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ON T-FUZZY $\mathcal{M}\Gamma$-SUBGROUPS OF $\mathcal{M}\Gamma$-GROUPS

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Abstract. In this paper, we introduce the notion of T-fuzzy $\mathcal{M}\Gamma$-subgroup of $\mathcal{M}\Gamma$-group, and give its characterization.

1. Introduction

The notion of fuzzy subsets was formulated by Zadeh [10], and since then fuzzy subsets have been applied to various branches of Mathematics and computer science. Using this concept, Chang [4] generalized some of the basic concepts of general topology, and Rosenfeld [7] applied it to the theory of groupoids and groups, and many researchers [4, 5] applied the concept of fuzzy sets to the elementary theory of Γ-rings. In [2], Booth introduced the concept of Γ-near-rings which is due to Satyanarayana [8]. Also Booth and Groenewald [3] studied radical theory of a Γ-near-ring, and introduced the notion of $\mathcal{M}\Gamma$-group. In this paper, we introduce the notion of T-fuzzy $\mathcal{M}\Gamma$-subgroup of $\mathcal{M}\Gamma$-group, and give its characterization.

2. Preliminaries

All near-rings considered in this paper will be right distributive. A Γ-*near-ring* is a triple $(M, +, \Gamma)$ where:

(i) $(M, +)$ is a (not necessarily abelian) group;
(ii) Γ is a nonempty set of binary operators on M such that, for each $\alpha \in \Gamma$, $(M, +, \alpha)$ is a near-ring;
(iii) $x\alpha(y\beta z) = (x\alpha y)\beta z$ for all $x, y, z \in M$, $\alpha, \beta \in \Gamma$. If, in addition, it holds that

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(iv) $x\alpha 0 = 0$ for all $x \in M$;
then the Γ-near-ring M is said to be *zero symmetric.*

DEFINITION 2.1. [3] Let G be an additive group. If, for all $a, b \in M$, $\alpha, \beta \in \Gamma$ and $x \in G$ it holds that

(i) $a\alpha x \in G$;
(ii) $a\alpha(b\beta x) = (a\alpha b)\beta x$;
(iii) $(a + b)\alpha x = a\alpha x + b\alpha x$

then G is called an $\mathcal{M}\Gamma$-*group.*

In what follows, let M denotes the Γ-near-ring, and G denotes the $\mathcal{M}\Gamma$-group unless otherwise specified.

DEFINITION 2.2. [3] A subgroup K of G for which $a\alpha k \in K$ for all $a \in M$, $\alpha \in \Gamma$, $k \in K$, is called an $\mathcal{M}\Gamma$-*subgroup* of G.

We now review some fuzzy logic concepts. A fuzzy set in a set G is a function $\mu : G \to [0, 1]$. We shall use the notation $U(\mu; t)$, called a *level subset* of μ, for $\{x \in G | \mu(x) \geq t\}$ where $t \in [0, 1]$.

A fuzzy set μ in G is called a *fuzzy* $\mathcal{M}\Gamma$-*subgroup* of G if

(i) $\mu(x - y) \geq \min\{\mu(x), \mu(y)\}$ for all $x, y \in G$.
(ii) $\mu(a\alpha y) \geq \mu(y)$ for all $a \in M, y \in G$ and $\alpha \in \Gamma$.

DEFINITION 2.3. [1] By a *t-norm* T, we mean a function $T : [0, 1] \times [0, 1] \to [0, 1]$ satisfying the following conditions:

(T1) $T(x, 1) = x$,
(T2) $T(x, y) \leq T(x, z)$ if $y \leq z$,
(T3) $T(x, y) = T(y, x)$,
(T4) $T(x, T(y, z)) = T(T(x, y), z)$,

for all $x, y, z \in [0, 1]$.

PROPOSITION 2.4. *Every t-norm T has a useful property:*

$$T(\alpha, \beta) \leq \min(\alpha, \beta)$$

for all $\alpha, \beta \in [0, 1]$.

