주기적 외란의 제거를 위한 빠른 오프라인 학습 제어

A Fast Off-line Learning Approach to the Rejection of Periodic Disturbances

장정국†, 김남국*, 이호성**

Jung-Kook Chang, Nam-Guk Kim and Ho-Seong Lee

Abstract

The recently-developed off-line learning control approaches for the rejection of periodic disturbances utilize the specific property that the learning system tends to oscillate in steady state. Unfortunately, the prior works have not clarified how closely the learning system should approach the steady state to achieve the rejection of periodic disturbances to satisfactory level. In this paper, we address this issue extensively for the class of linear systems. We also attempt to remove the effect of other aperiodic disturbances on the rejection of the periodic disturbances effectively. In fact, the proposed learning control algorithm can provide very fast convergence performance in the presence of aperiodic disturbance. The effectiveness and practicality of our work is demonstrated through mathematical performance analysis as well as various simulation results.

Key Words : Disturbance rejection, Periodic disturbance, Off-line learning control, Iterative update.

1. Introduction

Many engineering applications often face unavoidable disturbances which can degrade system performance seriously. In particular, periodic disturbances are inherent in rotating machinery. For an example, in a data storage system, there are periodic disturbances due to the eccentricity of tracks on a disk. This periodic disturbance generally occurs at frequencies that are integer multiples of the frequency of rotation of the disk and can be a considerable source of tracking error. Since the fundamental period of the periodic disturbances is known, much control effort has been usually expended to compensate for these periodic disturbances.

Recently, numerous control design methods have been developed specifically for eliminating periodic disturbances. Generally these methods generate the control input, whereby the system asymptotically tracks the periodic disturbance in the output. One of them is repetitive control which is based on the well-known internal model principle. A modified repetitive controller was proposed, which can enhance system stability at the

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$\dagger$ Servo Gr., Storage Div., Samsung Electronics Co., Ltd.

* Servo Gr., Storage Div., Samsung Electronics Co., Ltd

** Servo Gr., Storage Div., Samsung Electronics Co., Ltd.

  E-mail : jungkook.chang@samsung.com

  TEL : (031)200-0239

cost of tracking performance at high frequencies[1]. And, it was applied to a disk drive system and has proved its usefulness in improvement of tracking performance as well as elimination of harmonic components in spectrum of position error [2]-[4]. However, it is commonly known that repetitive control usually tends to amplify the effect of nonrepeatable disturbances whose frequencies are located near those of periodic disturbances. Also, design engineers should consider some tradeoff between system stability and tracking performance as an important factor.

On the other hand, the AFC (Adaptive Feedforward Concellation) method considered simply rejects sinusoidal disturbances at the input of the plant by adding the negative of their values at all times[5]-[8]. It is shown that it is equivalent in some sense to the well-known internal model principle[6]. So one can design an AFC controller easily by using the well-developed linear control theory. Unfortunately, the AFC method requires intensive computation when rejecting periodic disturbances with many harmonic components. Also, it need be designed carefully, considering some tradeoff between convergence rate and system stability.

Some off-line learning control methods to periodic disturbance rejection were recently proposed in [12]-[14] for a class of linear systems and in [9]-[11] for a class of nonlinear systems. Like AFC method, these methods also estimate the magnitude and phase of the harmonic components but the estimation carries out off-line at every fundamental period. So they have no stability problem if the original closed-loop system is stable. Unfortunately, it is practically impossible for a system to

reach its steady state within a finite time. Nonetheless, the prior works have not clarified how closely the learning system should approach the steady state to achieve the rejection of periodic disturbance to satisfactory level.

In this paper, we address this issue extensively for a class of linear systems. In the proposed learning control method, the update time is adjusted adaptively through computation of the so-called dwell time at each iteration. We also attempt to remove the effect of other aperiodic disturbances on the rejection of the periodic disturbances effectively. Thereby, the proposed learning control method can provide very fast convergence in the presence of aperiodic disturbances. The effectiveness and practicality of the proposed method is demonstrated through mathematical performance analysis as well as various simulation results. Our result may be viewed as practically and theoretically more complete evolution of the prior works in [12]- [14].

