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단일 분산시스템의 강인안정성 해석

(Stability Robustness of Unified Decentralized Systems)

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요 약

이 논문에서는 델타연산자를 사용하는 단일접근법에 의해 단일분산시스템에 대한 변동경계치의 새로운 결과를 제시하였다. 시스템 불확실성이 존재하는 경우에 대한 새로운 강인 안정성 경계치를 이용하여 단일 분산시스템의 강인 안정성 해석이 수행되었다. 새로운 단일 안정성 경계치는 단일 리아프노프 행렬 방정식에 근거하여 개발되었다. 또한 새로운 단일 경계치가 적용되었을 때 시스템이 그 안정성을 유지함을 나타내었고 예제가 이 제안된 결과를 입증하기 위해 제시되었다.

Abstract

In this paper, new results for perturbation bounds for unified decentralized systems by a unified approach using δ (defined as a shift operator at unified approach) are presented. Robust stability analysis of unified decentralized system is investigated by new robust stability bound under system uncertainties. New unified stability bounds are developed based on the unified Lyapunov matrix equation. It is shown that the system maintains its stability when new unified bounds are applied. Numerical example is presented to illustrate the proposed analysis.

Keywords: Lyapunov matrix inequalities, similarity transformation, bound estimates, stability

I. INTRODUCTION

In recent years, many researchers have considerably explored the stability robustness of a system under perturbations. Since the differences between mathematical models and real systems cause poor system performance, it is important that robust feedback control systems must be designed considering system uncertainties. Thus, this problem, i.e., feedback stabilization of uncertain systems has been one of main topics in control theory[1]-[3]. In this paper, design and analysis of robust feedback control systems are presented using state-space approach for continuous-time, discrete-time, and unified systems. In [1], robust stability bounds are presented for uncertain continuous-time systems. In [4], Kolla et al.presented the results for robust stability bounds of discrete-time systems under perturbations. Now, motivated by [1] and [4], the new robust stability bounds for unified systems with system uncertainties are derived. Also the output feedback control for

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unified systems is not informed yet while the state feedback control for unified systems is reported already[7]. In the literature, no research for this unified bound problem is reported yet. And a research for robustness of output feedback control is not shown yet.

Ⅱ. MAIN RESULTS

In this section, a perturbation bound of unified systems with system uncertainties is proposed. The bound is developed using unified approach, Lyapunov theory and singular value analysis.Let us consider a linear dynamic system that has linear perturbations as follows

$$\rho x(\tau) = (A_\rho + M)\, x(\tau) \tag{1}$$

where $A_\rho \in \Re^{n\times n}$ is an time-invariant asymptotically stable matrix and $M \in \Re^{n\times n}$ is a perturbation matrix. To derive a perturbation bound, we assume that the matrix M can be given in the following form

$$M = \sum_{i=1}^{m} \gamma_i M_i \tag{2}$$

where M_i are constant matrices and γ_i are uncertain parameters. Also we define $P_{1i} \in \Re^{n\times n}$, $P_2 \in \Re^{mn\times mn}$, and $P_{3i} \in \Re^{n\times n}$ as the following form

$$P_{1i} = (A_\rho^T P + PA_\rho + hA_\rho^T PA_\rho)/2, \tag{3}$$

$$P_2 = \begin{bmatrix} (M_1^T PM_1) & (M_1^T PM_2)_s & \dots & (M_1^T PM_m)_s \\ (M_1^T PM_2)_s & (M_2^T PM_2) & \dots & (M_2^T PM_m)_s \\ \vdots & \vdots & & \vdots \\ (M_1^T PM_m)_s & (M_2^T PM_m)_s & \dots & (M_m^T PM_m) \end{bmatrix} \tag{4}$$

$$P_{3i} = (A_\rho^T PM_i)_s \tag{5}$$

where $(N)_s = \dfrac{(N+N^T)}{2}$ and P is the solution of the unified Lyapunov equation

$$A_\rho^T P + PA_\rho + hA_\rho^T PA_\rho + Q_\rho = 0 \tag{6}$$

Inspired by the theorems presented in [1] and [4], the following theorem is proposed:

Theorem 1: By manipulating the unified Lyapunov function and using (2), we have

$$2\sum_{i=1}^{m}|\gamma_i|\sigma_{\max}(P_{1i}) + h\sum_{i=1}^{m}|\gamma_i|^2\sigma_{\max}(P_2) + 2h\sum_{i=1}^{m}|\gamma_i|\sigma_{\max}(P_{3i}) < \sigma_{\min}(Q_\rho) \tag{7}$$

