diagnostics for the quasi-likelihood estimates before data analysis.

3. Local Influence for Quasi-likelihood

We define a new quasi-likelihood displacement based on the maximum-likelihood displacement (Cook, 1986) and then adapt the local influence method to the quasi-likelihood estimates.

Let $\mathbf{w} = (w_1, \ldots, w_n)^T$ be an $n \times 1$ vector of small perturbations. Specific perturbation schemes are studied in the next section. We denote the quasi-likelihoods for the unperturbed and perturbed models by $Q(\boldsymbol{\beta})$ and $Q(\boldsymbol{\beta}|\mathbf{w})$, respectively. Similar to the maximum-likelihood displacement (Cook, 1986), we define the quasi-likelihood displacement as

$$QD(\mathbf{w}) = 2\{Q(\hat{\boldsymbol{\beta}}) - Q(\hat{\boldsymbol{\beta}}_{\mathbf{w}})\},$$

where $\hat{\beta}$ and $\hat{\beta}_{\mathbf{w}}$ are the quasi-likelihood estimators of $\boldsymbol{\beta}$ under the unperturbed and perturbed models, respectively.

Let $\mathbf{w} = \mathbf{w}_0 + a\boldsymbol{l}$ with $\|\boldsymbol{l}\| = 1$ and scalar a. Here $\mathbf{w}_0$ is called the null point that represents no perturbation of the data or the statistical model. We considered the surface $(\mathbf{w}^T, QD(\mathbf{w}))^T$. Since $\partial QD(\mathbf{w})/\partial w_i|_{\mathbf{w}=\mathbf{w}_0} = 0, i = 1, \ldots, n$ from the property of the quasi-likelihood estimator at the null point, the normal curvature of this surface becomes

$$C_{\boldsymbol{l}} = 2|\boldsymbol{l}^T \ddot{\mathbf{F}} \boldsymbol{l}|, \tag{3.1}$$

where

$$\ddot{\mathbf{F}} = -\left.\frac{\partial^2 Q(\hat{\boldsymbol{\beta}}_{\mathbf{w}})}{\partial \mathbf{w} \partial \mathbf{w}^T}\right|_{\mathbf{w}=\mathbf{w}_0} = \boldsymbol{\Delta}^T \{-\ddot{\mathbf{Q}}_{\boldsymbol{\beta}\boldsymbol{\beta}}(\hat{\boldsymbol{\beta}})\}^{-1} \boldsymbol{\Delta} \tag{3.2}$$

in which $\boldsymbol{\Delta} = \partial^2 Q(\boldsymbol{\beta}|\mathbf{w})/\partial\boldsymbol{\beta}\partial\mathbf{w}^T$ is a $p \times n$ matrix evaluated at $\boldsymbol{\beta} = \hat{\boldsymbol{\beta}}$ and $\mathbf{w} = \mathbf{w}_0$, $\ddot{\mathbf{Q}}_{\boldsymbol{\beta}\boldsymbol{\beta}}(\hat{\boldsymbol{\beta}}) = \partial^2 Q(\boldsymbol{\beta})/\partial\boldsymbol{\beta}\partial\boldsymbol{\beta}^T$ is a $p \times p$ Hessian matrix evaluated at $\boldsymbol{\beta} = \hat{\boldsymbol{\beta}}$.

There are several ways in which (3.1) can be used to study the local influence in practice. First we consider the extremes $C_{max} = \mathrm{Max}_{\boldsymbol{l}} C_{\boldsymbol{l}}$ which is obtained by the maximum absolute eigenvalue of $\ddot{\mathbf{F}}$. The direction vector $\boldsymbol{l}_{max}$ corresponding to C_{max} indicates how to perturb the postulated model to obtain the greatest local changes in the quasi-likelihood displacement. We should pay attention to observations corresponding to the large element of the direction vector $\boldsymbol{l}_{max}$. Then the index plot of $\boldsymbol{l}_{max}$ may be helpful in finding large direction cosines of the direction vector corresponding to the maximum normal curvature of the surface $(\mathbf{w}^T, QD(\mathbf{w}))^T$.

We consider another influence measure (Zhu and Zhang, 2004) for the i^{th} observation

$$\ddot{F}_{d_i} = \sum_{j=1}^{p} \lambda_j \gamma_{ji}^2, \tag{3.3}$$

where $\{(\lambda_j, \boldsymbol{\gamma}_j)|j = 1, \ldots, n\}$ are the eigenvalue-eigenvector pairs of $\ddot{\mathbf{F}}$ with $\lambda_1 \geq \cdots \geq \lambda_p \geq \lambda_{p+1} = \cdots = \lambda_n = 0$ and $\{\boldsymbol{\gamma}_j = (\gamma_{j1}, \ldots, \gamma_{jn})^T|j = 1, \ldots, n\}$ is the associated orthonormal basis. Equation (3.3) equals almost to the i^{th} diagonal elements of $\ddot{\mathbf{F}}$. Thus this influence measure expresses the local sensitivity to the quasi-likelihood by the perturbations.