2. Main Result

The proposed learning control scheme is depicted in Fig. 1, where $K_C(s)$ is a stabilizing controller designed so as to stabilize the closed-loop system and to satisfy design specifications. We assume that there are two classes of bounded disturbances : (i) specific periodic disturbance $d_P(t)$ to reject and (ii) other periodic and aperiodic disturbances $d_N(t)$. We also assume that the plant is a linear system with Laplace transfer function $P(s)$. And $\hat{d}_P(t)$ is the estimate of $d_P(t)$ estimated off line and is used as feedforward term which is updated once at each iteration rather than continuously in time.

Since $d_P(t)$ is periodic with a known period T, it can be expressed as the following Fourier series representation :

$$d_P(t) = \sum_{i=1}^{N} \text{Re}\left[c_i e^{-j\omega_i t} \right] \tag{1}$$

where $\omega_i \triangleq 2\pi n_i / T$ and $c_i \in \mathbb{C}, i = 1, 2, \cdots, N$. Here, N and the n_i are nonnegative integers. In this context, we also can express $\hat{d}_P(t)$ as follows :

$$\hat{d}_P(t) = \hat{d}_P^k(t) \triangleq \sum_{i=1}^{N} \text{Re}\left[\hat{c}_{i,k} e^{-j\omega_i t} \right], \tag{2}$$

$$\text{if } t \in [t_k, t_{k+1}), k = 0, 1, 2, \cdots$$

where $t_0 \triangleq 0$ and $\hat{c}_{i,k} \in \mathbb{C}, i = 1, 2, \cdots, N$. Let

$$\Delta\hat{d}_P^l(t) \triangleq \sum_{i=1}^{N} \text{Re}\left[\Delta\hat{c}_{i,l} e^{-j\omega_i t} \right] v(t - t_l), \tag{3}$$

$$l = 0, 1, 2, \cdots$$

where $v(t)$ is the unit step function and $\Delta\hat{c}_{i,l} \triangleq \hat{c}_{i,l} - \hat{c}_{i,l-1}, l = 0, 1, 2, \cdots$. but $\hat{c}_{i,-1} \triangleq c_i$ Then, $\hat{d}(t)$ can be written in the following form :

$$\hat{d}_P(t) = \sum_{l=0}^{\infty} \Delta\hat{d}_P^l(t) \tag{4}$$

On the other hand, from Fig. 1 and (4), we can easily derive the following expression of the position error e(t) :

$$E(s) = S(s)R(s) + H(s)D_N(s) - \sum_{l=0}^{\infty} H(s)\Delta\hat{D}_P^l(s) \tag{5}$$

Where

$$S(s) \triangleq \frac{1}{1+P(s)K_c(s)}, H(s) \triangleq \frac{P(s)}{1+P(s)K_c(s)}, E(s) \triangleq \mathcal{L}\left[e(t) \right],$$

$$R(s) \triangleq \mathcal{L}\left[r(t) \right], \quad D_N(s) \triangleq \mathcal{L}\left[d_N(t) \right], \quad \text{and}$$

$$\Delta\hat{D}_P^l(s) \triangleq \mathcal{L}\left[\Delta\hat{d}_P^l(t) \right]. \text{ We denote the nominal value}$$

of $H(j\omega)$ by $\hat{H}(j\omega)$. Also, we define a kind of periodic impulse response $h_N^{-1}(t)$ by

$$h_N^{-1}(t) \triangleq \frac{2}{T} \sum_{i=1}^{N} \text{Re}\left[\frac{e^{-j\omega_i t}}{H^*(j\omega_i)} \right], \tag{6}$$

We also define

$$\gamma_N^{-1} \triangleq \sup_{0 \leq t \leq T} \left| h_N^{-1}(t) \right| \tag{7}$$

For further developments, we need to make the following assumptions.

A1) All poles of $S(s)$ and $H(s)$ are in the left-half comples plane and $S(0) = 0$

A2) There exists a known constant δ_0 such that

$$\sup_{0 \leq t \leq T} \left| \Delta\hat{d}_P^0(t) \right| \leq \delta_0 \tag{8}$$

A3) The learning system is initially relaxed at $t = 0$.

