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설비 생존곡선 추정을 위한 혼합형 Weibull 함수의 활용

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A Study on the Application of Mixed Weibull Function to Estimate Survivor Curves of Industrial Property

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일반 투자안의 의사결정에서와 마찬가지로 산업설비의 경제성 분석에서도 가장 중요한 결정 요소 중의 하나가 설비의 생존곡선 추정이다. 설비의 자산 가치가 감소하는 원인은 여러 가지가 있으나, 여러 원인 중 물리적 훼손이 과거의 산업설비에서는 가장 중요한 원인이었으므로 기존의 생존모형 분석에서는 Iowa 생존곡선을 이용하여 설비의 생존곡선을 추정하였다. 그러나 새로운 기술상의 변화로 인한 첨단 생산시스템의 설비교체 분석시에는 적합지 않다. 따라서, 본 연구에서 제안된 혼합형 Weibull 함수를 이용하여 설비의 폐기 형태를 추정함으로써 설비들의 실제적인 생존곡선을 정확하게 파악할 수 있다.

Keywords : Life Analysis, Life Estimation, Iowa Type Curves, Mixed Weibull Function

1. Introduction

The estimation of mortality characteristics of industrial property is an important adjunct to engineering valuation and depreciation estimates. Once the importance of depreciation estimates is determined, it is desirable to understand the processes upon which these estimates are based. The processes upon which estimates are based can be generally classified into two distinct procedures : life analysis and life estimation.

Life analysis is the process of aggregating and analyzing the historical record of property for the purpose of obtaining information about the mortality characteristics of property. Life estimation may be thought of as the use of judgement in applying the results of life analysis to estimate the mortality characteristics of industrial property. Life analysis and life estimation are different processes since the former is concerned which an analysis of the past whereas the latter is generally concerned with a prediction of the future, but not exclusively.

Property life analysts have adapted certain of techniques from the field of actuarial science to summarize their data [10]. The usual outcome of the most frequently used of these techniques, the retirement and the property's age. Typically, this relationship is given as either conditional failure for survival rates at various ages. the latter in graphical form produce a survivor curve that indicates the portion of any group of like-aged items remaining in service at a given age.

Since these analytical results are many times erratic and generally of less than full life cycle, mathematical or graphical procedures are utilized to smooth and extrapolate the data as

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necessary to make it more useful. A number of techniques are used to accomplish this, including a variety of mathematical equations and sets of standardized survivor curves [11, 12].

Probably the most widely recognized and used systems of standardized curves is that known as the Iowa type curves. It was in June 1931 that the Iowa Engineering Experiment Station of Iowa State College published Bulletin 103 by Robley Winfrey and Edwin B. Kurtz [18]. This publication was the original one in a series from Iowa State College (now Iowa State University) dealing with the life characteristics of physical property and their relationships to valuation and depreciation. Between 1931 and 1935, Winfrey continued data collection and added 111 property group curves to the 65 used 1931. From these 176 curves, 18 standard Iowa type curves emerged in 1935 when Winfrey wrote the still existent Iowa Engineering Experiment Station Bulletin 125 [19]. A revised edition of Bulletin 125 was made available in 1967 by the Iowa State University Engineering Research Institute, successor to the Iowa Engineering Experiment Station. The revision included the addition of four origin modal (O type) standard curves empirically developed by Frank Couch [1]; thus the total array was increased to 22. Nine additional standard curve types have since been recognized by some users, although their development did not necessitate further work with actual property data. The single square curve and the eight so-called "half curves" have, in a pragmatic way, been made part of the system by use.

The object of this study is to analyze the ability of the Weibull distribution to describe industrial property mortality characteristics. Thirty three sets of data were used in this study. These data were collected by Russo [15] to revalidate the Iowa curves in use under current economic, technological, and managerial conditions.

2. Survivor Functions and Weibull Function

2.1 Some Survivor Functions

Theoretically, any proper distribution might be used as a survivor function. In real situation, however, certain families of distributions are especially useful for fitting survival data. Properties of exponential, Weibull, Log-normal, Gamma, generalized Gamma, and Log-logistic distributions are discussed by Jonson and Kot z[8], who also give extensive bibliographies on these distributions. Some of these distributions are also discussed by Cox [2], Mann et al. [9] and Gross and Clark [4]. The generalized Gamma distribution was introduced by Stacy [16] and has been discussed by Parr and Webster [13], Harter [6], Hagar and Bain [5]. The generalized F-distribution is discussed by Prentice [14].

