The DOA Estimation of Wide Band Moving Sources

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***Abstract*—** In this paper, a new method is proposed for tracking the direction-of-arrival (DOA) of the wideband moving source incident on uniform linear array sensors. DOA is estimated by focusing transformation matrices. To update focusing matrices along with new data snap shots, we use the FAST (Fast Approximate Subspace Tracking) method. Present focusing matrices are constructed by previous signal and its orthogonal basis vectors as well as present signal and its orthogonal basis vectors, which are the left and right singular vectors of the inner product of two approximated matrices. Simulation results are shown to illustrate the performance of the proposed method.

Index Terms —Wideband, Focusing matrix, Subspace tracking, Direction of Arrival.

I. INTRODUCTION

THIS paper will address the problem of estimating the direction-of-arrival (DOA) of a wideband moving source efficiently. A wideband signal has a bandwidth comparable to the carrier frequency. Several methods for estimating DOA using an array of sensors have been proposed [1], [3], [5].

The signal subspace techniques have been used for array signal processing, because the signal subspace is the span of the location vectors of the array. Since each location vector is a function of frequency in the wideband case, the method for narrow band cannot be used for wideband signal directly. For wideband signals, the signal subspaces at different frequencies do not overlap, and as a result, the observation vectors at the frequency bins cannot be added directly to each other. Hung and Kaveh [1] proposed focusing method, observation signals at each frequency bins except carrier frequency bin are focused to carrier frequency bin by focusing matrices. When sources move rapidly, Hung and Kaveh's method is not adequate. The focusing technique in the batch mode requires that the N transformation matrices, corresponding to the N frequency bins, have to be calculated. As a result, there is high computational burden, and the estimated DOA is averaged with the processing block size.

The objective of this paper is to present a method that can track the DOA of wideband incoherent signals incident on uniform linear array sensors. To update the focusing matrices at each snap shot, it is necessary to modify left and right singular vector according to each signal snap shot. Many subspace tracking algorithms may be found in the literature [2], [4]. We use the approximate subspace tracking method. The performance of the proposed method is illustrated by some simulation experiments.

II. FORMULATION

A. Signal Model

In the wideband DOA tracking formulation proposed here, we model all wideband sources to have an ideal band pass power spectrum over a given bandwidth [3]. The signal $x^m(t)$ received by the mth sensor of an M-sensor array, from the pth source located at θ_p, can be written as

$$x^m(t) = \sum_{p=1}^{P} \int_{f_l}^{f_h} e^{j2\pi f(t+\tau_m(\theta_p))} ds_p(f) \qquad (1)$$

where f_l and f_h are the limit of the frequency support of the signal, $dS_p(f)$ is a measure of the signal spectrum at frequency f, and $\tau_m(\theta_p)$ denotes the propagation delay corresponding to the mth sensor with the direction of θ_p. Assuming sources are uncorrelated, the (m,n)th element of the spatial covariance matrix of the received signal from the pth source is given by

$$[\mathbf{R}_p]^{m,n} = E[x^{mp}(t)\{x^{np}(t)\}^H]$$
$$= \int_{f_l}^{f_h}\int_{f_l}^{f_h} e^{j2\pi(f\lambda_m(\theta_p)-f'\tau_n(\theta_p))} E(dS_p(f)dS_p^*(f')). \qquad (2)$$

Because of the assumption, we can write

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$$E(dS_p(f)dS_p^*(f')) = \frac{\rho_p}{f_h - f_l}\delta(f - f') \quad (3)$$

where ρ_p denotes the power spectral density distributed uniformly over the range of frequency support, and $\delta(f)$ is the Dirac delta function. Substituting (3) into (2), and with some manipulation, the array covariance matrix may be written by

$$[\mathbf{R}_p]^{m,n} = \rho_p e^{j\pi(f_h+f_l)\tau_{mn}(\theta_p)} \operatorname{sinc}(\pi(f_h - f_l)(\tau_m(\theta) - \tau_n(\theta))). \quad (4)$$

Assuming that the additive noise at each sensor is white Gaussian, the overall received data covariance matrix is given by

$$\mathbf{R} = \sum_{p=1}^{P} \mathbf{R}_p + \sigma^2 \mathbf{I} \quad (5)$$

where $\mathbf{R}_p$ is given by (4), and $\mathbf{I}$ is the identity matrix. Equation (5) contains the direction of arrival information of the sources.

