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CONGRUENCES ON TERNARY SEMIGROUPS

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Abstract. In this paper we introduce the notion of congruence on a ternary semigroup and study some interesting properties. We also introduce the notions of cancellative congruence, group congruence and Rees congruence and characterize these congruences in ternary semigroups.

1. Introduction

The notion of congruence was first introduced by Karl Fredrich Gauss in the beginning of the nineteenth century. Congruence is a special type of equivalence relation which plays a vital role in the study of quotient structures of different algebraic structures. We study the quotient structure of ternary semigroup by using the notion of congruence in ternary semigroup. The notion of ternary semigroup was known to S. Banach. He showed, by an example, that a ternary semigroup does not necessarily reduce to an ordinary semigroup. In [4], J. Los studied some properties of ternary semigroup and proved that every ternary semigroup can be embedded in a semigroup. In [6], F. M. Sioson studied ideal theory in ternary semigroups. He also introduced the notion of regular ternary semigroups and characterized them by using the notion of quasi-ideals. In [5], M. L. Santiago developed the theory of ternary semigroups and semiheaps.

The main purpose of this paper is to classify congruences on ternary semigroups and study some basic properties of congruences on ternary semigroups. We also introduce the notions of cancellative congruence, group congruene and Rees congruence and characterize these congruences in ternary semigroups.

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2. Preliminaries

Definition 2.1. *A non-empty set S together with a ternary operation, called ternary multiplication, denoted by juxtaposition, is said to be a ternary semigroup if* $(abc)de = a(bcd)e = ab(cde)$ *for all* $a, b, c, d, e \in S$.

Definition 2.2. *An element e of a ternary semigroup S is called*

(i) *a left identity (or left unital element) if* $eex = x$ *for all* $x \in S$.

(ii) *a right identity (or right unital element) if* $xee = x$ *for all* $x \in S$.

(iii) *a lateral identity (or lateral unital element) if* $exe = x$ *for all* $x \in S$.

(iv) *a two-sided identity (or bi-unital element) if* $eex = xee = x$ *for all* $x \in S$.

(v) *an identity (or unital element) if* $eex = exe = xee = x$ *for all* $x \in S$.

Example 2.3. *Let* $\mathbf{Z}_{\mathbf{0}}^{-}$ *be the set of all non-positive integers. Then with the usual ternary multiplication,* $\mathbf{Z}_{\mathbf{0}}^{-}$ *forms a ternary semigroup with zero element '0' and identity element '−1'.*

Definition 2.4. *A ternary semigroup S is said to be commutative if* $x_1x_2x_3 = x_{\sigma(1)}x_{\sigma(2)}x_{\sigma(3)}$ *for every permutation* σ *of* $\{1, 2, 3\}$ *and* $x_1, x_2, x_3 \in S$.

Definition 2.5. *A non-empty subset T of a ternary semigroup S is called a ternary subsemigroup if* $t_1t_2t_3 \in T$ *for all* $t_1, t_2, t_3 \in T$.

Definition 2.6. *A ternary subsemigroup I of a ternary semigroup S is called*

(i) *a left ideal of S if* $SSI \subseteq I$.

(ii) *a lateral ideal of S if* $SIS \subseteq I$.

(iii) *a right ideal of S if* $ISS \subseteq I$.

(iv) *a two-sided ideal of S if I is both left and right ideal of S.*

(v) *an ideal of S if I is a left, a right, a lateral ideal of S.*

An ideal I of a ternary semigroup S is called a proper ideal if $I \neq S$.

Definition 2.7. *A ternary semigroup S is said to be*

(i) *left cancellative (LC) if* $abx = aby \Longrightarrow x = y$ *for all* $a, b, x, y \in S$.

(ii) *right cancellative (RC) if* $xab = yab \Longrightarrow x = y$ *for all* $a, b, x, y \in S$.

(iii) *laterally cancellative (LLC) if* $axb = ayb \Longrightarrow x = y$ *for all* $a, b, x, y \in S$.