To gain the influence information about the quasi-likelihood estimator $\hat{\boldsymbol{\beta}}$ based on the local influence we need some derivatives of the quasi-likelihood function with respect to the parameters. Equation (2.1) is the quasi-score function for $\boldsymbol{\beta}$ evaluated at $\boldsymbol{\beta} = \hat{\boldsymbol{\beta}}$, which is rewritten by

$$\frac{\partial Q}{\partial \boldsymbol{\beta}} = \mathbf{X}^T \dot{\mathbf{E}}/\phi,$$

where $\dot{\mathbf{E}} = \text{diag}(\dot{e}(\hat{\eta}_1), \ldots \dot{e}(\hat{\eta}_n))$ and $\dot{e}(\hat{\eta}_i) = \dot{k}(\hat{\eta}_i)(Y_i - \hat{\mu}_i)/V(\hat{\mu}_i)$ for $i = 1, \ldots, n$. Here $\hat{\eta}_i = \mathbf{x}_i^T \hat{\boldsymbol{\beta}}$ and $\dot{k}(\hat{\eta}_i) = \partial \mu_i / \partial \eta_i|_{\eta_i = \hat{\eta}_i} = \partial k(\eta_i)/\partial \eta_i|_{\eta_i = \hat{\eta}_i}$. Furthermore, we obtained the Hessian matrix

$$-\ddot{\mathbf{Q}}_{\boldsymbol{\beta}\boldsymbol{\beta}}(\hat{\boldsymbol{\beta}}) = \mathbf{X}^T \ddot{\mathbf{E}} \mathbf{X}, \tag{3.4}$$

where $\ddot{\mathbf{E}} = \text{diag}(\ddot{e}(\hat{\eta}_1), \ldots, \ddot{e}(\hat{\eta}_n))$ and $\ddot{e}(\hat{\eta}_i) = [\dot{k}^2(\hat{\eta}_i) - \ddot{k}(\hat{\eta}_i)(Y_i - \hat{\mu}_i)]/V(\hat{\mu}_i)$. Here $\ddot{k}(\hat{\eta}_i)$ is the second derivative of the $k(\eta_i)$ with respect to η_i evaluated at $\eta_i = \hat{\eta}_i$. If $k(u) = u$ or we use the expectation of the Hessian matrix instead of the Hessian matrix, then (3.4) reduces to $-\ddot{\mathbf{Q}}_{\boldsymbol{\beta}\boldsymbol{\beta}}(\hat{\boldsymbol{\beta}}) = \mathbf{X}^T \mathbf{V}^{-1} \mathbf{X}$, where $\mathbf{V}$ is the $n \times n$ diagonal matrix with the i^{th} element $V(\hat{\mu}_i)$ (Thomas and Cook, 1986).

To construct $\ddot{\mathbf{F}} = \boldsymbol{\Delta}^T (\mathbf{X}^T \ddot{\mathbf{E}} \mathbf{X})^{-1} \boldsymbol{\Delta}$ we need the matrix $\boldsymbol{\Delta}$ which is determined by the perturbation scheme. The influence diagnostic measure is calculated as follows. First we specify a perturbation scheme and derive $\boldsymbol{\Delta}$. Then we use the direction of the maximum normal curvature or the diagonal elements of $\ddot{\mathbf{F}}$.

For example, we consider the normal linear model for the normally distributed response variable. Natural choices are given as $g(\mu_i) = \mu_i, V(\mu_i) = 1, \phi = \sigma^2$. Then we have $k(\hat{\eta}_i) = \hat{\mu}_i, \dot{k}(\hat{\eta}_i) = 1, \ddot{(}\hat{\eta}_i) = 0$ and $\ddot{\mathbf{E}} = -\mathbf{I}$, that is the same as the results of Cook (1986) for the normal linear model.

4. Perturbation Schemes

4.1. Case Weight Perturbation

We consider a case weight perturbation which modifies the weight given to each case in the quasi-likelihood. Cook (1986) asserted that case weight perturbation generalizes the case deletion by allowing fractional addition and deletion.