By A1), we see that the impulse responses $s(t) \triangleq \mathcal{L}'\left[S(s) \right], \quad h(t) \triangleq \mathcal{L}'\left[H(s) \right]$ satisfy the following inequalities for some positive constants $\alpha_i, \beta_i, i = s, h$:

$$\left| s(t) \right| \leq \alpha_s e^{-\beta_s t}, \left| h(t) \right| \leq \alpha_h e^{-\beta_h t}, t \geq 0 \tag{9}$$
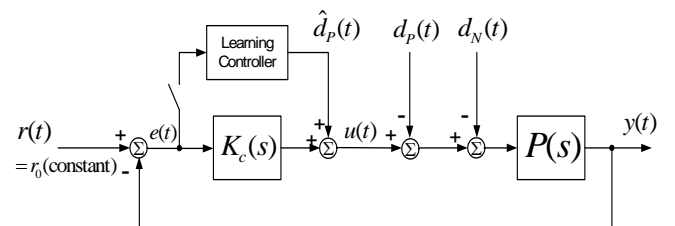

Fig. 1 Blockdiagram representation of the proposed learning control scheme

Now, we are ready to describe precisely the update scheme of our off-line learning control algorithm. In what follows, $\varepsilon > 0$ is a design parameter to be discussed later, while the subscript k is used to denote the iteration number and z^* denotes the conjugate of a complex number z.

Step 1) Set $k = 0$ and $\hat{d}_P(t) = \hat{d}_P^0(t)$ (initial guess).

Step 2) Wait for the dwell time τ_k determined by

$$\tau_k \geq \begin{cases} \max\left\{\dfrac{1}{\beta_s}\ln\dfrac{r_0\alpha_s}{\beta_s^2\varepsilon_k}, \dfrac{1}{\beta_h}\ln\dfrac{\delta_0\alpha_h}{\beta_h^2\varepsilon_k}, 0\right\}, & \text{if } k = 0 \\ \max\left\{\dfrac{1}{\beta_h}\ln\dfrac{\delta_k\alpha_h}{\beta_h^2\varepsilon_k}, 0\right\} & \text{otherwise} \end{cases} \quad (10)$$

where

$$\delta_k \triangleq \sup_{0 \leq t \leq T}\left|\Delta\hat{d}_P^k(t)\right|, \quad (11)$$

$$\varepsilon_k \triangleq \frac{\varepsilon}{1 + e^{-\beta_s kT} - e^{-\beta_s(k+1)T} - e^{-\beta_h(k+1)T}} \quad (12)$$

Step 3) Let $t_{k+1} \triangleq t_k + \tau_k + T$. Save the time history of the position error $e(t)$ on the time interval $[t_k + \tau_k, t_{k+1}]$ by letting $e_{ss}^k(\overline{t}) \triangleq e(\overline{t} + t_k + \tau_k)$ for each $\overline{t} \in [0, T]$. Then, determine $\hat{d}_P^{k+1}(t)$ by

$$\hat{d}_P^{k+1}(t) \triangleq \hat{d}_P^k(t) + \frac{1}{k+1}\int_0^T e_{ss}^k(\tau)\hat{h}_N^{-1}(t-\tau)d\tau. \quad (13)$$

Step 4) At $t = t_{k+1}$, set $\hat{d}_P(t) = \hat{d}_P^{k+1}(t)$ and increase k by one. Then jump to Step 2.

For better understanding of the update scheme stated above, we depict its timing diagram in Fig. 2. The kth estimate $\hat{d}_P^k(t)$ is determined by the (k-1)th estimate $\hat{d}_P^{k-1}(t)$ and the steady-state response of the closed-loop system with the feedforward term $\hat{d}_P^{k-1}(t)$. It should be clear that larger difference between $\hat{d}_P^{k-1}(t)$ and $\hat{d}_P^k(t)$ brings larger transient response of position error. The dwell time τ_k in (10) determines the minimum time after which the transient response becomes sufficiently small. At the kth update time t_k, we then wait for the dwell time τ_k after which the learning systems is ready for data acquisition. This clearly illustrates the fundamental difference between the proposed learning control algorithm and the others in the prior literature.