Then, the unified system (1) is stable if

i) Continuous-time case: From (7), for continuous-time, we set $h=0$. Then, we have

$$\sum_{i=1}^{m}|\gamma_i|\sigma_{\max}(P_{1i}) < \frac{\sigma_{\min}(Q_\rho)}{2} \tag{8}$$

ii) Discrete-time case: From (7), we obtain

$$|\gamma_i| < -\frac{\frac{1}{h}\sigma_{\max}(P_{1i}) + \sigma_{\max}(P_{3i})}{m\sigma_{\max}(P_2)} + \left[\left(\frac{\frac{1}{h}\sigma_{\max}(P_{1i}) + \sigma_{\max}(P_{3i})}{m\sigma_{\max}(P_2)}\right)^2 + \frac{\frac{1}{h}\sigma_{\max}(Q_\rho)}{m\sigma_{\max}(P_2)}\right]^{1/2} \tag{9}$$

Proof: Let $V(x,\tau) = x^T P x$ be a Lyapunov function and the unified system is defined as

$$\rho x(\tau) = A_\rho x(\tau) \tag{10}$$

where $\rho = \begin{Bmatrix} d/dt & continuous-time \\ \delta & discrete-time \end{Bmatrix}$.

For the asymptotically stable nominal matrix A_p, the unified Lyapunov equation

$$A_\rho^T P + PA_\rho + hA_\rho^T PA_\rho + Q = 0 \tag{11}$$

gives the uniqu11e symmetric solution $P > 0$ for any given symmetric matrix $Q > 0$. Using [5, pp66],

$$\frac{dV}{d\tau}=(\rho x)^T Px+x^T P(\rho x)^T+h(\rho x)^T P(\rho x)^T$$
$$=(A_\rho x)^T Px+x^T P(A_\rho x)+h(A_\rho x)^T P(A_\rho x)$$
$$=x^T(A_\rho^T P+PA_\rho+hA_\rho^T PA_\rho)x=x^T(-Q)x \quad (12)$$

Now, consider the following linear unified system with linear perturbations:

$$\rho x(\tau)=(A_p+M)x(\tau) \quad (13)$$

Then,

$$\frac{dV}{d\tau}=$$
$$[(A_\rho+M)x]^T Px+x^T P[(A_\rho+M)x]$$
$$+h[(A_\rho+M)x]^T P[(A_\rho+M)x]$$
$$=x^T(A_\rho+M)^T Px+x^T P(A_\rho+M)x$$
$$+hx^T(A_\rho+M)^T P(A_\rho+M)x$$
$$=x^T[A_\rho^T P+PA_\rho+hA_\rho^T PA_\rho+M^T P+PM$$
$$+h(A_\rho^T PM+M^T PA_\rho+M^T PM)]x \quad (14)$$

From (11), (14) can be rewritten as

$$\frac{dV}{d\tau}=x^T[M^T P+PM+h\left(A_\rho^T PM+M^T PA_\rho+M^T PM\right)-Q]x \quad (15)$$

Then, $V(x)$ is a Lyapunov function and the system (13) is stable if

$$M^T P+PM+h(A_\rho^T PM+M^T PA_\rho+M^T PM)-Q<0 \quad (16)$$

Equation (16) is satisfied if

$$\sigma_{\max}[M^T P+PM+h(A_\rho^T PM+M^T PA_\rho+M^T PM)]<\sigma_{\min}(Q) \quad (17)$$

Using (17), (3) - (6), and $M=\sum_{i=1}^{m}\gamma_i M_i$, we have (7).

From (7),

$$0>h\sum_{i=1}^{m}|k_i|^2\sigma_{\max}(P_2)$$
$$+2\sum_{i=1}^{m}|k_i|[\sigma_{\max}(P_{1i})+h\sigma_{\max}(P_{3i})]-\sigma_{\min}(Q)$$
$$=\sum_{i=1}^{m}|k_i|^2\sigma_{\max}(P_2)$$
$$+2\sum_{i=1}^{m}|k_i|\left[\frac{1}{h}\sigma_{\max}(P_{1i})+\sigma_{\max}(P_{3i})\right]-\frac{\sigma_{\min}(Q)}{h} \quad (18)$$

Considering the two roots of the quadratic equation formed with the equality in (18), we have (8) for $M\neq 0,\ \sigma_{\max}(M)>0$. The bound (8) is exactly same as that derived by [1] due to the definition of unified systems. This completes the proof.