(iv) *cancellative if S is left, right and laterally cancellative.*

Definition 2.8. *An element a of a ternary semigroup S is said to be invertible in S if there exists an element b in S such that $abx = bax = xab = xba = x$ for all $x \in S$.*

Definition 2.9. *A ternary semigroup S is called a ternary group if for $a, b, c \in S$, the equations $abx = c$, $axb = c$ and $xab = c$ have solutions in S.*

Remark 2.10. In a ternary group S, for $a, b, c \in S$, the equations $abx = c$, $axb = c$ and $xab = c$ have unique solutions in S.

Definition 2.11. *An element a in a ternary semigroup S is called regular if there exists an element x in S such that $axa = a$.*

A ternary semigroup S is called regular if all of its elements are regular.

Proposition 2.12. *Every lateral ideal of a regular ternary semigroup S is a regular ternary semigroup.*

Proof. Let L be a lateral ideal of a regular ternary semigroup S. Then for each $a \in L$, there exists $x \in S$ such that $a = axa$. Now $a = axa = axaxa = a(xax)a = aba$, where $b = xax \in L$. This implies that L is a regular ternary semigroup. □

Note 2.13. Every ideal of a regular ternary semigroup S is a regular ternary semigroup.

Definition 2.14. [5] *A pair (a, b) of elements in a ternary semigroup S is said to be an idempotent pair if $ab(abx) = abx$ and $(xab)ab = xab$ for all $x \in S$.*

Definition 2.15. [5] *Two idempotent pairs (a, b) and (c, d) of a ternary semigroup S are said to be equivalent, in notation we write $(a, b) \sim (c, d)$, if $abx = cdx$ and $xab = xcd$ for all $x \in S$.*

Theorem 2.16. [5] *The following conditions in a ternary semigroup S are equivalent :*

(i) *S is regular and cancellative;*

(ii) *S is regular and the idempotent pairs in S are all equivalent;*

(iii) *Every element of S is invertible in S;*

(iv) *S is a ternary group;*

(v) *S contains no proper one-sided ideals.*

Definition 2.17. *Let S and T be two ternary semigroups and $f : S \longrightarrow T$ be a mapping. Then the mapping $f : S \longrightarrow T$ is called a homomorphism of S into T if $f(abc) = f(a)f(b)f(c)$ for all $a, b, c \in S$.*

Moreover, if both the ternary semigroups S and T have identity elements e_S and e_T respectively, then $f(e_S) = e_T$.

A homomorphism $f : S \longrightarrow T$ is called a monomorphism if it is one-one.

A homomorphism $f : S \longrightarrow T$ is called an epimorphism if it is onto.

A homomorphism $f : S \longrightarrow T$ is called an isomorphism if it is both one-one and onto and in this case we say that the ternary semigroups S and T are isomorphic and write $S \simeq T$.

THEOREM 2.18. *Let f be a homomorphism from a regular ternary semigroup S onto a ternary semigroup T. Then T is a regular ternary semigroup i.e. the homomorphic image of a regular ternary semigroup is still a regular ternary semigroup.*

Proof. Suppose $b \in T$. Since f is onto, there exists $a \in S$ such that $f(a) = b$. Again, since S is regular, there exists $x \in S$ such that $a = axa$. Therefore, $b = f(a) = f(axa) = f(a)f(x)f(a) = byb$, where $y = f(x) \in T$. This shows that T is a regular ternary semigroup. □

3. Congruences on ternary semigroups

DEFINITION 3.1. *An equivalence relation ρ on a ternary semigroup S is said to be a*

(i) left congruence if $a\rho b \Longrightarrow (sta)\rho(stb)$ for all $a, b, s, t \in S$.

(ii) right congruence if $a\rho b \Longrightarrow (ast)\rho(bst)$ for all $a, b, s, t \in S$.

(iii) lateral congruence if $a\rho b \Longrightarrow (sat)\rho(sbt)$ for all $a, b, s, t \in S$.

(iv) congruence if $a\rho a'$, $b\rho b'$ and $c\rho c' \Longrightarrow (abc)\rho(a'b'c')$ for all a, a', b, b', $c, c' \in S$.

PROPOSITION 3.2. *An equivalence relation ρ on a ternary semigroup S is a congruence if and only if it is a left, a right and a lateral congruence on S.*

Proof. Let ρ be a congruence on S. If $a\rho b$ and $s, t \in S$, then $s\rho s$ and $t\rho t$ by reflexivity and hence $(sta)\rho(stb)$, $(ast)\rho(bst)$ and $(sat)\rho(sbt)$, since ρ is a congruence on S. Thus ρ is a left, a right and a lateral congruence on S.