When fitting survivor functions, the choice of the form of survivor distribution(i.e., the family to which it belongs) is supposed to have already been decided. Our interest is then in estimating values of the parameters appearing in the mathematical formula for the survivor function of the family of distributions considered.

2.2 The Weibull Distribution

The Weibull distribution, named after its conceiver, Waloddi Weibull [17], is the most widely used survivor functions. Weibull proposed a cumulative distribution function of the form

$$P(x) = 1 - \exp(-np(x)) \quad \cdots\cdots (1)$$

where, P(x) = probability of an event,
n = number of possible events,
p(x) = function of the population,
x = some measure of each individual in the population.

Equation (1) does not have much meaning until p(x) is defined. weibull specified that p(x) be a positive, non-decreasing function, vanishing at μ(μ is not necessary zero). The simplest form satisfying these condition is;

$$np(x) = \frac{(x-\mu)^m}{x_0} \quad \cdots\cdots (2)$$

In most reliability application, P(x) is considered to be the retirement function, x_0(change to $\frac{1}{\alpha}$) is said to be proportional to the reciprocal mean time to failure, μ is set to zero, m is changed to β, and x is considered to be age. After making all necessary substitutions into equation (2), the Weibull distribution for the retirement function or cumulative function becomes,

$$F(x) = 1 - \exp(-\alpha x^{\beta}) \quad \cdots\cdots (3)$$

where, x = age of time
α = scale parameter
β = shape parameter.

Rearranging equation (3) gives the survivor function :

$$S(x) = \exp(-\alpha x^{\beta}) \quad (4)$$

Differencing equation (4) gives the retirement frequency function or the probability density function as

$$f(x) = \alpha\beta x^{\beta-1} \exp(-\alpha x^{\beta}) \quad (5)$$

The hazard rate or the retirement rate is defined as the retirement frequency function divided by the survivor function :

$$h(x) = \alpha\beta x^{\beta-1} \quad (6)$$

3. Procedures

The procedures necessary to carry out the objectives of this study can be generally classified into three categories : i) data selection, ii) estimating Weibull parameters to data, and iii) testing the resulting Weibull sets against the Iowa type survivor curves.

3.1 Data

The data used in this study are the same data as were collected by Russo [15]. Aged mortality data were collected for over 1,500 accounts from the various types of industries : gas transmission, water, electric, roads and highways, and commercial. Of the 1,500 accounts collected, 490 were selected that had latest experience years and that had survivor curves extending below 20%. A clustering procedures was then employed to produce thirty three subgroups of curves with related shapes within each modal group. Finally, thirty three curves obtained by averaging the subgroups of curves and were used in this study.

3.2 Estimation of Weibull parameters

In this study, the survivor function is the medium for estimating the parameters and an iterative process is investigated. The graphical method gives fast but rough estimate, and consequently is not explored in detail.

3.2.1 The modified Gauss-Newton iteration

This procedure developed by Hartley [7] determines parameters α and β for the sum of squares,

$$Q(\alpha, \beta) = \sum_{i=1}^{n} [Y_i - f(x_i, \alpha, \beta)]^2 \quad (7)$$

where, Y_i = observed value

$$f(x_i, \alpha, \beta) = \exp(-\alpha x^{\beta})$$

minimizing.

Note that $f(x_i, \alpha, \beta)$ represents the survivor function, not the retirement frequency function. One of the necessary assumptions for facilitation of the procedures is that the partial derivatives of S(x) with respect to α and β are continuous:

$$\frac{df}{d\alpha} = f_1(x_i, \alpha) = -x^{\beta}\exp(-\alpha x^{\beta}) \quad (8)$$

and

$$\frac{df}{d\beta} = f_2(x_i, \beta) = -\alpha x^{\beta}\log x \exp(-\alpha x^{\beta}) \quad (9)$$

With the above condition satisfied, equation (7) can be differentiated a follows :

$$\frac{dQ}{d\alpha} = Q_1(x, \alpha) = -2\sum_{i=1}^{n}[Y_i - f(x_i, \alpha, \beta)]\ f_1(x_i, \alpha) \quad (10)$$

and

$$\frac{dQ}{d\beta} = Q_2(x, \beta) = -2\sum_{i=1}^{n}[Y_i - f(x_i, \alpha, \beta)]\ f_2(x_i, \beta) \quad (11)$$