Under the above assumptions, it can be shown that the position error $e(t)$ can be divided into three components : steady-state response, transient response, and aperiodic disturbance response as follows :

$$e(t) = e_{ss}(t) + e_{tr}(t) + v_N(t) \quad (14)$$

where

$$e_{ss}(t) \triangleq \int_0^\infty s(\tau)r(t-\tau)d\tau$$
$$- \sum_{l=0}^\infty\left(\sum_{i=1}^N \text{Re}\left[H^*(j\omega_i)\Delta\hat{c}_{i,l}e^{-j\omega_i t}\right]\right)v(t-t_l) \quad (15)$$

$$e_{tr}(t) \triangleq -\int_t^\infty s(\tau)r(t-\tau)d\tau$$
$$+ \sum_{l=0}^\infty\left(v(t-t_l)\int_{t-t_l}^\infty h(\tau)\Delta\hat{d}_P^l(t-\tau)d\tau\right) \quad (16)$$

$$v_N(t) \triangleq \int_0^t h(\tau)d_N(t-\tau)d\tau \quad (17)$$

Then, the following Theorem 1 describes the convergence property of the proposed algorithm.

Theorem 1 : Further, suppose that (i) the time average of $v_N e^{j\omega_i t}$ is zero, $i = 1, 2, \cdots, N$ and (ii) there is no modeling uncertainty such that $\hat{H}(j\omega_i) = H(j\omega_i)$, $i = 1, 2, \cdots, N$. Then, the update scheme described by Steps 1)-4) assures that

$$\lim_{k\to\infty}\left|\hat{c}_{i,k} - c_i\right| \leq \frac{2\varepsilon}{|H(j\omega_i)T|}, \quad i = 1, 2, \cdots, N \quad (18)$$

$$\lim_{k\to\infty}\sup_{t_k \leq t \leq t_{k+1}}\left|d_P(t) - \hat{d}_P^k(t)\right| \leq \gamma_N^{-1}\varepsilon \quad (19)$$

$$\lim_{k\to\infty}\sup_{t_k+\tau_k \leq t \leq t_{k+1}}|e(t)| \leq \varepsilon\left[\frac{2N}{T} + \frac{\beta_h}{1-e^{-\beta_h T}}\right]$$
$$+ \lim_{k\to\infty}\sup_{t_k+\tau_k \leq t \leq t_{k+1}}|v_N(t)| \quad (20)$$
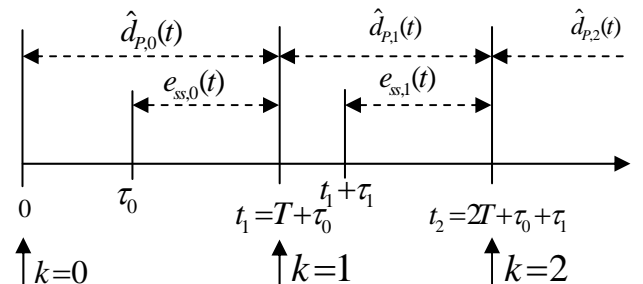

Fig. 2 Timing diagram of the proposed update scheme

The proof of Theorem 1 is omitted because of limited space.

Note from (10)-(12), (19), and (20) that through the choice of ε, we can make not only the estimation error of the periodic disturbance $d_P(t)$ but also its effect on the position error as small as desired but at the cost of longer dwell time. In fact, Theorem 1 suggests that the value of the design parameter ε need not be too small for the magnitude of $v_N(t)$. As can be seen from (17) and (20), our off-line learning approach to rejection of periodic disturbance can enable us to design $K_C(s)$ independently only for the reduction of aperiodic-disturbance effect on the position error. Also, note from Theorem 1 that the effect of the aperiodic disturbance on the estimation error tends to disappear as $k \to \infty$. This quite desirable feature is due to our unique choice of the update gain as $1/(k+1)$ in (13).

Finally, we address the issue of practical implementation of the proposed learning control algorithm. Let T_S be a sampling time. For computational simplicity, we assume that $T = MT_S$ for a positive integer and $M \gg 1$. Then, we can show that the iteration equation in (13) can be approximated well by

$$\hat{d}_P^{k+1}(nT_S) = \hat{d}_P^k(nT_S)$$
$$+ \frac{1}{2(k+1)} \sum_{l=0}^{M-1} [e_{ss}^k(lT_S) h_N^{-1}((n-1)T_S)$$
$$+ e_{ss}^k((l+1)T_S) h_N^{-1}((n-l-1)T_S)],$$
$$n = 0, 1, \cdots. \quad (21)$$

Thus, the proposed learning control algorithm can be converted in the form of a FIR filter, where $h_N^{-1}(mT_S)$, $m = 0, 1, \cdots, (M-1)$ can be pre-computed and stored in the microprocessor memory.