Let the perturbed quasi-likelihood be

$$Q(\boldsymbol{\beta}|\mathbf{w}) = \sum_{i=1}^{n} w_i Q(\mu_i, Y_i). \tag{4.1}$$

Simple calculation follows

$$\boldsymbol{\Delta}_c = \mathbf{X}^T \dot{\mathbf{E}} / \hat{\phi},$$

where $\dot{\mathbf{E}}$ is the $n \times n$ diagonal matrix with the i^{th} element $\dot{e}_i(\hat{\eta}_i) = \dot{k}(\hat{\eta}_i)(y_i - \hat{\mu}_i)/V(\hat{\mu}_i)$. Then the direction of the maximum normal curvature can be obtained by the eigenvector of $\ddot{\mathbf{F}} = \dot{\mathbf{E}}\mathbf{X}(\mathbf{X}^T\ddot{\mathbf{E}}\mathbf{X})^{-1}\mathbf{X}^T\dot{\mathbf{E}}$ associated with the largest eigenvalue.

The Cook's distance for GLMs is given by McCullagh and Nelder (1989, p. 407). Similarly we obtain the following results based on the reduced data omitting the i^{th} observation

$$\hat{\boldsymbol{\beta}}_{(i)} - \hat{\boldsymbol{\beta}} = -(\mathbf{X}^T\mathbf{S}\mathbf{X})^{-1}\mathbf{x}_i \frac{(y_i - \hat{\mu}_i)s_i}{(1 - h_i)\dot{k}(\hat{\eta}_i)},$$

where $\hat{\boldsymbol{\beta}}_{(i)}$ denotes the estimate based on the reduced data, s_i and h_i are the i^{th} diagonal element of $\mathbf{S} = \dot{\mathbf{K}}\mathbf{V}^{-1}\dot{\mathbf{K}}$ and $\mathbf{H} = \mathbf{S}\mathbf{X}(\mathbf{X}^T\mathbf{S}\mathbf{X})^{-1}\mathbf{X}^T$, respectively. Here $\dot{\mathbf{K}}$ is the $n \times n$ diagonal matrix with the i^{th} element $\dot{k}(\hat{\eta}_i)$. Thus the Cook's distance for the quasi-likelihood estimator of GLMs can be written as

$$C_i = \frac{h_i}{(1 - h_i)^2} \frac{(y_i - \hat{\mu}_i)^2}{V(\hat{\mu}_i)} \frac{1}{p\hat{\phi}}. \tag{4.2}$$

The i^{th} diagonal element of $\ddot{\mathbf{F}}$ becomes $\dot{k}(\hat{\eta}_i)^2(y_i - \hat{\mu}_i)^2\mathbf{x}_i^T(\mathbf{X}^T\ddot{\mathbf{E}}\mathbf{X})^{-1}\mathbf{x}_i/(V^2(\hat{\mu}_i)\hat{\phi}^2)$, which can be an alternative influence measure of the Cook's distance. Furthermore the i^{th} element of $\ddot{\mathbf{F}}$ is approximately equal to d_i in (3.3) and it can be seen in Section 5.

4.2. Perturbation of Covariates

Cook (1986) proposed a general scheme for perturbing the whole design matrix $\mathbf{X}$ in linear regression models. Some authors have studied the perturbation of covariates (Thomas and Cook, 1989). This perturbation can be considered from the well known fact that minor perturbations of the covariates in linear regression can seriously influence the least squares estimates when collinearity is present (Suárez Rancel and González Sierra, 2001).

Let $\mathbf{x}_t^c$ be the t^{th} column vector of $\mathbf{X}$. Consider the perturbation

$$\mathbf{x}_{tw}^c = \mathbf{x}_t^c + u_t \mathbf{w}, \tag{4.3}$$

where the perturbation vector $\mathbf{w}$ is the $n \times 1$ vector and u_t is a scalar. This perturbation means that the t column of the covariates is modified by adding some minor perturbation vector which is scaled by u_t (Thomas and Cook, 1989). Note that the perturbation of covariates is not meaningful for indicator variables.