1) System Description

Let the system be a decentralized singularly perturbed system given by

$$\rho x(\tau)=A_\rho x(\tau)+\sum_{i=1}^{k}B_{\rho i}u_i(\tau)$$
$$y_i(\tau)=C_{\rho i}x(\tau) \quad (19)$$

which has the following form for singularly perturbed unified systems

$$\rho x(\tau)=A_{\rho 11}x(\tau)+A_{\rho 12}\zeta(\tau)+\sum_{i=0}^{k}B_{\rho 1i}u_i(\tau)$$
$$\rho\mu\zeta(\tau)=A_{\rho 21}x(\tau)+A_{\rho 22}\zeta(\tau)+\sum_{i=0}^{k}B_{\rho 2i}u_i(\tau) \quad (20)$$
$$y_i(\tau)=C_{\rho 1i}x_i(\tau)+C_{\rho 2i}\zeta(\tau)$$

where $x\in\Re^n$ and $\zeta\in\Re^m$ are state vectors, $u_i\in\Re^r$ is a control vector, and A_p, B_{pi}, C_{pi} are the constant matrices of appropriate dimension. μ is a small positive parameter, $0<\mu\ll 1$. In this decentralized unified system, A_{p22} is assumed to be a nonsingular matrix, $B_{p1i}=0$, and $C_{p1i}=0$. Also, the associated performance index with system (20) is given by [6]

$$J_i=\frac{1}{2}\int_0^\infty[x_i^T(\tau)Q_{\rho 1i}x_i(\tau)+\zeta_i^T(\tau)Q_{\rho 2i}\zeta_i(\tau)+u_i^T(\tau)R_{\rho i}u_i(\tau)]\ d\tau \quad (21)$$

where

$$Q_{\rho i}=\begin{bmatrix} Q_{\rho 1i} & 0 \\ 0 & Q_{\rho 2i}\end{bmatrix}=Q_{\rho i}^{T} \text{ and } R_{\rho i}=R_{\rho i}^{T}\geq 0 .$$

2) Robustness of State Feedback Control

The singularly perturbed system in (20) can be simplified as shown in [8]. Hence, the new reduced-order model of each subsystem has the following form

$$\begin{aligned} \rho\, x_{si}(\tau) &= A_{\rho 0i}x_{si}(\tau)+B_{\rho 0i}u_{si}(\tau) \\ y_{si}(\tau) &= C_{\rho 0i}x_{si}(\tau)+D_{\rho 0i}u_{si}(\tau) \end{aligned} \tag{22}$$

where

$$\begin{aligned} A_{\rho 0i} &= A_{\rho 11}-A_{\rho 12}A_{\rho 22}^{-1}A_{\rho 21} \\ B_{\rho 0i} &= B_{\rho 1i}-A_{\rho 12}A_{\rho 22}^{-1}B_{\rho 2i} \\ C_{\rho 0i} &= C_{\rho 1i}-C_{\rho 2i}A_{\rho 22}^{-1}A_{\rho 21} \\ D_{\rho 0i} &= -C_{\rho 2i}A_{\rho 22}^{-1}B_{\rho 2i} \end{aligned}$$

Also by applying $\mu=0$ to (21), the performance index becomes the reduced LQ problem given by

$$\begin{aligned} J_{i} = \frac{1}{2}\mathop{S}_{0}^{\infty}[\, x_{si}^{T}(\tau)Q_{\rho 0i}x_{si}(\tau)+2x_{si}^{T}(\tau)E_{\rho 0i}u_{si}(\tau) \\ +u_{si}^{T}(\tau)R_{\rho 0i}u_{si}(\tau)] \quad d\tau \end{aligned} \tag{23}$$

where

$$\begin{aligned} Q_{\rho 0i} &= Q_{\rho 1i}+A_{\rho 21}^{T}(A_{\rho 22}^{-1})^{T}Q_{\rho 2i}A_{\rho 22}^{-1}A_{\rho 21} \\ E_{\rho 0i} &= A_{\rho 21}^{T}(A_{\rho 22}^{-1})^{T}Q_{\rho 2i}A_{\rho 21}^{-1}B_{\rho 2i} \\ R_{\rho 0i} &= R_{\rho i}+B_{\rho 2i}^{T}(A_{\rho 22}^{-1})^{T}Q_{\rho 2i}A_{\rho 22}^{-1}B_{\rho 2i} \end{aligned}$$