3. Simulation Results

In this section, we present some simulation results to illuminate further the effectiveness of the proposed learning control method. For our simulation work, we have assumed that $P(s)$ and $K_C(s)$ are given by

$$P(s) = \frac{2000}{s(s+100)}, \quad K_c(s) = 116\frac{s+150}{s+880} \quad (22)$$

Then, the 98\% settling time of the position error response to a step disturbance input is 0.022(s). Other data used in simulation are as follows :

$$\delta_0 = 25, \quad \hat{d}_P^0(t) = 0, \quad T = 0.1, \quad N = 3, \quad n_i = i,$$
$$c_1 = 2 + j8, \quad c_2 = -3 + j3, \quad c_3 = -4 + j5 \quad (23)$$

Based on the simulation data, we can calculate or choose the following values:

$$\beta_s = 290, \quad \alpha_s = 5000, \quad (24)$$
$$\beta_h = 165, \quad \alpha_h = 17.5, \quad \gamma_N^{-1} = 1489$$

We assume that $d_N(t)$ contains two higher-order harmonic components, a periodic disturbance with different period, and a white Gaussian noise w(t) with mean zero and variance 50 as follows.

$$d_N(t) = 2\sin 80\pi t + 2\sin 20\sqrt{3}\pi t + \sin 2000\pi t + w(t) \quad (25)$$

The simulation results are summarized in Figs. 3-6 for the four cases : (i) $r_0 = 100$, $\varepsilon = 0.001$, (ii) $r_0 = 100$, $\varepsilon = 0.0002$, (iii) $r_0 = 10$, $\varepsilon = 0.001$, (iv) $r_0 = 10$, $\varepsilon = 0.0002$. The dwell time $\tau_k, k = 0, 1, 2, \cdots$ determined by (10) at each iteration for these four cases were as follows :

(i) $\tau_0 = 0.0323, \tau_1 = 0.0187, \tau_2 = \tau_3 = \cdots = 0$
(ii) $\tau_0 = 0.0379, \tau_1 = 0.0285, \tau_2 = 0.0080$,
$\tau_3 = 0.0049, \tau_4 = 0.0024, \tau_5 = \tau_6 \cdots = 0$
(iii) $\tau_0 = 0.0244, \tau_1 = 0.0186, \tau_2 = \tau_3 = \cdots = 0$
(iv) $\tau_0 = 0.0307, \tau_1 = 0.0284, \tau_2 = 0.0076$,
$\tau_3 = 0.0043, \tau_4 = 0.0022, \tau_5 = \tau_6 \cdots = 0$

The above results show that the dwell time is reduced very rapidly as the iteration number increases. This is mainly because the proposed learning control method is based on the concept of system inversion and can estimate the specific periodic disturbance in one iteration in the absence of other periodic and aperiodic disturbance $d_N(t)$.

It can be observed from Figs. 3-6 that, during the initial iteration (k=0), the estimated periodic disturbance is zero since $\hat{d}_P^0(t) = 0$. As the iteration number is increased, however, the estimation errors converge to zero even in the presence of $d_N(t)$, while the position error does not due to the effect of $d_N(t)$. Comparing the simulation result in Fig. 3 (Fig. 5, respectively) with that in Fig. 4 (Fig. 6, respectively), we see that choosing ε too small increases only the estimation time unnecessarily without reducing the position error less. In fact, the simulation results can be explained well by

Theorem 1.