For a t-norm T on $[0, 1]$, denote by Δ_T the set of element $\alpha \in [0, 1]$ such that $T(\alpha, \alpha) = \alpha$, i.e., $\Delta_T := \{\alpha \in [0, 1] \mid T(\alpha, \alpha) = \alpha\}$.

DEFINITION 2.5. Let T be a t-norm. A fuzzy set μ in G is said to satisfy *idempotent property with respect to* T if $\mathrm{Im}(\mu) \subseteq \Delta_T$.

3. Fuzzy $\mathcal{M}\Gamma$-subgroups of $\mathcal{M}\Gamma$-groups

DEFINITION 3.1. Let T be an t-norm. A fuzzy set μ in G is called an *T-fuzzy $\mathcal{M}\Gamma$-subgroup* of G if

(F1) $\mu(x-y) \geq T(\mu(x), \mu(y))$ for all $x, y \in G$.

(F2) $\mu(a\alpha y) \geq \mu(y)$ for all $a \in M, y \in G$ and $\alpha \in \Gamma$.

PROPOSITION 3.2. *Let T be a t-norm. If μ is an idempotent T-fuzzy $\mathcal{M}\Gamma$-subgroup of G, then we have $\mu(0) \geq \mu(x)$ for all $x \in G$.*

Proof. For every $x \in G$, we have

$$\mu(0) = \mu(x-x) \geq T(\mu(x), \mu(x)) = \mu(x).$$

This completes the proof. □

PROPOSITION 3.3. *If μ is an idempotent T-fuzzy $\mathcal{M}\Gamma$-subgroup of G, then the set*

$$G^{\omega} = \{x \in G \mid \mu(x) \geq \mu(\omega)\}$$

is an $\mathcal{M}\Gamma$-subgroup of G.

Proof. Let $x, y \in G^{\omega}$. Then $\mu(x) \geq \mu(\omega)$ and $\mu(y) \geq \mu(\omega)$. Since μ is an T-fuzzy $\mathcal{M}\Gamma$-subgroup of G, it follows that

$$\mu(x-y) \geq T(\mu(x), \mu(y)) \geq T(\mu(\omega), \mu(\omega)) = \mu(\omega).$$

Now let $a \in M, \alpha \in \Gamma$ and $k \in G^{\omega}$. Then $\mu(a\alpha k) \geq \mu(k) \geq \mu(\omega)$. Thus, we have $\mu(x-y) \geq \mu(\omega)$ and $\mu(a\alpha k) \geq \mu(\omega)$, that is., $x-y \in G^{\omega}$ and $a\alpha k \in G^{\omega}$. This completes the proof. □

COROLLARY 3.4. *Let T be a t-norm. If μ is an idempotent T-fuzzy $\mathcal{M}\Gamma$-subgroup of G, then the set*

$$\mu_G = \{x \in G \mid \mu(x) = \mu(0)\}$$

is an $\mathcal{M}\Gamma$-subgroup of G.

Proof. From the Proposition 3.2, $\mu_G = \{x \in G \mid \mu(x) = \mu(0)\} = \{x \in G \mid \mu(x) \geq \mu(0)\}$, hence μ_G is an $\mathcal{M}\Gamma$-subgroup of G from the Proposition 3.4. □

DEFINITION 3.5. Let G and G' be $\mathcal{M}\Gamma$-groups. A map $\theta : G \to G'$ is called a *$\mathcal{M}\Gamma$-group homomorphism* if $\theta(x+y) = \theta(x) + \theta(y)$ and $\theta(a\alpha x) = a\alpha\theta(x)$ for all $a \in M, \alpha \in \Gamma$ and $x \in G$.

DEFINITION 3.6. Let $\theta : G \to G'$ be an $\mathcal{M}\Gamma$-group homomorphism of $\mathcal{M}\Gamma$-groups. For any fuzzy set μ in G', we define a fuzzy set μ^{θ} in G by $\mu^{\theta}(x) := \mu(\theta(x))$ for all $x \in G$.