The preliminary feedback control for (22) and (23) is given by

$$u_{si}(\tau)=-R_{\rho 0i}^{-1}E_{\rho 0i}^{T}x_{si}(\tau)+v_{i}(\tau) \tag{24}$$

to reduce (23) to standard format with no cross terms. Applying (24) to (22), we have

$$\rho \quad x_{si}(\tau)=A_{\rho 01i}x_{si}(\tau)+B_{\rho 0i}v_{i}(\tau) \tag{25}$$

and

$$J_{i}=\frac{1}{2}\mathop{S}_{0}^{\infty}[x_{si}^{T}(\tau)Q_{\rho 01i}x_{si}(\tau)+v_{i}^{T}(\tau)R_{\rho 0i}v_{i}(\tau)] \quad d\tau \tag{26}$$

where

$$\begin{aligned} A_{\rho 01i} &= A_{\rho 0i}-B_{\rho 0i}R_{\rho 0i}^{-1}E_{\rho 0i}^{T} \\ Q_{\rho 01i} &= Q_{\rho 0i}-E_{\rho 0i}R_{\rho 0i}^{-1}E_{\rho 0i}^{T} \end{aligned}$$

Then, the optimal feedback control for LQ problem is given by [6]

$$\begin{aligned} u_{si}(\tau) &= -(R_{\rho 0i}+hB_{\rho 0i}^{T}K_{i}B_{\rho 0i})^{-1} \\ &\quad \cdot(B_{\rho 0i}^{T}K_{i}(I+hA_{\rho 0i})+E_{\rho 0i}^{T})x_{si}(\tau) \\ &= G_{\rho 0i}x_{si}(\tau) \end{aligned} \tag{27}$$

where K_{i} is a stabilizing solution of the algebraic Riccati equation.

$$\begin{aligned} 0 &= A_{\rho 0i}^{T}K_{i}+K_{i}A_{\rho 0i}+hA_{\rho 0i}^{T}K_{i}A_{\rho 0i}+Q_{\rho 0i} \\ &\quad -[B_{\rho 0i}^{T}K_{i}(I+hA_{\rho 0i})+E_{\rho 0i}]^{T}(R_{\rho 0i}+hB_{\rho 0i}^{T}K_{i}B_{\rho 0i})^{-1} \\ &\quad \cdot[B_{\rho 0i}^{T}K_{i}(I+hA_{\rho 0i})+E_{\rho 0i}] \\ &= (I+hA_{\rho 0i})^{T}K_{i}(I+hA_{\rho 0i})-K_{i}+hQ_{\rho 0i} \\ &\quad -h[B_{\rho 0i}^{T}K_{i}(I+hA_{\rho 0i})+E_{\rho 0i}]^{T} \\ &\quad \cdot(R_{\rho 0i}+hB_{\rho 0i}^{T}K_{i}B_{\rho 0i})^{-1}[B_{\rho 0i}^{T}K_{i}(I+hA_{\rho 0i})+E_{\rho 0i}] \end{aligned} \tag{28}$$

Using (27) into (22), we have the following stable closed-loop system

$$\rho x_{si}(\tau)=(A_{\rho 0i}+B_{\rho 0i}G_{\rho 0i})x_{si}(\tau) \tag{29}$$

The objective here is to find the range of $|\gamma_{i}|$ that allows the unified feedback system to be stable.

3) Robustness of Output Feedback Control

Now, we focus on the analysis of output feedback system under perturbations. The design procedure for the optimal output feedback control for continuous and discrete-time systems is presented by [11]. Motivated by [11], the optimal output feedback control for unified systems is developed by [7]. In [7], if the admissible control is given by

$$u_i(\tau) = -F_{\rho i} y_i(\tau) \tag{30}$$

By using (30) into (19), we have the closed-loop system equation

$$x_i(\tau) = (A_\rho - B_{\rho i} F_{\rho i} C_{\rho i}) x_i(\tau) = A_{\rho ci} x_i(\tau) \tag{31}$$

Then, the cost function can be of the form

$$J_i = \frac{1}{2} \int_0^\infty [x_i^T(\tau)[Q_{\rho i} + C_{\rho i}^T F_{\rho i}^T R_{\rho i} F_{\rho i} C_{\rho i}] x_i(\tau)] d\tau \tag{32}$$