Conversely, suppose that ρ is a left, a right and a lateral congruence on S. Let $a\rho a'$, $b\rho b'$ and $c\rho c'$ hold. Then $(abc)\rho(a'bc)$ (since ρ is a right congruence on S), $(a'bc)\rho(a'b'c)$ (since ρ is a lateral congruence on S) and $(a'b'c)\rho(a'b'c')$ (since ρ is a left congruence on S). This implies that $(abc)\rho(a'b'c')$, by transitivity. Thus ρ is a congruence on S. □

Definition 3.3. If ρ is a congruence on a ternary semigroup S, then we can define a ternary multiplication on the quotient set S/ρ by $(a/\rho)(b/\rho)(c/\rho) = (abc)/\rho$ for all $a, b, c \in S$.

Clearly, this operation is well-defined because ρ is congruence :

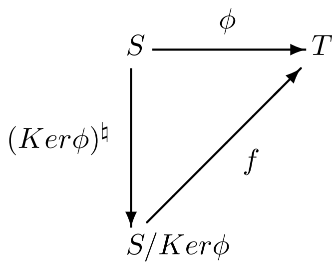
for all a, a', b, b', c, c' in S, $a\rho = a'\rho$, $b\rho = b'\rho$ and $c\rho = c'\rho \Longrightarrow a\rho a'$, $b\rho b'$ and $c\rho c' \Longrightarrow (abc)\rho(a'b'c') \Longrightarrow (abc)\rho = (a'b'c')\rho$

This ternary operation is also associative. Hence with this ternary operation, S/ρ forms a ternary semigroup.

Let ρ be a congruence on a ternary semigroup S. Then the mapping $f^{\natural}$ from S onto S/ρ given by $f^{\natural}(x) = x\rho$ is a homomorphism and we call it the natural homomorphism.

Remark 3.4. If S is a regular ternary semigroup and ρ is a congruence on S, then S/ρ is regular.

Theorem 3.5. *Let S and T be two ternary semigroups and $\phi : S \longrightarrow T$ be a homomorphism. Then $Ker\phi = \phi \circ \phi^{-1} = \{(a, b) \in S \times S : \phi(a) = \phi(b)\}$ is a congruence on S and there is a homomorphism $f : S/Ker\phi \longrightarrow T$ such that $Imf = Im\phi$ and the diagram*



is commutative.

Proof. Clearly, $Ker\phi$ is an equivalence relation.

To show that $Ker\phi$ is a congruence, suppose that $a(Ker\phi)a'$, $b(Ker\phi)b'$, $c(Ker\phi)c'$.

Then $\phi(a) = \phi(a')$, $\phi(b) = \phi(b')$ and $\phi(c) = \phi(c')$.

This implies that $\phi(abc) = \phi(a)\phi(b)\phi(c) = \phi(a')\phi(b')\phi(c') = \phi(a'b'c')$.

Consequently, $abc(Ker\phi)a'b'c'$ and hence $Ker\phi$ is a congruence on S.

Now we define $f : S/Ker\phi \longrightarrow T$ by $f(aKer\phi) = \phi(a)$ for all $a \in S$.

Then f is both well-defined and one-one, because

$aKer\phi = bKer\phi \Longleftrightarrow a(Ker\phi)b \Longleftrightarrow \phi(a) = \phi(b)$.

Also f is a homomorphism, since

$$\begin{aligned} f[(aKer\phi)(bKer\phi)(cKer\phi)] &= f[(abc)Ker\phi] \\ &= \phi(abc) \\ &= \phi(a)\phi(b)\phi(c) \\ &= f(aKer\phi)f(bKer\phi)f(cKer\phi) \end{aligned}$$

for all $a, b, c \in S$.

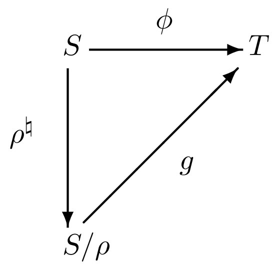
Clearly, $Imf = Im\phi$.

Again, from the definition of f, it is clear that

$[f \circ (Ker\phi)^{\natural}](a) = f[(Ker\phi)^{\natural}(a)] = f(aKer\phi) = \phi(a)$ for all $a \in S$.