Proposition 3.7. *Let $\theta : G \to G'$ be an $\mathcal{M}\Gamma$-group homomorphism of $\mathcal{M}\Gamma$-groups. If μ is an T-fuzzy $\mathcal{M}\Gamma$-subgroup of G', then μ^θ is an T-fuzzy $\mathcal{M}\Gamma$-subgroup of G.*

Proof. For any $x, y \in G$, we have

$$\begin{aligned}\mu^\theta(x-y) &= \mu(\theta(x-y)) = \mu(\theta(x)-\theta(y)) \\ &\geq T(\mu(\theta(x)), \mu(\theta(y))) = T(\mu^\theta(x), \mu^\theta(y))\end{aligned}$$

Let $a \in M, y \in G$ and $\alpha \in \Gamma$. Then

$$\mu^\theta(x\alpha y) = \mu(\theta(x\alpha y)) = \mu(a\alpha\theta(y)) \geq \mu(\theta(y)) = \mu^\theta(y).$$

This completes the proof. □

Proposition 3.8. *Let I be an $\mathcal{M}\Gamma$-subgroup of G and let μ be a fuzzy set in G defined by*

$$\mu(x) := \begin{cases} \alpha & \text{if } x \in I, \\ \beta & \text{otherwise }, \end{cases}$$

for all $x \in G$, where $\alpha, \beta \in [0,1]$ with $\alpha > \beta$. Then μ is a T-fuzzy $\mathcal{M}\Gamma$-subgroup of G where $T(\alpha, \beta) = \max(\alpha + \beta - 1, 0)$ for all $\alpha, \beta \in [0,1]$.

Proof. Let $x, y \in G$. If $x, y \in I$, then

$$\begin{aligned}T(\mu(x), \mu(y)) &= T(\alpha, \alpha) = \max(2\alpha - 1, 0) \\ &= \begin{cases} 2\alpha & \text{if } \alpha \geq \frac{1}{2}, \\ \beta & \text{if } \alpha < \frac{1}{2}, \end{cases} \\ &\leq \alpha = \mu(x-y),\end{aligned}$$

and for all $a \in M$ and $\alpha \in \Gamma$, we have $\mu(a\alpha y) = \mu(y) = \alpha$. If $x \in I$ and $y \notin I$ (or, $x \notin I$ and $y \in I$), then

$$\begin{aligned}T(\mu(x), \mu(y)) &= T(\alpha, \beta) = \max(\alpha + \beta - 1, 0) \\ &= \begin{cases} \alpha + \beta - 1 & \text{if } \alpha + \beta \geq \frac{1}{2}, \\ \beta & \text{otherwise }, \end{cases} \\ &\leq \beta = \mu(x-y),\end{aligned}$$

and for all $a \in M$ and $\alpha \in \Gamma$, we have $\mu(a\alpha y) \geq \beta = \mu(y)$. If $x \notin I$ and $y \notin I$, then

$$\begin{aligned} T(\mu(x),\mu(y)) &= T(\beta,\beta) = \max(2\beta-1,0) \\ &= \begin{cases} 2\beta-1 & \text{if } \beta \geq \frac{1}{2}, \\ 0 & \text{otherwise }, \end{cases} \\ &\leq \beta = \mu(x-y), \end{aligned}$$

and for all $a \in M$ and $\alpha \in \Gamma$, we have $\mu(a\alpha y) \geq \beta = \mu(y)$. Hence μ is an T-fuzzy $\mathcal{M}\Gamma$-subgroup of G. □

For any subset I of $\mathcal{M}\Gamma$-group G, $\mathcal{X}_I$ denote the characteristic function of I.