Next, expand a Taylor series for Q_1 about α_0 and Q_2 about β_0 which results in:

$$\sum_{i=1}^{n} f_1(x_i, \alpha_0)[f_1(x_i, \alpha_0)D_1 + f_2(x_i, \beta_0)D_2] = \sum_{i=1}^{n}[Y_i - f(x_i, \alpha_0, \beta_0)]f_1(x_i, \alpha_0) \quad (12)$$

and

$$\sum_{i=1}^{n} f_2(x_i, \beta_0)[f_1(x_i, \alpha_0)D_1 + f_2(x_i, \beta_0)D_2] = \sum_{i=1}^{n}[Y_i - f(x_i, \alpha_0, \beta_0)]f_1(x_i, \beta_0) \quad (13)$$

The corrections for α_0 and β_0 are proportional to the solutions D_i(i.e., D_1 and D_2) of the Gauss-Newton equations such that,

$$\alpha_1 = \alpha_0 + \nu D_1 \quad (14)$$

and

$$\beta_1 = \beta_0 + \nu D_2 \qquad (15)$$

where, α_0 = initial estimate of α

α_1 = corrected estimate of α

β_0 = initial estimate of β

β_1 = corrected estimate of β

ν = a value between 0 and 1

The estimate of ν is determined by successive trials of V evaluated from,

$$V = \frac{1}{2^{k-1}} \qquad (16)$$

where, k = trial number,

until the sum of squares $Q(\alpha_0+\nu D_1, \beta_0+\nu D_2)$ is less than $Q(\alpha_0, \beta_0)$. The ν equals the last V evaluated from equation (16) which gives an improvement (i.e., decrease) in the sum of squares. The corrected α_1 and β_1 served as starting values for next iteration. This process is repeated until sum if squares shows negligible improvement, i.e., net change less than 10^{-6}.

3.2.2 Mixure distribution

A mixture of two distributions, each belonging to the same known Weibull distributions, is proposed, and a simple graphical method for estimating the parameters of the mixed distribution is applied. The first step is to plot the observed survivor curve. From such a plot, we may get some indication whether we need a second component. Suppose that there are two components and their location parameters differ sufficiently (as compared with the scale parameters). Thus, the survivor function for lower (higher) values should reflect mainly variation of the component with lower (higher) value of location parameter [3]. Denoting the survivor functions of the two components by $S_i(x)$ (i= 1, 2), the mixture survivor function is

$$S(x) = \nu S_1(x) + (1-\nu) S_2(x) \qquad (17)$$

where ν (0< ν <1) is the proportion of the first component in the mixture survivor function. Under the stated assumptions, $S_1(x)$ corresponds overall to much shorter life time than $S_2(x)$. For small values of x, we then have $S_2(x)$=1, and

$$S(x) = \nu S_1(x) + (1-\nu) \qquad (18)$$

or

$$F(x) = \nu[1 - S_1(x)] = \nu F_1(x) \qquad (19)$$

On the other hand, for large values of x,

$S_1(x)=0$ and so,

$$S(x) = (1-\nu)\, S_2(x) \qquad (20)$$

From equation (20), we have

$$S_2(x) = \frac{S(x)}{1-\nu} \qquad (21)$$

and to estimate $S_2(x)$, we have plot

$$\log[-\log[(1-\nu)^{-1}S(x)]] = \log[\log(1-\nu) - \log S(x)] \qquad (22)$$

against $\log x$. We use a series of trial values of $\log(1-\nu)$ until we get the nearest approach(in our judgement) to a straight line. The lower part of the survivor curve will be estimated by $S_1(x)$. From the equation (19),

$$S_1(x) = 1 - \frac{F(x)}{\nu} \qquad (23)$$

Plotting $\log[-\log[1-\nu^{-1}F(x)]]$ against $\log x$ we would get a nearly straight line plot. A more symmetrical approach would be obtain simultaneously plots of :

$$\log[-\log[(1-\nu)^{-1}S(x)]]$$

against $\log x$ for higher value of x, and

$$\log[-\log[1-\nu^{-1}F(x)]]$$

against $\log x$ for lower value of x, and try to make both as nearly linear as possible by varying ν. The results are summarized in <Table 1>.