Consequently, $f \circ (Ker\phi)^{\natural} = \phi$ and hence the above diagram is commutative. □

THEOREM 3.6. *Let S and T be two ternary semigroups. Let ρ be a congruence on S and let $\phi : S \longrightarrow T$ be a homomorphism such that $\rho \subseteq Ker\phi$. Then there is a unique homomorphism $g : S/\rho \longrightarrow T$ such that $Img = Im\phi$ and the following diagram*



is commutative.

Proof. We define $g : S/\rho \longrightarrow T$ by $g(a\rho) = \phi(a)$ for all $a \in S$.

Then g is well-defined, because

$a\rho = b\rho \Longrightarrow a\rho b \Longrightarrow a(Ker\phi)b$ (since $\rho \subseteq Ker\phi$) $\Longrightarrow \phi(a) = \phi(b)$ for all $a, b \in S$.

It is easy to show that g is a homomorphism such that $Img = Im\phi$.

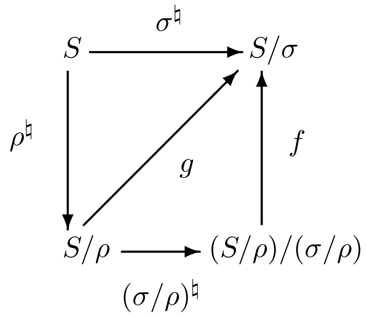
We can also easily show that $g \circ \rho^{\natural} = \phi$ and hence the above diagram is commutative.

Now we show that g is also unique. If possible, let $g' : S/\rho \longrightarrow T$ be a mapping defined by $g'(a\rho) = \phi(a)$ for all $a \in S$.

Now $g'(a\rho) = \phi(a) = [g \circ \rho^{\natural}](a) = g(\rho^{\natural}(a)) = g(a\rho)$ for all $a\rho \in S/\rho$ implies that $g' = g$. □

THEOREM 3.7. *Let ρ and σ be two congruences on a ternary semigroup S such that $\rho \subseteq \sigma$. Then $\sigma/\rho = \{(x\rho, y\rho) \in S/\rho \times S/\rho : (x, y) \in \sigma\}$ is a congruence on S/ρ and $(S/\rho)/(\sigma/\rho) \simeq S/\sigma$.*

Proof. We consider the following figure :



Clearly, σ/ρ is an equivalence relation on S/ρ. Let $(a_1\rho)(\sigma/\rho)(b_1\rho)$, $(a_2\rho)(\sigma/\rho)(b_2\rho)$ and $(a_3\rho)(\sigma/\rho)(b_3\rho)$. Then $a_1\sigma b_1$, $a_2\sigma b_2$ and $a_3\sigma b_3$. Since σ is a congruence on S, we have $(a_1a_2a_3)\sigma(b_1b_2b_3)$. This implies that $(a_1a_2a_3)\rho(\sigma/\rho)(b_1b_2b_3)\rho$ and hence σ/ρ is a congruence on S/ρ.

Note that σ/ρ is the kernel of g. From Theorem 3.5, it follows that there is an isomorphism $f : (S/\rho)/(\sigma/\rho) \longrightarrow S/\sigma$ defined by $f((a\rho)(\sigma/\rho)) = a\sigma$ for all $a \in S$ and the above diagram is commutative. □

PROPOSITION 3.8. *Let S be a ternary semigroup. If ρ_1 and ρ_2 are two left congruences (resp. right congruences, lateral congruences, congruences) of S, then $\rho_1 \circ \rho_2$ is a left congruence (resp. right congruence, lateral congruence, congruence) of S.*

Proof. Let ρ_1 and ρ_2 be two left congruences of S. Suppose $a(\rho_1 \circ \rho_2)b$ holds. Then there exists $c \in S$ such that $a\rho_1 c$ and $c\rho_2 b$ hold. Since ρ_1, ρ_2 are left congruences of S, it follows that $(sta)\rho_1(stc)$ and $(stc)\rho_2(stb)$ hold for all $s, t \in S$. This implies that $(sta)(\rho_1 \circ \rho_2)(stb)$ hold for all $s, t \in S$ and hence $\rho_1 \circ \rho_2$ is a left congruence of S.