Corollary 3.9. *Let $I \subseteq G$. Then I is an $\mathcal{M}\Gamma$-subgroup of G if and only if $\mathcal{X}_I$ is an T-fuzzy $\mathcal{M}\Gamma$-subgroup of G.*

Proof. Let I be an $\mathcal{M}\Gamma$-subgroup of G. Then it is easy to show that $\mathcal{X}_I$ is an T-fuzzy $\mathcal{M}\Gamma$-subgroup of G. In fact, let $x, y \in I$, and so $x-y \in I$. Hence we have $\mathcal{X}_I(x-y) = 1 = T(\mathcal{X}_I(x), \mathcal{X}_I(y)) = T(1,1)$. Assume that $x \in I$ and $y \notin I$ (or $x \notin I$ and $y \in I$). Then $\mathcal{X}_I(x) = 1 > 0 = \mathcal{X}_I(y)$ (or $\mathcal{X}_I(x) = 0 < 1 = \mathcal{X}_I(y)$). It follows that $\mathcal{X}_I(x-y) \geq 0 = T(\mathcal{X}_I(x), \mathcal{X}_I(y)) = T(1,0) = 0$. Now let $a \in M, \alpha \in \Gamma$. If $y \in I$, then we have $a\alpha y \in I$. Hence $\mathcal{X}_I(a\alpha y) = 1 = \mathcal{X}_I(y)$. If $y \notin I$, then $\mathcal{X}_I(a\alpha y) \geq \mathcal{X}_I(y) = 0$. Conversely, Let $\mathcal{X}_I$ be an T-fuzzy $\mathcal{M}\Gamma$-subgroup of G. Let $x, y \in I$. Then we have $\mathcal{X}_I(x-y) \geq T(\mathcal{X}_I(x), \mathcal{X}_I(y)) = T(1.,1) = 1$, and so $x-y \in I$. Now let $a \in M, \alpha \in \Gamma$ and $y \in I$. Hence $\mathcal{X}_I(a\alpha y) \geq \mathcal{X}_I(y) = 1$, and so $a\alpha y \in I$. □

Theorem 3.10. *Let T be a t-norm. Then every idempotent T-fuzzy $\mathcal{M}\Gamma$-subgroup of G is a fuzzy ideal of G.*

Proof. Let μ be an idempotent T-fuzzy $\mathcal{M}\Gamma$-subgroup of G. Then $\mu(x-y) \geq T(\mu(x),\mu(y))$ for all $x, y \in G$. Since μ satisfies the idempotent property, we have

$$\begin{aligned} \min(\mu(x),\mu(y)) &= T(\min(\mu(x),\mu(y)), \min(\mu(x),\mu(x))) \\ &\leq T(\mu(x),\mu(y)) \leq \min(\mu(x),\mu(y)). \end{aligned}$$

It follows that

$$\mu(x-y) \geq T(\mu(x),\mu(y)) = \min(\mu(x),\mu(y))$$

so that μ is a fuzzy ideal of G. ending the proof. □

Theorem 3.11. *Let μ be an T-fuzzy $\mathcal{M}\Gamma$-subgroup of G and let $\alpha \in \Gamma$ be such that that $T(\alpha,\alpha) = \alpha$. Then $U(\mu;\alpha)$ is either empty or an $\mathcal{M}\Gamma$-subgroup of G for all $x \in G$.*

Proof. Let $x, y \in U(\mu;\alpha)$. Then we have $\mu(x) \geq \alpha$ and $\mu(y) \geq \alpha$, and so

$$\mu(x-y) \geq T(\mu(x), \mu(y)) \geq T(\alpha, \alpha) = \alpha,$$

which implies that $x - y \in U(\mu;\alpha)$. Now let $a \in M, y \in U(\mu;\alpha)$ and $\gamma \in \Gamma$. Then we have $\mu(a\gamma y) \geq \mu(y) \geq \alpha$, so $a\gamma y \in U(\mu;\alpha)$. Hence $U(\mu;\alpha)$ is an $\mathcal{M}\Gamma$-subgroup of G. ending the proof. □

Since $T(1,1) = 1$, we have the following corollary.