3.3 Statistical testing

3.3.1 Test curves and testing

Just as with the choice of the original data for the study, the test curves were randomly chosen to represent as great industrial property types as possible. The data from totally separate companies for testing were as follows: electric utilities, gas utilities, telephone company. Thus, a total of 25 test curves, all 0% surviving, were prepared for testing. Preparation involved formulation of survivor curves by the retirement rate method so that graphic comparisons with Weibull fitting and Iowa type curves could be accomplished.

<Table 1> Results of mixed Weibull function

curve type	%age	α	β	sum of squared residuals
1	99	1.6400E-05	2.69342750	1555.2
	246	1.2416E-02	0.78716639	5421.4
2	177	5.0000E-07	2.87662931	7016.4
	300	1.9802E-01	0.59517092	1.2
3	89	5.5100E-06	2.57294586	4732.2
	210	3.3297E-04	1.62888544	3196.0
4	71	1.0000E-09	4.72984100	28212.7
	300	6.5280E-03	1.23400996	427.8
5	129	6.0620E-05	2.04552907	3006.2
	300	5.7900E-02	0.65617938	195.6
6	274	2.5067E-04	1.72146922	7888.1
7	300	5.0909E-03	1.09247138	1023.5
8	300	3.1632E-03	1.23736426	6020.9
9	69	8.0220E-10	4.51405023	62660.0
	300	4.2564E-01	0.31517573	363.0
10	59	6.5700E-05	2.03095583	204.1
	300	2.1853E-04	1.82667388	3237.6
11	69	2.8770E-05	1.66828862	6.7
	300	1.3190E-05	2.43244803	14546.6
12	192	1.1684E-04	1.82211558	3855.7
	299	7.1500E-07	2.87805121	52.1
13	300	3.6118E-04	1.66723086	155.7
14	300	2.0856E-03	1.20791464	4643.5
15	300	1.6945E-05	2.31743486	3694.5
16	96	2.2363E-07	3.18053984	4812.7
	229	3.7136E-01	0.25677454	2673.3
17	97	2.2037E-05	1.83043340	465.5
	161	6.1000E-08	3.37331470	4834.2
18	92	3.4442E-04	1.47894475	32.7
	237	1.6407E-08	3.67853893	1853.5
19	300	5.2083E-04	1.58487959	1705.3
20	91	5.5429E-07	2.60293137	215.7
	241	8.4365E-10	4.45958204	7248.0
21	300	2.6897E-05	2.21888493	6301.4
22	78	4.7473E-07	2.98830121	2381.8
	223	7.8930E-05	1.86905974	2292.8
23	135	1.8225E-04	1.44563099	781.9
	300	2.5705E-01	0.11655903	803.8
24	238	2.1904E-03	1.17952908	3965.5
25	212	4.5000E-07	2.42269125	5018.2
26	173	3.8900E-06	2.49838409	8632.6
27	98	2.3326E-06	2.65814990	208.5
	261	8.5453E-03	0.9919594	3988.7
28	102	3.9020E-05	1.81040189	114.5
	172	6.9683E-15	6.72062184	700.5
29	190	6.0802E-04	1.46094362	4220.2
	300	4.4999E-02	0.85740228	122.0
30	170	2.2980E-05	2.22001376	9260.1
	300	5.7636E-02	0.87360292	0.8
31	300	8.0112E-05	1.96587733	2196.0
32	300	2.7317E-05	2.21227444	4293.7
33	87	5.6590E-05	1.80716709	249.1
	295	1.6700E-06	2.82777044	15467.3

The test involved computer fitting of the 25 test curves to both the Weibull fitting array and Iowa type curves to determine which produced the best fit for each teat curve. The test curves were held constant in each case and the Weibull fitting and Iowa type curves adjusted until the closest fit were found. Since the average service life of each test curve could be easily determined by the area under the survivor curve, a starting point for each curve of the curve sets was established. Thus, with this average service life starting point, curve set shapes determined by average service lives, adjacent to the starting point average service life, could be investigated until the least area between each test curve and the Weibull fitting and Iowa type curve was found. The test curve average service life was used simply as a device to start the investigation of which standard curve best fit the test curve. The final criteria of best fit were the total area between the curves. All areas were considered to be positive in sign, so full variation was indicated. Thus, for each test curve the area between the test curve and the fit curves of each set were projected.

