

## Fluxon resonance steps in the $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_{8+x}$ single crystals

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We observed discrete fluxon-flow resonance steps in high magnetic fields in a stack of Josephson junctions with lateral size of  $1.5 \times 17 \mu\text{m}^2$ . The measurement sample was prepared by sandwiching a stack of  $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_{8+x}$  intrinsic Josephson junctions between two Au electrodes by using the double-side-cleaving technique. This technique allowed us to isolate the intrinsic Josephson junction structures from the inductive interference of the basal stack. The resonance steps observed are in good agreement with the collective Josephson fluxon dynamics that are in resonance with the plasma oscillation modes inside the stacked Josephson junctions.

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### INTRODUCTION

The discovery of the intrinsic Josephson coupling in highly anisotropic  $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_{8+x}$  (Bi-2212) single crystals has stimulated intensive studies on the fluxon dynamics in relation with the inductive inter-junction coupling [1–3]. Since the thickness of  $\text{CuO}_2$  superconducting layers of Bi-2212 (0.3 nm) is much thinner [4] than the London penetration depth  $\lambda_L$  (a few  $\mu\text{m}$ ) a strong inductive inter-junction coupling is expected [5]. Recently, a quasi-one dimensional annular-type mesa structure with its width comparable to the Josephson penetration depth  $\lambda_J$  is employed for the investigation of the coherent fluxon motion in terms of the inductive coupling theory.

Most of fluxon dynamics studies to date, however, were made using mesa structures on the surface of single crystals containing intrinsic Josephson junctions. Due to strong inter-junction coupling of Josephson fluxons both in a mesa and the basal stack situated underneath the mesa, the presence of the basal stack easily distorts the fluxon-flow characteristics in the mesa itself [9]. In order to observe more ideal fluxon dynamics as predicted by the theories, one thus needs to remove the basal stack, while reducing its width down comparable to  $\lambda_J$  as well. In this study, we removed the basal stack using the double-side-cleaving technique [6] and examined resonance phenomena in Josephson fluxon dynamics of stacked intrinsic Josephson junctions in terms of the inductive coupling theory.

### THEORETICAL BACKGROUND

Resonance phenomena take place in a Josephson junction between oscillations of the gauge-invariant phase difference  $\gamma$  between the two superconducting layers (*i.e.*, the AC Josephson effect) and the electromagnetic field in the junction. For instance, the Shapiro steps arise from the resonance between the alternating current (AC) effect and the applied external microwaves [7]. The Fiske steps, on the other hand, are the resonance between the

AC effect and the cavity electric-wave resonant modes. Zero-field steps are the resonance between AC effect and the oscillatory motion of pinned Josephson vortices without an external field. The fluxon-flow resonance is due to the resonance between the Josephson vortices induced by an external field and the cavity electric wave resonant modes (*i.e.*, plasma oscillation modes) [8]. In this study, focus is placed on this fluxon-flow resonance steps.

The Swihart velocity, the phase velocity of small-amplitude electromagnetic oscillation modes in a single Josephson junction, is given by  $\bar{c} = c\sqrt{l/\epsilon_r d}$ . Here,  $c$  is the velocity of light in vacuum;  $l$ ,  $\epsilon_r$ , and  $d$  ( $\equiv l + 2\lambda$ ) are the thickness, the relative dielectric constant, and the effective thickness of the insulating barrier, respectively.  $\lambda$  is the London penetration depth of each superconducting layer. One combines the Josephson relation with the Maxwell equation in a Josephson junction to obtain the sine-Gordon equation. For a spatially uniform  $\gamma$  without the dissipative term in the sine-Gordon equation, one can define the Josephson plasma frequency as  $\omega_J = \bar{c}/\lambda_J = \sqrt{2eI_c/\hbar C}$ , where  $C$  is the total junction capacitance and  $\lambda_J = \sqrt{\hbar C^2/8\pi e d J_c}$  is the Josephson penetration depth. Thus, the Swihart velocity is determined by the Josephson plasma frequency.