B. Focusing Matrix

We divide the frequency support interval of the signal into J frequency bands. The signal in (1), after passing the jth filter bank, is written by

$$\mathbf{X}_j = [x_j(1),\ x_j(2),\ \ldots,\ x_j(w)] \quad (6)$$

where $x_j(\zeta) = [x_j^1(\zeta),\ x_j^2(\zeta),\ \ldots,\ x_j^M(\zeta)]^T$,

w is the observation window size and M is the number of antennas. The reconstructed focused data for each filter bank is written by

$$\mathbf{Y}_j = \mathbf{T}_j\ \mathbf{X}_j \quad (7)$$

where $\mathbf{T}_j$ is called the focusing matrix. From the unitary invariance of the Coherent Signal Subspace Method (CSM) developed by Hung and Kaveh [1], the $\mathbf{T}_j$, $j = 1, \ldots, J$ are selected from

$$\min_{\mathbf{T}_j} \| \mathbf{A}_0 - \mathbf{T}_j\ \mathbf{A}_j \|, \quad (8)$$
$$\text{subject to } \mathbf{T}_j^H\ \mathbf{T}_j = \mathbf{I}$$

where $\| \cdot \|$ is the Frobenius matrix norm. The solution to this minimization is given by

$$\mathbf{T}_j = \mathbf{U}_j\ \mathbf{V}_j^H \quad (9)$$

where $\mathbf{U}_j$ and $\mathbf{V}_j$ are the left and right singular vectors of $\mathbf{A}_0\ \mathbf{A}_j^H$. Matrices $\mathbf{A}_0$ and $\mathbf{A}_j$ are assumed to be known. These array response vectors are calculated using a beamformer or Capon method ahead of the focusing. The focusing matrix transforms the orthonormal basis spanning the signal subspace at each frequency band f_j to an orthonormal basis spanning the signal subspace at the carrier frequency band f_0.

III. TRACK THE FOCUS MATRIX

We assume that the size of the observation matrix is fixed to be w. Then the filtered data at each frequency band are written by

$$\mathbf{X}_{0\_old} = [x_0(1),\ x_0(2),\ \ldots,\ x_0(w)] \quad (10)$$

$$\mathbf{X}_{j\_old} = [x_j(1),\ x_j(2),\ \ldots,\ x_j(w)] \quad (11)$$

where $\mathbf{X}_{0\_old}$ is the filtered data, which include the carrier frequency, and $\mathbf{X}_{j\_old}$ is the filtered data at the jth filter bank. We select the d dominant left singular vectors of $\mathbf{X}_{0\_old}\ \mathbf{X}_{0\_old}^H$, and denoted them by $\mathbf{U}_{0\_old}$, and d dominant right singular vectors of $\mathbf{X}_{0\_old}\ \mathbf{X}_{j\_old}^H$ is written by $\mathbf{V}_{j\_old}$. Then the focusing matrix may be written as follow

$$\mathbf{T}_{j\_old} = \mathbf{U}_{0\_old}\ \mathbf{V}_{j\_old}^H. \quad (12)$$

The columns of $\mathbf{U}_{0\_old}$ are basis vectors for the signal subspace in the carrier frequency band, and the columns of $\mathbf{V}_{j\_old}$ are basis vectors for the orthogonal subspace data cross-covariance matrix between the carrier frequency band and another frequency band. Because we select the d dominant basis vectors to construct the signal subspace and the orthogonal subspace, the squared Frobenius norm error in the approximation of the original data is given by

$$\varepsilon_{left} = \| \mathbf{X}_{0\_old} - \mathbf{U}_{0\_old}\ \mathbf{U}_{0\_old}^H\ \mathbf{X}_{0\_old} \|_F^2 \quad (13)$$

$$\varepsilon_{right} = \| \mathbf{X}_{j\_old} - \mathbf{V}_{j\_old}\ \mathbf{V}_{j\_old}^H\ \mathbf{X}_{j\_old} \|_F^2. \quad (14)$$

When we receive a new data snap shot $x_0(w+1)$, $x_j(w+1)$, we shall have the new matrices at each frequency band,

$$\mathbf{X}_{o\_new} = [x_0(2),\ x_o(3),\ \ldots,\ x_0(w+1)] \quad (15)$$

$$\mathbf{X}_{\text{j\_new}} = [x_j(2),\ x_j(3),\ . \ . \ . \ ,\ x_j(w+1)]. \quad (16)$$

We form M x (*d+1*) matrix $\mathbf{A}$ with the previous left singular vectors and new snap shot as

$$\mathbf{A} = [\ \mathbf{U}_{\text{0\_old}} \quad q\ ]\ \mathbf{C} \quad (17)$$

where

$$\mathbf{C} = \begin{bmatrix} a_0(2) & a_0(3) & a_0(4) & . \ . \ . & a_0(w+1) \\ 0 & 0 & 0 & & \alpha \end{bmatrix}$$

$$a_0(i) = \mathbf{U}^{\text{H}}_{\text{0\_old}}\ x_0(i)$$

$$z_0 = x_0(w+1) - \mathbf{U}_{\text{0\_old}}\ \mathbf{U}^{\text{H}}_{\text{0\_old}}\ x_0(w+1)$$