COROLLARY 3.12. *If μ is an T-fuzzy $\mathcal{M}\Gamma$-subgroup of G, then $U(\mu;1)$ is either empty or an $\mathcal{M}\Gamma$-subgroup of G.*

For a family $\{\mu_\alpha \mid \alpha \in \Lambda\}$ fuzzy sets in G, define the join $\bigvee_{\alpha\in\Lambda} \mu_\alpha$ and the meet $\bigwedge_{\alpha\in\Lambda} \mu_\alpha$ as follows:

$(\bigvee_{\alpha\in\Lambda} \mu_\alpha)(x) = \sup\{\mu_\alpha(x) \mid \alpha \in \Lambda\}$, $\quad (\bigwedge_{\alpha\in\Lambda} \mu_\alpha)(x) = \inf\{\mu_\alpha(x) \mid \alpha \in \Lambda\}$,

for all $x \in G$, where Λ is any index set.

THEOREM 3.13. *The family of T-fuzzy $\mathcal{M}\Gamma$-subgroups of G is a completely distributive lattice with respect to meet "$\wedge$" and join "$\vee$".*

Proof. Since $[0,1]$ is a completely distributive lattice with respect to the usual ordering in $[0,1]$, it is sufficient to show that $\bigvee_{\alpha\in\Lambda} \mu_\alpha$ and $\bigwedge_{\alpha\in\Lambda} \mu_\alpha$ are T-fuzzy $\mathcal{M}\Gamma$-subgroups of G for a family of T-fuzzy $\mathcal{M}\Gamma$-subgroups $\{\mu_\alpha \mid \alpha \in \Lambda\}$. For any $x, y \in G$, we have

$$\begin{aligned}
(\bigvee_{\alpha\in\Lambda} \mu_\alpha)(x-y) &= \sup\{\mu_\alpha(x-y) \mid \alpha \in \Lambda\} \\
&\geq \sup\{T(\mu_\alpha(x), \mu_\alpha(y)) \mid \alpha \in \Lambda\} \\
&\geq T(\sup\{\mu_\alpha(x) \mid \alpha \in \Lambda\}, \sup\{\mu_\alpha(y) \mid \alpha \in \Lambda\}) \\
&= T((\bigvee_{\alpha\in\Lambda} \mu_\alpha)(x), (\bigvee_{\alpha\in\Lambda} \mu_\alpha)(y)),
\end{aligned}$$

$$\begin{aligned}
(\bigwedge_{\alpha\in\Lambda} \mu_\alpha)(x-y) &= \inf\{\mu_\alpha(x-y) \mid \alpha \in \Lambda\} \\
&\geq \inf\{T(\mu_\alpha(x), \mu_\alpha(y)) \mid \alpha \in \Lambda\} \\
&\geq T(\inf\{\mu_\alpha(x) \mid \alpha \in \Lambda\}, \inf\{\mu_\alpha(y) \mid \alpha \in \Lambda\}) \\
&= T((\bigwedge_{\alpha\in\Lambda} \mu_\alpha)(x), (\bigwedge_{\alpha\in\Lambda} \mu_\alpha)(y)).
\end{aligned}$$

Now let $a \in M, y \in G$ and $\alpha \in \Gamma$. Then

$$\begin{aligned}(\bigvee_{\alpha\in\Lambda} \mu_\alpha)(a\alpha y) &= \sup\{\mu_\alpha(a\alpha y) \mid \alpha \in \Lambda\} \\ &\geq \sup\{\mu_\alpha(y) \mid \alpha \in \Lambda\} \\ &= (\bigvee_{\alpha\in\Lambda} \mu_\alpha)(y), \\ (\bigwedge_{\alpha\in\Lambda} \mu_\alpha)(x\alpha y) &= \inf\{\mu_\alpha(x\alpha y) \mid \alpha \in \Lambda\} \\ &\geq \inf\{\mu_\alpha(y) \mid \alpha \in \Lambda\} \\ &= (\bigwedge_{\alpha\in\Lambda} \mu_\alpha)(y),\end{aligned}$$

Hence $\bigvee_{\alpha\in\Lambda} \mu_\alpha$ and $\bigwedge_{\alpha\in\Lambda} \mu_\alpha$ are T-fuzzy $\mathcal{M}\Gamma$-subgroups of G, completing the proof. □