We first consider the fluxon resonance states for a single Josephson junction. If a fluxon penetrates into the insulating barrier in a long single Josephson junction of length  $L$ , the phase difference  $\gamma$  changes with time with the frequency of  $u\bar{c}/L$ , where  $u$  is the velocity of the fluxon in units of  $\bar{c}$ . Fluxon velocity increases with increasing the bias currents. In the relativistic fluxon velocity limit ( $u \approx 1$ ), the DC voltage across a junction becomes  $V = \Phi_0 \bar{c}/L$ , where  $\Phi_0$  is the flux quantum. Since the fluxon cannot propagate faster than the Swihart velocity, the current-voltage characteristics (IVC) of a single long Josephson junction exhibits cut-off voltages. For multiple fluxons generated in a junction the number of fluxons per junction is given by  $HdL/\Phi_0$  ( $\equiv N_f$ ). Thus, the resonance voltage step occurs at  $V_n = n\Phi_0 \bar{c}/L = \bar{c}Hd$ , where  $H$  and  $n$  are the external field and the number of fluxons, respectively. The relation can be alternatively expressed as  $V_n = m\Phi_0 \bar{c}/2L$ , where  $m = 2n$ .

Because of the reflection of fluxons at the edges of a Josephson junction of length  $L$ , the junction behaves like a string and supports standing-wave modes of the electromagnetic field inside the junction. For an open-ended transmission line the standing wave for the  $n$ th mode is  $e_n(x, t) = e_n e^{i\omega_n t} \cos(n\pi x/L)$ , where  $\omega_n = n\pi\bar{c}/L$ . This leads to infinite number of self-resonant modes. Locking of fluxon motion takes place as a self-resonance mode frequency coincides with a Josephson current frequency due to the fluxon motion. Since the velocity of a single fluxon corresponding to the lowest resonant Josephson mode frequency of  $\bar{c}/2L$  is  $\bar{c}/2$  the voltage drop due to locking of the fluxon motion occurs at  $\Phi_0\bar{c}/2L$ . The next higher resonant voltage is  $\Phi_0\bar{c}/L$ , which corresponds to the single-fluxon velocity of  $\bar{c}$ . Thus, two resonating voltages exist for a single fluxon in a junction, which is similar to the Fiske steps in the short-junction limit. When the Josephson frequency matches with the frequency of one of the junction modes zero frequency currents appears, which is the Fiske steps. If an external magnetic field is applied in parallel with the junction line the Josephson current must be modulated in the junction line direction, even in the absence of fluxons, with the resonant voltages of  $V_n = n\Phi_0\bar{c}/2L$ . This is similar to the fluxon case but the number of resonating values is infinity. In the case of fluxon motion, however, fluxons resonate with the junction modes and the number of resonating voltages is limited by the number of fluxons as discussed above. Thus, the resonating frequency values are given by

$$f_m = m\bar{c}/2L, m = 1, 2, \dots, 2N_f \quad (1)$$

where  $N_f$  is the number of fluxons in a junction. In other words, fluxons resonate with the Josephson plasma oscillation modes, because  $\bar{c}$  is determined by the Josephson plasma frequency  $\omega_J$ . If the resonating states appear over the whole multi-layer Josephson junctions at the same time, the resonating frequency occurs for

$$f_{nm} = mc_n/2L, m = 1, 2, \dots, 2N_f, n = 1, 2, \dots, N \quad (2)$$

where  $N$  is the number of junctions and  $c_n$  is the  $n$ th-mode Swihart velocity. If the thickness of superconducting layers is much thinner than the London penetration depth, Josephson current of each junction can be coupled with each other by the inductive coupling. This situation is similar to the coupled harmonic oscillators, where the characteristic frequency (eigenvalues) can be obtained by the tri-diagonal matrix with the general solution of positions expressed as the linear combination of the eigenvectors. In the system of multiple Josephson junctions, we can also set up a tri-diagonal matrix through the coupled sine-Gordon equation and get the eigenvalue equation composed of a tri-diagonal matrix. The induced resonance frequency is given by

$$\omega = \omega_J \frac{\pi m}{L} \frac{1}{\sqrt{\frac{1}{\lambda_l^2} + \frac{2}{\lambda_k^2} [1 - \cos(\pi n/(N+1))]} \quad (3)$$