Now, the design problem is to choose the gain F_{pi} so that the cost J_i is minimized with respect to (31). Then, the closed-loop Lyapunov equation is given by

$$\begin{aligned} \varphi_i \equiv S_i A_{\rho ci} + A_{\rho ci}^T S_i + h A_{\rho ci}^T S_i A_{\rho ci} + Q_{\rho i} \\ + C_{\rho i}^T F_{\rho i}^T R_{\rho i} F_{\rho i} C_{\rho i} = 0 \end{aligned} \tag{33}$$

Notice that since, in many design problems, the initial condition $x_i(0)$ is not known, it is usual to use the expected value of J_i instead of J_i itself. Thus, the cost can be rewritten as

$$E\{J_i\} = \frac{1}{2} E\{x_i^T(0) S_i x_i(0)\} = \frac{1}{2} tr(S_i \Omega_i) \tag{34}$$

where $\Omega_i \in \Re^{n \times n}$ is symmetric matrix and defined by

$$\Omega_i = E\{x_i(0) \quad x_i^T(0)\}.$$

In order to solve this problem, the Lagrange multiplier approach is introduced as

$$\mathrm{H}_i = tr(S_i \Omega_i) + tr(\varphi_i L_i) \tag{35}$$

where $L_i \in \Re^{n \times n}$ is a symmetric matrix that should be determined. Now, our constrained optimal control problem becomes the simpler problem of minimizing Lagrange multiplier H_i with no constraints. Then, we should solve the following coupled equations with respect to S_i, L_i, F_{pi}. To solve this, we first set the partial derivatives of H_i equal to zero.

$$\begin{aligned} 0 = \frac{\partial \mathrm{H}_i}{\partial L_i} = S_i A_{\rho ci} + A_{\rho ci}^T S_i + h A_{\rho ci}^T S_i A_{\rho ci} \\ + Q_{\rho i} + C_{\rho i}^T F_{\rho i}^T R_{\rho i} F_{\rho i} C_{\rho i} \end{aligned} \tag{36}$$

$$0 = \frac{\partial \mathrm{H}_i}{\partial S_i} = A_{\rho ci} L_i + L_i A_{\rho ci}^T + h A_{\rho ci} A_{\rho ci}^T + \Omega_i \tag{37}$$

$$\begin{aligned} 0 = \frac{1}{2} \frac{\partial \mathrm{H}_i}{\partial F_{\rho i}} = R_{\rho i} F_{\rho i} C_{\rho i} L_i C_{\rho i}^T \\ - B_{\rho i}^T S_i (I + h A_{\rho ci}) L_i C_i^T \end{aligned} \tag{38}$$

where $A_{\rho ci} = A_\rho - B_{\rho i} F_{\rho i} C_{\rho i}$, $\Omega_i = E\left\{x_i(0) \quad x_i^T(0)\right\}$.

From (38), we have the optimal output feedback gain

$$F_{\rho i} = R_{\rho i}^{-1} B_{\rho i}^T S_i (I + h A_{\rho ci}) L_i C_{\rho i}^T (C_{\rho i} L_i C_{\rho i}^T)^{-1} \tag{39}$$

In the literature, the procedure of obtaining the optimal output feedback gain for unified systems is not reported yet. Hence, this procedure and gain (81) are new. Notice that by setting the sampling period $h = 0$ the output feedback gain for continuous-time systems is obtained.

Then, using the control law $u_i(\tau) = -F_{\rho i} y_i(\tau)$, the system (19) becomes the closed-loop system

$$x_i(\tau) = (A_\rho - B_{\rho i} F_{\rho i} C_{\rho i}) x_i(\tau) = A_{\rho ci} x_i(\tau) \tag{40}$$

To analyze the stability of the output feedback system, let the perturbed system be described by

$$\rho x_i(\tau) = (A_{\rho ci} + M) x_i(\tau) \tag{41}$$

where

$$A_{ci} = A_\rho - B_{\rho i} F_{\rho i} C_{\rho i}, M = \begin{bmatrix} \gamma_1 & 0 & \cdots & 0 \\ 0 & \gamma_2 & \cdots & 0 \\ \vdots & \vdots & \gamma_3 & \vdots \\ 0 & 0 & \cdots & \gamma_4 \end{bmatrix}$$

The focus here is to investigate the range of $|\gamma|$ such that the unified output feedback system maintains its stability.