$$\alpha = \| z_0 \|$$

$$q = \frac{z_0}{\alpha}\ .$$

The approximation error of $\| \mathbf{X}_{\text{0\_new}} - \mathbf{A} \|^2_{\text{F}}$ is at or below the error value ε_{left}. A new constructed matrix $\mathbf{A}$ is a good approximation and contains the previous basis vector and present data snap shot basis information. The (*d+1*) x w matrix $\mathbf{C}$ forms the space orthogonal $\mathbf{U}_{\text{0\_old}}$. The same method is applied to $\mathbf{X}_{\text{j\_new}}$; we make the matrix $\mathbf{G}$ such as

$$\mathbf{G} = [\ \mathbf{V}_{\text{0\_old}} \quad o\ ]\ \mathbf{D} \quad (18)$$

where

$$\mathbf{D} = \begin{bmatrix} a_j(2) & a_j(3) & a_j(4) & . \ . \ . & a_j(w+1) \\ 0 & 0 & 0 & & \beta \end{bmatrix}$$

$$a_j(i) = \mathbf{V}^{\text{H}}_{\text{j\_old}}\ x_j(i)$$

$$z_j = x_j(w+1) - \mathbf{V}_{\text{j\_old}}\ \mathbf{V}^{\text{H}}_{\text{j\_old}}\ x_j(w+1)$$

$$\beta = \| z_j \|$$

$$o = \frac{z_j}{\beta}\ .$$

The error between original data matrix and approximated matrix is $\| \mathbf{X}_{\text{j\_new}} - \mathbf{G} \|^2_{\text{F}} \le \varepsilon_{right}$, $\mathbf{G}$ is a good approximation to $\mathbf{X}_{\text{j\_new}}$. The matrices $\mathbf{C}$ and $\mathbf{D}$ are space orthogonal $\mathbf{U}_{\text{0\_old}}$, $x_0(w+1)$ and $\mathbf{V}_{\text{j\_old}}$, $x_j(w+1)$ respectively. We can form the new (*d+1*) x (*d+1*) matrix $\mathbf{R}_{\text{c}}$ that contains the information of the space orthogonal $\mathbf{U}_{\text{0\_old}}$ and $\mathbf{V}_{\text{j\_old}}$, as follow

$$\mathbf{R}_{\text{c}} = \mathbf{C}\ \mathbf{D}^{\text{H}}. \quad (19)$$

After the singular value decomposition of the new matrix $\mathbf{R}_{\text{c}}$, we select *d* dominant left and right singular vectors as

$$\mathbf{U}_{\text{RC}} = [u_{RC}(1)\ u_{RC}(2), \ldots, u_{Rc}(d)] \quad (20)$$

$$\mathbf{V}_{\text{RC}} = [v_{RC}(1)\ v_{RC}(2), \ldots, v_{Rc}(d)]. \quad (21)$$

Now we can make the singular vectors with respect to the new data snap shot by

$$\mathbf{U}_{\text{0\_new}} = [\ \mathbf{U}_{0\_old} \quad q\ ]\ \mathbf{U}_{\text{RC}} \quad (22)$$

$$\mathbf{V}_{\text{j\_new}} = [\ \mathbf{V}_{j\_old} \quad o\ ]\ \mathbf{V}_{\text{RC}}. \quad (23)$$

Finally, new focusing matrix are updated by

$$\mathbf{T}_{\text{j\_new}} = \mathbf{U}_{\text{0\_new}}\ \mathbf{V}^{\text{H}}_{\text{j\_new}}\ . \quad (24)$$

Transformed observation data by the focusing matrix at each frequency band are added, and the DOA is estimated using a high-resolution parameter estimator such as MUSIC.

IV. SIMULATION RESULTS

To examine the effectiveness of the proposed method for tracking the direction of arrival of wideband signal, we assume that incoherent source incident on a linear-equally-spaced eight array-sensors. The source in the supported frequency has the uniform spectral density, with an 80% bandwidth relative to the carrier frequency.

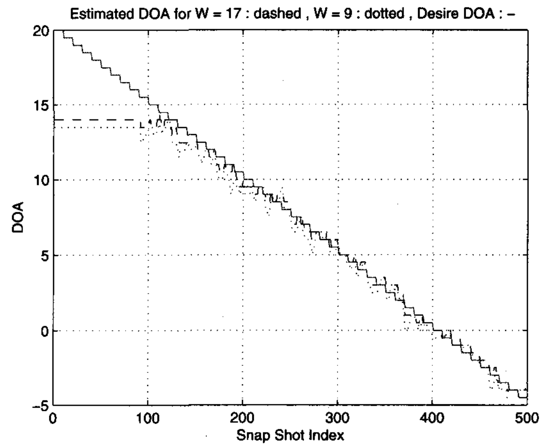

Fig. 1 DOA trajectories of the linearly changed angle with 10 dB SNR. Solid line denotes the true angle. Dashed and dotted lines are for the 17 and 9 snap shot window sizes respectively

Symbol rate is twice the carrier frequency. We sample the received data with 10 times of the symbol rate. For simplicity, three filter banks are used for non-overlapped equal-sized bandwidth. We assume that angles of the sources are stationary in the over sample period. To estimate the DOA of the source, we choose that the processing block size is 10 over-sampled, initial angle of the signal is 20°, the angle of the source changes linearly with -0.5° / symbol, and 50 symbols are used.