Similarly, we can prove the remaining cases. □

From Proposition 3.8, it can be easily prove by induction the following result :

COROLLARY 3.9. *Let S be a ternary semigroup. If ρ_1, ρ_2, ..., ρ_n are left congruences (resp. right congruences, lateral congruences, congruences) of S, then $\rho_1 \circ \rho_2 \circ ... \circ \rho_n$ is a left congruence (resp. right congruence, lateral congruence, congruence) of S.*

PROPOSITION 3.10. *The union of a non-empty family of congruences on a ternary semigroup S is a congruence on S.*

Proposition 3.11. *The intersection of a non-empty family of congruences on a ternary semigroup S is a congruence on S.*

The set of congruences on a ternary semigroup S is denoted by $\mathcal{C}(\mathcal{S})$.

Theorem 3.12. *Let S be a ternary semigroup. Then $(\mathcal{C}(\mathcal{S}), \subseteq)$ is a complete lattice.*

Definition 3.13. *A congruence ρ on a ternary semigroup S is called a ternary group congruence on S if S/ρ is a ternary group.*

Remark 3.14. If a congruence ρ is contained in every ternary group congruence on S, then ρ is called the minimum ternary group congruence on S.

Definition 3.15. *A congruence ρ on a ternary semigroup S is called a cancellative congruence on S if S/ρ is a cancellative ternary semigroup.*

Lemma 3.16. *Let S be a ternary semigroup. Then a congruence ρ on S is a cancellative congruence if and only if $(cda)\rho(cdb) \Longrightarrow a\rho b$, $(cad)\rho(cbd) \Longrightarrow a\rho b$ and $(acd)\rho(bcd) \Longrightarrow a\rho b$ for all $a, b, c, d \in S$.*

Proof. Let ρ be a cancellative congruence on S. Then S/ρ is a cancellative ternary semigroup. Suppose that $(cda)\rho(cdb)$ holds. Then $(cda)\rho = (cdb)\rho \Longrightarrow (c\rho)(b\rho)(a\rho) = (c\rho)(d\rho)(b\rho) \Longrightarrow a\rho = b\rho$ (Since S/ρ is cancellative) $\Longrightarrow a\rho b$. Similarly, we can show that $(cad)\rho(cbd) \Longrightarrow a\rho b$ and $(acd)\rho(bcd) \Longrightarrow a\rho b$ for all $a, b, c, d \in S$.

Converse follows by reversing the above argument. □

Remark 3.17. The intersection of a family of cancellative congruences on a ternary semigroup S is cancellative.

Note that this is a minimum cancellative congruence on S.

Proposition 3.18. *Every cancellative congruence on a regular ternary semigroup is a ternary group congruence.*

Proof. Let S be a regular ternary semigroup and ρ be a cancellative congruence on S. Then S/ρ is a regular and cancellative ternary semigroup. Consequently, S/ρ is a ternary group, by Theorem 2.16. This implies that ρ is a ternary group congruence on S. □

From Remark 3.17 and Proposition 3.18, we have the following result:

Corollary 3.19. *Every regular ternary semigroup possesses a minimum ternary group congruence.*

Definition 3.20. *Let S be a ternary semigroup and I be any ideal of S.*

We define $\rho_I = (I \times I) \cup i_S$, where i_S is the identity relation of S.

Note that $x\rho_I y$ if and only if either $x = y$ or both x and y belong to I.

Clearly, ρ_I is a congruence on S and $S/\rho_I = \{I\} \cup \{\{x\} : x \in S \setminus I\}$.

We call ρ_I as Rees congruence on S and the quotient S/ρ_I as Rees factor ternary semigroup of S determined by I.