4. Conclusion

In this paper, we have presented a learning control method, in which the update time is adjusted adaptively at each iteration. Its excellent estimation performance has been demonstrated by both mathematical analysis and simulation results. In this paper, we have considered only the case of no plant uncertainty. Nonetheless, it can be shown with much more complicated mathematical analysis that if $\left|1-H(j\omega_i)/\hat{H}(j\omega_i)\right|<1, i=1,2,\cdots,N,$ the proposed method still can provide good estimation performance even in the presence of plant uncertainty.
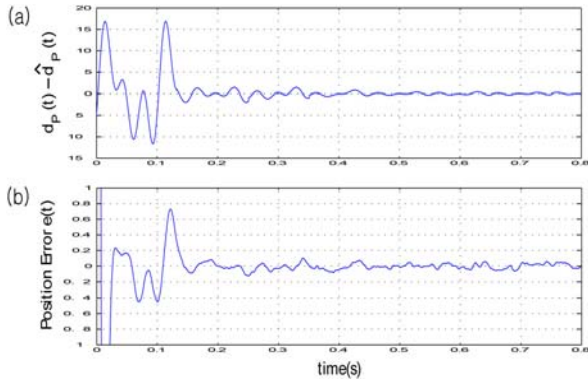

Fig. 3 Estimation performance of the proposed method in case of $r_0=100$ and $\varepsilon=0.001$. (a) time history of estimation error $d_P(t)-\hat{d}_P(t)$, (b) time history of position error $e(t)$
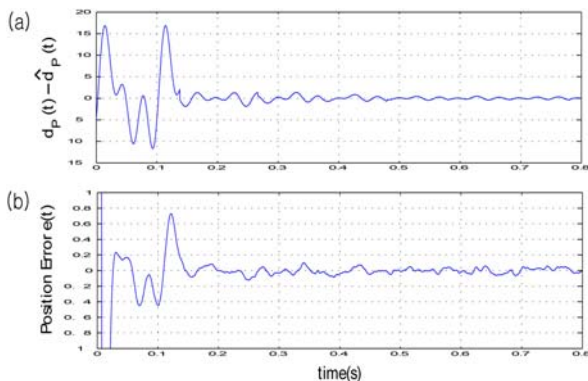

Fig. 4 Estimation performance of the proposed method in case of $r_0=100$ and $\varepsilon=0.0002$. (a) time history of estimation error $d_P(t)-\hat{d}_P(t)$, (b) time history of position error $e(t)$
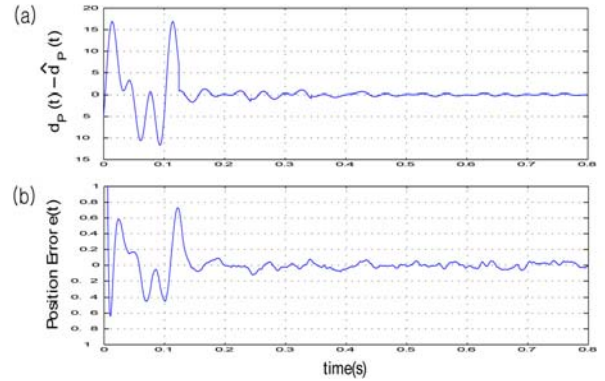

Fig. 5 Estimation performance of the proposed method in case of $r_0=10$ and $\varepsilon=0.001$. (a) time history of estimation error $d_P(t)-\hat{d}_P(t)$, (b) time history of position error $e(t)$
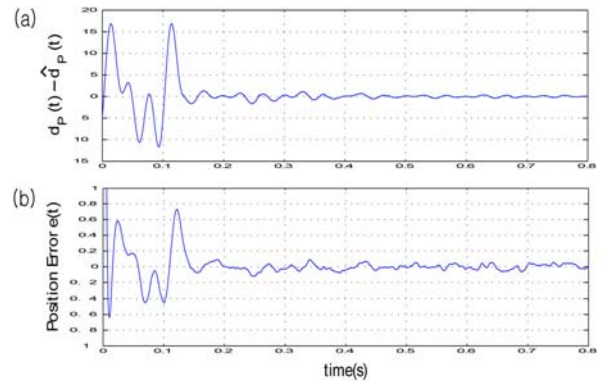

Fig. 6 Estimation performance of the proposed method in case of $r_0=10$ and $\varepsilon=0.0002$. (a) time history of estimation error $d_P(t)-\hat{d}_P(t)$, (b) time history of position error $e(t)$

It also should be noted that the proposed learning approach to disturbance rejection can be directly extended with slight modification to asymptotic tracking.

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