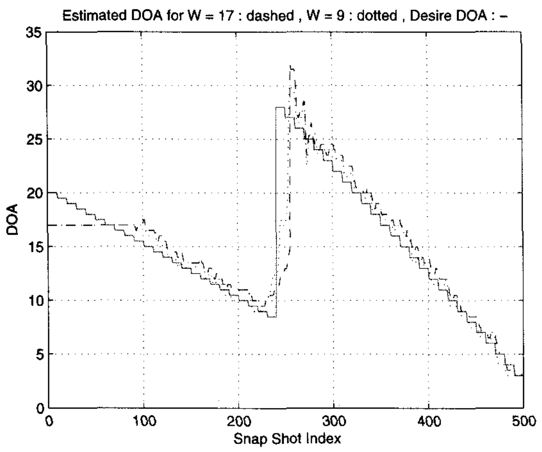

Fig. 2 DOA trajectories of an abruptly changed angle with 10 dB SNR. True angle is denoted with solid line, the dashed and dotted lines are for the 17 and 9 snap shot window sizes respectively.

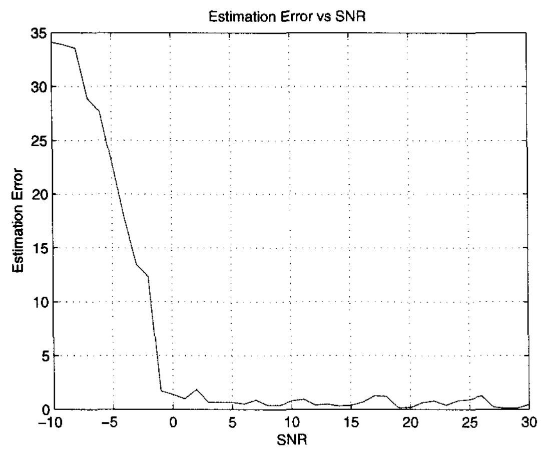

Fig. 3 RMS tracking error with respect to SNR, for the case of 10 snap shot window size, and 50 symbols interval.

When SNR is 10dB, Fig 1 shows the DOA trajectories, solid line represents the true angle, and the cases of the 17 and 9 processing snap shot sizes are marked with dashed and dotted line respectively.

To investigate the ability of the DOA tracking for the abrupt change, we let the true angle change from 7° to 27° at the 23rd symbol. The tracking trajectories are shown in Fig 2. The larger processing window size, the higher tracking error occurs at the discontinuous point because of the averaging effect at that point.

We figure the root-mean-square error of estimated angle versus SNR in Fig 3. From that, the proposed method tracks the angle of the source properly at higher than 0dB SNR.

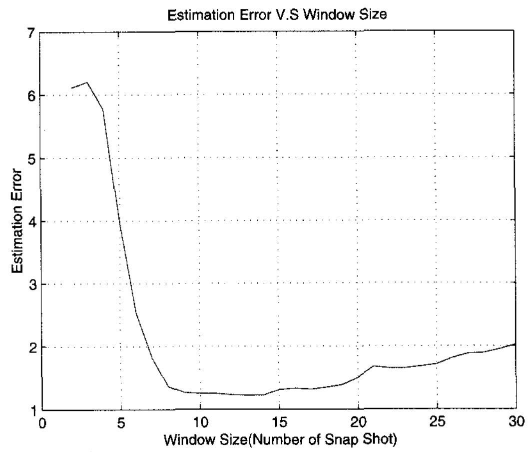

Fig. 4 RMS tracking error with respect to window size with 10dB SNR and 50 symbols interval

We investigate how many previous snap shots are needed to track the DOA at the current snap shot. From Fig 4 we can see that the window size greater than the number of over samples and less than twice of that would be required.

V. CONCLUSIONS

We present a new method to track the DOA of a wideband moving source by updating the focusing matrices as a new snap-shot arrives. Using the previous left and right singular vector bases, and current snap shot, we construct the new approximation matrices at carrier frequency band and other frequency bands. Present focusing matrices are updated by the previous left and right basis vectors and left and right basis vectors of the inner product matrix of two approximated matrices.

By simulation, we show that the proposed method tracks the DOA of wideband signals well even in low SNR environment.

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