THEOREM 3.14. *Let T be a t-norm and let μ be an idempotent fuzzy set in G. If each non-empty upper level set $U(\mu;\alpha)$ of μ is an $\mathcal{M}\Gamma$-subgroup of G, then μ is an idempotent T-fuzzy $\mathcal{M}\Gamma$-subgroup of G.*

Proof. Suppose each non-empty upper level set $U(\mu;\alpha)$ of μ is an $\mathcal{M}\Gamma$-subgroup of G. Then we first show that

$$\mu(x-y) \geq \min(\mu(x), \mu(y)) \text{ for all } x, y \in G.$$

In fact, if not then there exist $x_0, y_0 \in G$ such that $\mu(x_0 - y_0) < \min(\mu(x_0), \mu(y_0))$. Taking

$$\alpha_0 := \frac{1}{2}(\mu(x_0 - y_0) + \min(\mu(x_0), \mu(y_0))),$$

we get $\mu(x_0 - y_0) < \alpha_0 < \min(\mu(x_0), \mu(y_0))$ and thus $x_0, y_0 \in U(\mu;\alpha)$ and $x_0 - y_0 \notin U(\mu;\alpha)$. This is a contradiction. Hence

$$\mu(x-y) \geq \min(\mu(x), \mu(y)) \geq T(\mu(x), \mu(y))$$

for all $x, y \in G$. Now if (F2) is not true, then $\mu(a_0\gamma y_0) < \mu(y_0)$ for some $a_0 \in M, y_0 \in G$ and $\gamma \in \Gamma$. Taking $s_1 := \frac{1}{2}(\mu(a_0\gamma y_0) + \mu(y_0))$, then $0 \leq s_1 < \mu(y_0)$ and $\mu(a_0\gamma y_0) < s_1$. Hence $y_0 \in U(\mu; s_1)$ and $a_0\gamma y_0 \notin U(\mu; s_1)$, a contradiction. This completes the proof. □

PROPOSITION 3.15. *Let T be a t-norm and let μ be a fuzzy set in G with $Im(\mu) = \{\alpha_1, \alpha_2, \alpha_3, \cdots, \alpha_n\}$ where $\alpha_i < \alpha_j$ whenever $i > j$. Suppose that there exists a chain of $\mathcal{M}\Gamma$-subgroups of G:*

$$G_0 \subset G_1 \subset \cdots \subset G_n = G$$

such that $\mu(\tilde{G}_k) = \alpha_k$, *where* $\tilde{G}_k = G_k \backslash G_{k-1}$ *and* $G_1 = 0$ *for* $k = 0, 1, \cdots, n$. *Then* μ *is an* T*-fuzzy* $\mathcal{M}\Gamma$*-subgroup of* G.

Proof. Let $x, y \in G$. If x and y belong to the same $\tilde{G}_k$, then $\mu(x) = \mu(y) = \alpha_k$ and $x - y \in G_k$. Hence

$$\mu(x-y) \geq \alpha_k = \min\{\mu(x), \mu(y)\} \geq T(\mu(x), \mu(y)).$$

Let $x \in \tilde{G}_i$ and $y \in \tilde{G}_j$ for every $i \neq j$. Without loss of generality we may assume that $i > j$. Then $\mu(x) = \alpha_i < \alpha_j = \mu(y)$ and $x - y \in G_i$. It follows that

$$\mu(x-y) \geq \alpha_i = \min\{\mu(x), \mu(y)\} \geq T(\mu(x), \mu(y)\}.$$

Now let $y \in G$. Then there exists G_k such that $y \in \tilde{G}_k$ for some $k \in \{0, 1, 2, \cdots\}$. For any $a \in M, y \in \tilde{G}_k$ and $\alpha \in \Gamma$, we have $a\alpha y \in G_k$ and so $\mu(a\alpha y) \geq \alpha_k \geq \mu(y)$. Hence μ is an T-fuzzy $\mathcal{M}\Gamma$-subgroup of G. □

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