Definition 3.21. *Let S and T be two ternary semigroups. A homomorphism $\phi : S \longrightarrow T$ is called a Rees homomorphism if the kernel of ϕ is a Rees congruence on S.*

Theorem 3.22. *Let S be a ternary semigroup and I be an ideal of S. Then the Rees quotient S/ρ_I is a ternary semigroup and S/ρ_I is a homomorphic image of S.*

Proof. Let $a\rho_I, b\rho_I, c\rho_I \in S/\rho_I$. We define $(a\rho_I)(b\rho_I)(c\rho_I) = (abc)\rho_I$. Then it can be easily verified that S/ρ_I is a ternary semigroup. Let us now consider the mapping $f : S \longrightarrow S/\rho_I$ defined by $f(a) = a\rho_I$ for all $a \in S$. Then f is a homomorphism from S onto S/ρ_I. Hence S/ρ_I is a homomorphic image of S. □

Theorem 3.23. *Let S be a ternary semigroup and I be an ideal of S. Then S is a regular ternary semigroup if and only if I and the Rees factor ternary semigroup S/ρ_I are both regular ternary semigroup.*

Proof. Let S be a regular ternary semigroup and I be an ideal of S. Then I is regular, by Note 2.13. Again, from Theorem 3.22, it follows that S/ρ_I is a homomorphic image of S. Thus from Theorem 2.18, it follows that S/ρ_I is regular.

Conversely, let I be an ideal of the ternary semigroup S and let I and S/ρ_I are both regular ternary semigroups. Let $a \in S$. Then $a\rho_I \in S/\rho_I$. Since S/ρ_I is a regular ternary semigroup, there exists $x\rho_I \in S/\rho_I$ such that $(a\rho_I)(x\rho_I)(a\rho_I) = a\rho_I$. This implies that $(axa)\rho_I = a\rho_I$ and hence either $a = axa$ or $a, axa \in I$. If $a = axa$, then a is regular. Again, if $a \in I$, then also a is regular, since I is regular. Thus S is a regular ternary semigroup. □

Theorem 3.24. *Let I be an ideal of a ternary semigroup S. Let $\mathcal{I}(\mathcal{S})$ be the set of all ideals of S containing I and $\mathcal{I}(\mathcal{S}/\rho_{\mathcal{I}})$ be the set of all ideals of S/ρ_I. Then there exists an inclusion preserving bijection from $\mathcal{I}(\mathcal{S})$ onto $\mathcal{I}(\mathcal{S}/\rho_{\mathcal{I}})$.*

Proof. We define a mapping $\phi : \mathcal{I}(\mathcal{S}) \longrightarrow \mathcal{I}(\mathcal{S}/\rho_{\mathcal{I}})$ by $\phi(I_1) = I_1/\rho_I$ for all $I_1 \in \mathcal{I}(\mathcal{S})$.

The mapping ϕ is well-defined and one-one, because

$$\begin{array}{rrcl} & \phi(I_1) & = & \phi(I_2) \\ \Longleftrightarrow & I_1/\rho_I & = & I_2/\rho_I \\ \Longleftrightarrow & \{I\} \cup \{\{x\} : x \in I_1 \setminus I\} & = & \{I\} \cup \{\{x\} : x \in I_2 \setminus I\} \\ \Longleftrightarrow & \{x : x \in I_1 \setminus I\} & = & \{x : x \in I_2 \setminus I\} \\ \Longleftrightarrow & I \cup \{x : x \in I_1 \setminus I\} & = & I \cup \{x : x \in I_2 \setminus I\} \\ \Longleftrightarrow & I_1 & = & I_2. \end{array}$$

Let J be an ideal of S/ρ_I. Let $I' = I \cup \{x : x \notin I$ and $x\rho_I \in J\}$. We show that I' is an ideal of S. Let $a' \in I'$ and $s, t \in S$. If $a'st \in I$, then $a'st \in I'$. Suppose that $a'st \notin I$. Then $a' \notin I$. Now $(a'st)\rho_I = (a'\rho_I)(s\rho_I)(t\rho_I)$ implies that $a'st \in I'$. Similarly, we can show that $sa't \in I'$ and $a'st \in I'$. Thus I' is an ideal of S. We shall show that $\phi(I') = J$. By definition of ϕ, we have $\phi(I') = I'/\rho_I$. But $I'/\rho_I = \{I\} \cup \{\{x\} : x \in I' \setminus I\}$. Clearly, $\{I\} \in J$. Again, for $I' \setminus I$, $x\rho_I \in J$. Thus $I'/\rho_I \subseteq J$. Also, it can be easily shown that $J \subseteq I'/\rho_I$. Thus $I'/\rho_I = J$ i.e. $\phi(I') = J$ and hence ϕ is onto. Clearly, ϕ is inclusion preserving. Hence the theorem. □

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