3.3.2 Statistical treatments of test results

Three statistical procedures were chosen for analysis of the 25 test curves fit to Weibull and the Iowa type curves. The procedures were chosen so that conclusions could be drawn as to which curve set best fit the test curves. Each of the three procedures was applied to the area differences as a reflection of dispersion from the fitting tests.

(1) sign test

This test is non-parametric. While it does not reflect relative or absolute magnitudes of differences in population means, it does reflect direction in that larger or smaller member of a pair of observations is identified.

The sign test was applied to areas as follows : Define,

$$A_W - A_I = D_{W-I}$$

where, A_W = the area between each teat curve and its best fit Weibull fitting

A_I = the area between each teat curve and its best fit Iowa type curve

Let P equal the probability that $D_{W-I} \geq 0$ (i.e., Iowa type curve fitting is better); then test the hypotheses :

$$H_0 : P \leq 0.5$$
$$H_1 : P > 0.5$$

The test statistics :

$$Z = \frac{x-n(0.5)}{\sqrt{n(0.5)(0.5)}}$$

where, n = sample size

x = number of times D_{W-I} >0 in a sample of size n

follows standard normal distribution, $N(0, 1)$. Reject H_0 if Z value is substantially greater than a critical value, depends on the level of significance desired.

(2) Wilcoxon test for paired observations

This test is non-parametric and is more sensitive than the sign test in that it reflects the relative magnitude of differences in population central tendencies. It is accomplished by ranking the differences between paired values and applying the ranking in statistics as outlined below. Let CW_{W-I} be the true central tendency for the paired differences for the population between Iowa and Weibull fitting ; then test the hypothesis :

$$H_0 : CW_{W-I} \leq 0$$
$$H_1 : CW_{W-I} > 0$$

The test statistic :

$$Z = \frac{T - CT_W}{\sigma_W}$$

where, T = the total of the rank numbers with positive difference when all differences are ranked from 1 (smallest) to n (largest),

$$CT_W = \frac{n(n+1)}{4},$$
$$\sigma_W^2 = \frac{n(n+1)(2n+1)}{24},$$

follows standard normal distribution, $N(0, 1)$. Reject H_0 if Z value is substantially greater than a critical value, depends on the level of significance desired.

(3) Parametric test for paired observations

This test is parametric and reflects the actual magnitude of differences in population means. It concentrates on and compares mean optimal area of Weibull fitting and Iowa type vurves and somewhat better discriminating powers than other two tests or rejecting H_0 if H_0 truly deserves to be rejected when normality can be assumed. Let μ_I and μ_W be the true means for population when the Iowa curves and Weibull fittings, respectively, are used; then test the hypothesis:

$$H_0 : \mu_I - \mu_W = \mu_{W-I} \leq 0$$
$$H_1 : \mu_I - \mu_W = \mu_{W-I} > 0$$

The test statistic :

$$T = \frac{AD_{W-I}}{\sqrt{\frac{S_d^2}{n}}}$$

where, S_d = sample standard deviation,

AD_{W-I} = the mean of the differences of areas,

follows t-distribution. Reject H_0 if T value is substantially greater than a critical value, depends on the level of significance desired.

4. Results and discussion

4.1 Subjective analysis

Subjective analysis involved a thorough review of fitting test curves to the Iowa type curves and the Weibull distribution. The results of fitting test curves are summarized in <Table 2>.

The following was evident concerning the 25 test curves fit to the Iowa type curves and the Weibull fittings.

1. Based on the area difference and visual review of the fits, the Weibull fitting produced 10 best fits, the Iowa type curves 15 best fits. This is reflected in the remarks column of <Table 2>.
2. Of the 10 best curves best fit by the Weibull fitting, 4 were left modal, 3 symmetrical modal, and 3 right modal.
3. Of the 15 best curves fit by the Iowa type curve, 2 were origin modal, 6 left modal, 1 symmetrical modal, and 6 right modal.
4. Of the total array of the 31 Iowa type curves, 15 curves did not produce any fits, no matter whether best fit or not. These non-fit curves included 9 symmetrical modal, 1 left modal, 2 right modal, 2 origin modal, and the square curve.
5. Of the total array of the 33 Weibull fittings, 17 curves did not produce ant fits, no matter whether best fit or not.