The perturbation (4.2) affects the quasi-likelihood $Q(\boldsymbol{\beta}|\mathbf{w})$ only through $\boldsymbol{\eta}$. When the t^{th} column of $\mathbf{X}$ is perturbed, μ_{ij} is perturbed to $\mu_{ij,\mathbf{w}} = k(\eta_{ij,\mathbf{w}})$. The partial derivative of the quasi-likelihood Q evaluated at $\hat{\boldsymbol{\beta}}$ and $\mathbf{w}_0 = \mathbf{0}_n$ follows that

$$\frac{\partial^2 Q}{\partial w_i \partial \beta_j}|_{\hat{\boldsymbol{\beta}},\mathbf{w}_0} = \frac{u_t}{\hat{\phi}}\{\hat{\beta}_t x_{ij}\ddot{e}(\hat{\eta}_i) + \delta_{jt}\dot{k}(\hat{\eta}_i)\frac{(y_i - \hat{\mu}_i)}{V(\hat{\mu}_i)}\}.$$

Thus we obtained

$$\boldsymbol{\Delta}_t = \frac{u_t}{\hat{\phi}}\mathbf{A}_t,$$

where $\mathbf{A}_t = (\hat{\beta}_t \mathbf{X}^T\ddot{\mathbf{E}} + \mathbf{1}_{(t)}\dot{\mathbf{e}}^T)$. Here $\mathbf{1}_{(t)}$ is the p−dimensional column vector with its the t element equal to 1 and the others being zero and $\dot{\mathbf{e}}^T = (\dot{e}_1, \ldots, \dot{e}_n)$. Then the local influence measures can be obtained from the eigenvectors of $\mathbf{A}_t(\mathbf{X}^T\ddot{\mathbf{E}}\mathbf{X})^{-1}\mathbf{A}_t^T$.

4.3. Perturbation of Response

We consider the perturbation which altering the vector of response $\mathbf{y}$ by adding a vector $\mathbf{w}$. Note that such a perturbation may be meaningless if the response is discrete (Emerson *et al.*, 1984). Since y_i have a different variance, it needs a scaling of the perturbation vector $\mathbf{w}_i$ by an estimate of the standard deviation of $\mathbf{y}_i$. Thus we obtain the i^{th} perturbed response vector

$$y_{i,w} = y_i + \sqrt{V(\hat{\mu}_i)\hat{\phi}}w_i \quad \text{for } i = 1, \ldots, n.$$

Simple calculation yields

$$\boldsymbol{\Delta}_y = \hat{\phi}^{-1/2}\mathbf{X}^T\dot{\mathbf{K}}\mathbf{V}^{-1/2}.$$

The eigenvector of $\mathbf{V}^{-1/2}\dot{\mathbf{K}}\mathbf{X}(\mathbf{X}^T\ddot{\mathbf{E}}\mathbf{X})^{-1}\mathbf{X}^T\dot{\mathbf{K}}\mathbf{V}^{-1/2}$ associated with the largest eigenvalue gives the information about the influence diagnostics for $\hat{\boldsymbol{\beta}}$.

5. Examples

In this section we illustrate how the local influence measures given in Sections 3 and 4 provide influence information about the quasi-likelihood estimates of the regression coefficients in GLMs. We investigate the influence of observations on the regression coefficients using the direction vectors l_{max} and d_i in (3.2) and (3.3) under some perturbations.

Allison and Cicchetti (1976) presented the data on the sleep behavior of 45 mammals after excluding non-available observations. The response variable describes the proportion of sleep spent dreaming and the predictors are the weight of the body and the brain, the lifespan, the gestation period and the three constructed indices measuring vulnerability to predation, exposure while sleeping and overall danger. Considering the response that is not binomial suggests the binomial GLM with the canonical logit link (Faraway, 2006, pp. 149–150). After backward elimination we determined the quasi-binomial model on the data

$$\hat{y} = -0.493 + 0.146 \log(body) - 0.287 \log(lifespan) - 0.173\ danger.$$

Case deletion gives that observations 2 and 11 are influential, especially ob-

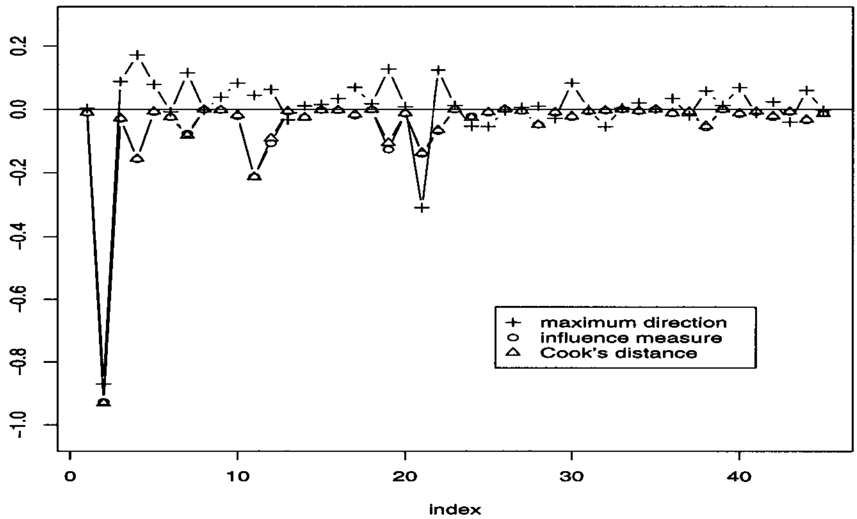

Figure 5.1: The local and global influence measures of case weights for the mammals sleep data.