where  $m=1, 2, \dots, 2N_f$ ,  $n=1, 2, \dots, N$ ,  $\lambda_k = [\Phi_0 d_{eff} / (2\pi\mu_0 \lambda_L^2 j_c)]^{1/2}$ ,  $\lambda_J = \sqrt{\Phi_0 / 2\pi\mu_0 j_c (t_{eff} + 2\lambda_L^2 / d_{eff})}$ ,  $1/\lambda_l^2 = 1/\lambda_J^2 - 2/\lambda_k^2$ ,  $t_{eff} = t + 2\lambda_L \tanh[d/(2\lambda_L)]$ , and  $d_{eff} = \lambda_L \sinh(d/\lambda_L)$ . Equating Eqs. (2) and (3), we can get the velocity corresponding to each plasma mode, when  $k_x$  is equal to  $2N_f$  as in the single-fluxon case as

$$c_n = \frac{\omega_J \lambda_J}{\sqrt{1 - 2s \cos[\pi n/(N+1)]}} \quad (4)$$

The dimensionless coupling parameter  $s$  stands for the inter-layer coupling strength, which is 0.5 for Bi-2212.

## EXPERIMENTS AND DISCUSSION

Slightly overdoped Bi-2212 single crystals were grown by the solid-state-reaction method [10]. For fabrication of the sample a single crystal was glued on a sapphire substrate using polyimide and were cleaved until optically clean surface was obtained. Then a 100-nm-thick Au film was thermally deposited on top of the crystal to protect the surface from oxidation and contamination during the further processes. A mesa with a Au layer of size  $1.5 \times 17 \mu\text{m}^2$  was patterned using positive photoresist (Shipley-1805) and Ar-ion-etching. The surface of patterned mesa was fixed to another sapphire substrate using polyimide and the basal part of the junctions was cleaved away to prepare a fresh surface, on which 100-nm-thick Au film was deposited immediately. This double-side-cleaving (DSC) technique allows one to prepare a stack of IJJs sandwiched between normal-metallic electrodes without the basal stack. A few- $\mu\text{m}$ -long portion on both ends of stack was etched away to get the bottom electrodes exposed for  $c$ -axis transport measurement. Finally, the Au-extension pads are attached by photolithography and Ar-ion etching. Using these processes a stack of size  $1.5 \times 17 \mu\text{m}^2$  was prepared. At the same time we fabricated the mesa structure on the base single crystal for the reference. The measurements were made in a two or four-terminal configuration using low pass filters connected to measurement electrodes. For the data of two terminal configuration, the contact resistance was subtracted numerically. The magnetic-field alignment to the plane of junctions is important because any field misalignment produces pancake vortices on the junction planes, which in turn act as pinning centers of Josephson vortices. The magnetic field alignment to the junction was done in a field of 2 tesla at 60 K with the alignment resolution of 0.01 degree. The details are described elsewhere.

In general, when a DC external magnetic field is applied in parallel with the planes of long Josephson junctions, the magnetic field penetrates into insulating layer in forms of fluxons, Josephson vortices. At the same time,

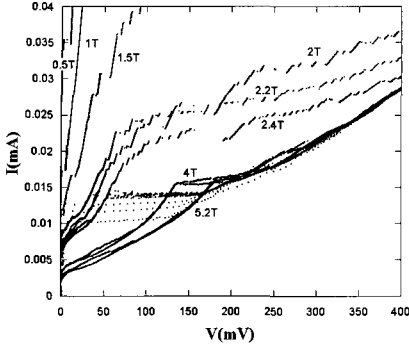


FIG. 1: Current-voltage characteristics of a stack of 55 multiple Josephson junctions with varying magnetic fields, showing fluxon-flow characteristics for high magnetic fields below the shoulder voltages as well as the quasiparticle tunneling branches for low magnetic fields.

in a  $c$ -axis tunneling bias current, the vortices move along the insulating layer by the Lorentz force. Figure 1 shows the typical IVC, in different magnetic fields up to 5.2 T, of a stack of intrinsic Josephson junctions ( $1.5 \times 17 \mu\text{m}^2$ ) made by the double-side-cleaving technique. The number of the junctions in the stack was 55 as obtained by dividing the sum gap by the average gap voltage per junction. The finite resistance is the fluxon-flow resistance of the field-induced fluxons. As in the figure, in a tesla-range high magnetic field, all hysteretic quasiparticle branches merge into a single curve gradually. The shoulder shown in each curve may correspond to the maximum velocity of the fluxon-flow motion. It is known that the maximum fluxon-flow velocity is limited by the propagation velocity of a corresponding Josephson plasma mode.