<Table 2> Results of fitting test curves

Test curve number	Weibull fitting		Iowa curves		Remark
	curve No.	area differ. (sq. %units)	curve No.	area differ. (sq. %units)	
1	2	1217.0	R1.5	1026.0	I
2	8	1243.0	O3	1338.0	W
3	28	908.0	R5	730.0	I
4	13	634.0	L0	599.0	I
5	29	1707.0	R0.5	1560.0	I
6	21	882.0	R2	671.0	I
7	13	1296.0	O2	1256.0	I
8	28	749.0	R5	540.0	I
9	32	695.0	S1.5	634.0	I
10	15	502.0	L2	495.0	I
11	21	869.0	L3	883.0	I
12	19	487.0	L0.5	250.0	I
13	13	408.0	L0	475.0	W
14	2	1130.0	R3	1365.0	W
15	7	1509.0	O2	1398.0	I
16	11	491.0	L4	343.0	I
17	3	1145.0	L2	1241.0	W
18	21	669.0	L2	705.0	W
19	29	1465.0	R1	1343.0	I
20	31	605.0	R0.5	637.0	W
21	15	311.0	L1.5	307.0	I
22	33	332.0	R3	408.0	W
23	18	814.0	R1.5	842.0	W
24	11	733.0	L3	533.0	I
25	33	464.0	L1	475.0	W

4.2 Statistical objective analysis

As outlined in the previous section of this study, statistical test were performed on the data that resulted from fitting the 25 test curves to the Weibull fittings and the Iowa type curves. The results for each of these statistical tests are summarized in <Table 3>.

<Table 3> Results of statistical tests

Test	calculated test value	criterion (α = 0.05)
sign test	Z = 1.0	1.64
Wilcoxon test	Z = -1.681	1.64
parametric test	T = 1.96	2.06

4.2.1 Results of sign test

This test failed to adduce evidence that the Iowa type curves produced better fits than the Weibull fitting, since Z value calculated was 1.0. The value of Z would have to have been substantially greater than 1.64 to reject the null hypothesis that indicated that the Weibull fitting produced at least as good or better fits than the Iowa type curve. Although not conclusive in itself, this test provides evidence that the Weibull fitting would produce at least as good fits a population of industrial property when considering areas.

4.2.2 Results of Wilcoxon test

This test is more sensitives than the sign test in reflecting relative magnitude of difference in population central tendencies. This relative magnitude of difference produced a Z value calculated as -1.68. Thus, the test failed to adduce evidence the Iowa type curves produced better fits than the Weibull fitting. The value of Z would have to have been substantially greater than 1.64 to reject the null hypothesis that indicated that the Weibull fitting produced at least as good or better fits than the Iowa type curve. Although not conclusive in itself, those two tests provide evidence that the Weibull fitting would produce at least as good fits a population of industrial property when considering areas.

4.2.3 Results of Parametric test

This test concentrated on and compared mean optimal areas of the Weibull fittings and Iowa curves and had somewhat better discriminating powers than the other two tests. This test also failed to adduce evidence the Iowa type curves produced better fits than the Weibull fitting, since the T value calculated was 1.96. The value of T would have to have been substantially greater than 2.06 to reject the null hypothesis that indicated that the Weibull fitting produced at least as good or better fits than the Iowa type curve. Although not conclusive in itself, those three tests provide strong evidence that the Weibull fitting would produce at least as good fits a population of industrial property when considering areas.

5. Conclusions

The study undertaken in this study was prompted because of the search for a relatively simple mathematical expression which would describe industrial property mortality characteristics. While there are undoubtedly many expressions which could be tried, for theoretical reasons the Weibull distribution was chosen. Because the Iowa type curves are widely used in the field of life estimation they were used as the standard against which to compare the Weibull distribution fitting. A total of 25 test accounts, all with 0% surviving, were prepared for testing. The tests involved fitting of each test curve to each set of curves to determine which set produced the best fit based on area difference between the test curve and the fit curves.

It appears from the results of this study that the Weibull distribution is an appropriate expression for describing industrial property mortality characteristics. The conclusions was based on the following evidences provided by the study :

1. The results of the statistical treatment and analysis of the test curves fit to the Weibull fitting and the Iowa type curves. Virtually all statistical evidence indicated that there is no significant difference between the two methods.
2. Based on the area difference and visual review of the fits, the Weibull fittings produced 10 best fits, the Iowa curves 15 best fits.

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