servation 2 is most influential. The Cook's distance C_i for each observation is depicted in Figure 5.1. We computed the local influence matrices $\ddot{\mathbf{F}}$ under the case weight perturbation (4.1). The index plots of $\boldsymbol{l}_{max}$ and $\ddot{F}_{d_i}$ are given in Figure 5.1. The largest eigenvalue of $\ddot{\mathbf{F}}$ is 20.5. For comparison we used the standardized values for three influence measures. Both C_i and $\ddot{F}_{d_i}$ have similar results which is described in Section 4.1. Figure 5.1 shows that observations 2 and 21 have a large local impact on the regression coefficients under the case weight perturbation (4.1) and observations 2 and 11 have a large global impact from deletion approach. Hence observation 2 has largest the influence from the local and global point of view.

Next we consider the perturbation to the individual covariates except the variable *danger*, because a categorical data is meaningless. From the figures of the plots $\boldsymbol{l}_{max}$ and $\ddot{F}_{d_i}$ for most of covariates, we found that observation 2 is most influential and observations 12 and 38 are minor for all covariates. In this article we present the result of local influence measures for covariate *log(lifespan)*. Figure 5.2 depicts the direction vector $\boldsymbol{l}_{max}$ to the maximum curvature and the diagonal elements $\ddot{F}_{d_i}$ of $\ddot{\mathbf{F}}$ under the small perturbation (4.3) for covariate *log (lifespan)*.

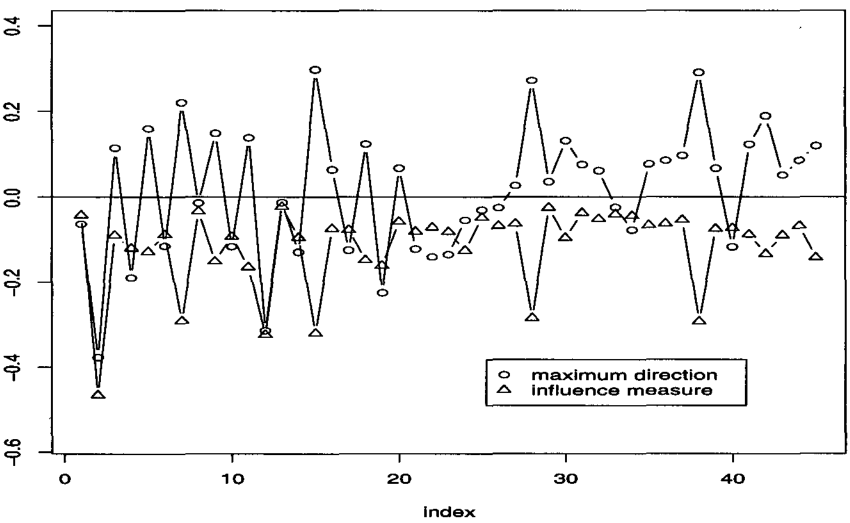

Figure 5.2: The index plots of the maximum direction vectors and diagonal elements of $\ddot{\mathbf{F}}$ under the perturbation of covariate *log(lifespan)* for the mammals sleep data.

It seems that the observations which have the same sign in the direction vector corresponding to the maximum curvature have joint reinforcing influence. Observations 2 and 12 have joint reinforcing influence, while observations 2 and 38 have joint canceling influence. The sign of the maximum direction vector means the direction of perturbation corresponding to a single explanatory variable. If we perturb the variable $lifespan$, observations 2 and 12 attract the regression coefficients in the same direction. However, observations 2 and 38 impact the model in the opposite direction. Thus the maximum direction vector gives joint influence information about multiple observations.

From the local influence measure for the mammal sleep data we may conclude that observation 2 has the largest impact on the estimation process for the quasi-binomial model.

6. Conclusions

We propose a local influence measure for the regression estimator of quasi-likelihood GLMs. We extend the work of Cook (1986) to the derivation of local influence measures under perturbation of case weight, covariate and response for the regression estimator in quasi-likelihood GLMs which is not based on likelihood functions but quasi-likelihood functions. We consider the local influence measures under several perturbation schemes. The study of a real data set shows the effectiveness of a new approach by comparison with deletion approaches, because these need a large amount of computing time and do not provide the influence information on the explanatory variables.

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