One can observe strange steps below the each shoulder voltage. These steps become more apparent for a higher field and is seen at the bias current below the return current  $I_r$ . Since we applied DC magnetic fields only in parallel with the junction planes either Shapiro steps nor zero field steps were observed. In the high field range as in this study, one cannot observe the pure Fiske steps, either. Thus, we analyze the data in terms of the fluxon-flow resonance model.

The system is supposed to have the plasma modes corresponding to the number of junctions of 55. Fluxons generated in a constant dc magnetic field move along the insulating layer with gradually increasing velocity with increasing the bias current. If the fluxon-flow velocity matches with any of the propagation velocity of electromagnetic waves, resonating coupling takes place between the fluxon-flow and the plasma oscillation modes. If the fluxon-flow velocity exceeds a resonating plasma propagation mode the Cherenkov radiation occurs and the fluxon mode switches to the next resonating plasma propagation

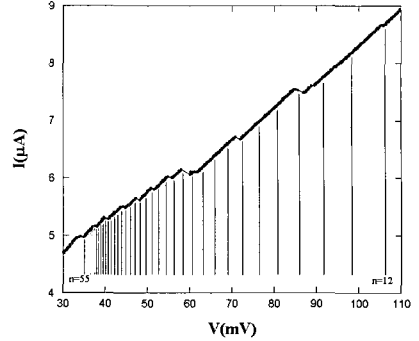


FIG. 2: Fluxon-flow current-voltage characteristics for  $H=5.2$  T, showing the steps at the mode-switching bias voltages. Vertical lines indicate the voltage positions where the mode switching is expected.

mode. This successive resonance switching takes place with the successive increase of the fluxon-flow velocity with increasing the bias current (refer to Fig. 2).

For the comparison of the experimental data to the analytic expectation, we obtain the Swihart velocity  $\bar{c}$  and the magnetic field corresponding the flux quantum. From the critical current density of  $1 \times 10^3 \text{ Acm}^{-2}$ , the Josephson penetration depth is estimated to be  $0.31 \mu\text{m}$ . The critical current of  $0.28 \text{ mA}$ , the return current  $8 \mu\text{A}$ , and the normal-state resistance of  $R_n=20 \Omega$ , lead to the plasma frequency of  $0.06 \text{ THz}$ . Using  $c_0=2\pi f\lambda_J$ , Swihart velocity is  $1.16 \times 10^5 \text{ m/s}$ . And the magnetic field corresponding to one flux quantum is  $810 \text{ G}$  for the junction length of  $17 \mu\text{m}$ .

The dispersion relation with  $c_0$  and number of junc-

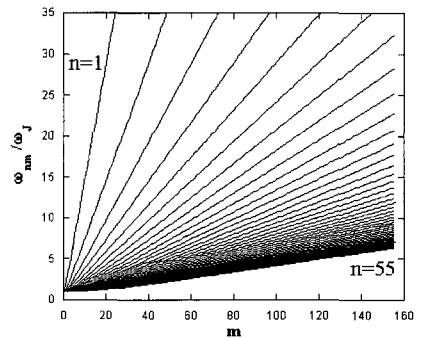


FIG. 3: Calculated dispersion relation, for modes with indices ranging from 1 to 55, of the collective plasma oscillation in a stack containing 55 intrinsic Josephson junctions.

tions are shown in the Figure 3. With this relations and Eq. (4), we compare our 5.2 T data ( $N_f=64$ ) in Figure 2. The theoretical resonance voltages and observed voltage-jump values are in qualitative agreement. A larger voltage jump is seen for a mode switching in the range of lower mode indices.

Figure 4 shows the overview of the normalized fluxon-flow voltages with varying the number of fluxons. The maximum-voltage shoulder position in the figure is the 7th plasma mode corresponding the velocity of  $4.2 \times 10^5$  m/s. In Fig. 4 the discontinuity starts appearing in the fluxon-flow line for the field higher than 2 T in the current regime smaller than  $I_r$ .

For a magnetic field of 2.2 T ( $N_f=27$ ) the spacing between two neighboring fluxons becomes the same as the diameter [11] of a fluxon itself  $2\lambda_J$ . This is close to the value of 2 T, where the discontinuity starts taking place in the fluxon-flow line. This indicates that the dense fluxon lattice is important in observing the resonance fluxon voltage. In the following the variation in the fluxon lattice over the stack of intrinsic Josephson junctions will be examined with increasing the fluxon velocity. One can find the solution of coherent transverse wave modes over a stack using the coupled sine-Gordon equation. If we set the gauge-invariant phase difference  $\gamma_{l+1,l}(x,t)=\gamma(x,t)$  and linearize the sin term, we get the transverse dispersion relation as  $\omega^2=\omega_J^2[1+(m^2\pi^2\lambda_m^2)/L^2]$ . This oscillation generates a collective transverse electromagnetic fields. This relation is obtained in the limit of vanishing value of  $\lambda_k$  in Eq. (3), corresponding to a weak coupling limit. On the other

hand, Bi-2212 stacked intrinsic Josephson junctions correspond to the strong coupling limit with the condition of  $N \gg 1$  and  $n=1$  in Equation (3). Thus in a stack consisting of  $N=55$ , the  $n=1$  vortex lattice mode is the rectangular lattice, which is resonant with the collective rectangular plasma mode. The  $n=55$  mode, on the other hand, is the triangle lattice mode. Thus, the shape of the vortex lattice follows the distribution of the eigen solution of the linearized coupled sine-Gordon equation. Josephson current across the  $m$ th junction is given by

$$j_m/j_c = \sin(2\pi N_f \Phi_0 x + \frac{\pi}{2} \text{sign}\{\sin[m\pi n/(N+1)]\}), \quad (5)$$

where the sign term is the eigenfunction. The mode number  $n$  runs from 1 to 55 in this sample stack containing 55 junctions. Different resonant modes can be realized in the multi-layered Josephson junction when the current flow along the  $c$  axis. Increasing the bias current, the fluxon energy along with the fluxon-flow velocity increases. The fluxons attract each other and form a rectangular lattice to reduce the fluxon screening current energy by cancelling the screening current in neighboring electrodes. The rectangular lattice corresponds to  $c_n$  resonant mode, which is stable for velocities  $c_n < v < c_{n-1}$ . With gradually increasing the bias current, the rectangular lattice becomes larger to form a collective rectangular lattice, which correspond to the  $c_1$  resonant mode. Whenever a rectangular lattice is formed by increasing the bias current, the corresponding resonant mode is excited and displayed in the IVC as kinks, steps, or voltage jumps.

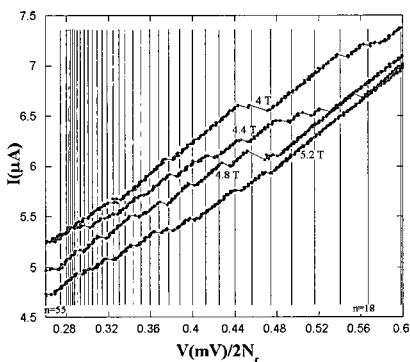


FIG. 4: Fluxon-flow current-voltage characteristics for a different values of applied magnetic fields as a function of bias voltage normalized by the number of Josephson vortices per junction, showing the steps at the mode-switching bias voltages. Vertical lines indicate the voltage positions where the mode switching is expected.

## CONCLUSION

We could observe the fluxon-flow resonance steps in the isolated stacks from basal Bi-2212 single crystal in high magnetic fields by measurements of all stacks without the basal part of the stack. A dense fluxon state with the inter-fluxon spacing smaller than  $\lambda_J$  is needed for the observation of distinct resonance steps. The width of stacks, however, is still 5 times larger than  $\lambda_J$ . More reduction of the width is required for the reduction of dimensionality close to one-dimensional limit.

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