Ⅲ. EXAMPLE

Let the system be a decentralized singularly perturbed system given by

$$\rho\, x(\tau) = A_\rho x(\tau) + \sum_{i=1}^{k} B_{\rho i} u_i(\tau)$$
$$y_i(\tau) = C_{\rho i} x(\tau) \tag{42}$$

where

$$A_\rho = \begin{bmatrix} A & continuous-time \\ A_\delta & discrete-time \end{bmatrix},\ B_\rho = \begin{bmatrix} B \\ B_\delta \end{bmatrix},\ C_\rho = \begin{bmatrix} C \\ C_\delta \end{bmatrix}$$

(1) Continuous-time case: By the definition of the unified system, when $h=0$, system (42) becomes a continuous system. Then, the correspondingsystem matrices of the continuous-time case are given by [8]

$$A = \begin{bmatrix} 1 & 0 & -0.64 & 0.02 \\ 0 & -0.5 & 0.345 & -1 \\ 200 & -524 & -265 & 0 \\ 500 & 200 & 0 & -700 \end{bmatrix},\ B_{i=1} = \begin{bmatrix} 0 \\ 0 \\ 81 \\ 0 \end{bmatrix},$$

$$B_{i=2} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 351 \end{bmatrix},\ C_{i=1} = \begin{bmatrix} 0 & 0 & -1.4 & 0 \end{bmatrix},$$

$$C_{i=2} = \begin{bmatrix} 0 & 0 & 0 & -1.2 \end{bmatrix}$$

i) Robustness of State Feedback Control

The numerical results for state feedback control are the same as those presented by [8]. Hence, those results are omitted here.

ii) Robustness of Output Feedback Control

The numerical results for output feedback control are the same as those presented in the previous example for the continuous-time system. Hence, those results are omitted here.

(2) Discrete-time case: By the definition of the unified system, when $h=0.6$, system (42) becomes discrete-time case of the unified system. Then, the corresponding system matrices of the discrete-time case are obtained as

$$A_\delta = \begin{bmatrix} 0.3990 & 0.8517 & -0.0038 & -0.0012 \\ -0.3060 & -0.9473 & 0.0017 & -0.0010 \\ 2.1582 & -0.7943 & -1.6730 & 0.0012 \\ 1.3875 & 0.8130 & -0.0023 & -1.6678 \end{bmatrix},$$

$$B_{\delta,i=1} = \begin{bmatrix} -0.1583 \\ 0.0763 \\ 0.2408 \\ -0.0910 \end{bmatrix},\ B_{\delta,i=2} = \begin{bmatrix} -0.1037 \\ -0.2895 \\ 0.4926 \\ 0.6795 \end{bmatrix}$$

$$C_{\delta,i=1} = \begin{bmatrix} 0 & 0 & -1.4 & 0 \end{bmatrix},\ C_{\delta,i=2} = \begin{bmatrix} 0 & 0 & 0 & -1.2 \end{bmatrix}$$

i) Robustness of State Feedback Control

To investigate the robust stability of (42), the subsystem is considered. The subsystem is given by

$$A_{\delta 0i} = \begin{bmatrix} -0.0937 & 1.6460 \\ -1.6922 & -0.0957 \end{bmatrix},\ B_{\delta 0i} = \begin{bmatrix} -0.3987 \\ 0.1679 \end{bmatrix}$$

$$C_{\delta 0i} = \begin{bmatrix} 0.4121 & -0.6652 \end{bmatrix},\ D_{\delta 0i} = 0.2015$$

The system poles are obtained by

$$\lambda_1 = -0.0947 + 1.6689i$$
$$\lambda_2 = -0.0947 - 1.6689i$$

The feedback control gain to stabilize the system is obtained by

$$G_{\delta 0i} = \begin{bmatrix} -3.9538 & -0.5695 \end{bmatrix} \tag{43}$$

Hence, we have the closed-loop system

$$\delta x_{si}(k) = \begin{bmatrix} -1.67 & 1.419 \\ -1.0283 & 0 \end{bmatrix} x_{si}(k) \tag{44}$$

which has the following system poles

$$\lambda_1 = -0.8350 + 0.8729i$$
$$\lambda_2 = -0.8350 - 0.8729i$$

To examine the robustness of the feedback control, we set the control in (43) as follows

$$G_{\delta 0i} = \begin{bmatrix} -3.9538 + \gamma_1 & -0.5695 + \gamma_2 \end{bmatrix}$$

which yields the perturbed system of the form

$$\delta x_{si}(\tau) = (A_{\delta c0i} + M)x_{si}(\tau) \tag{45}$$

where $A_{\delta c0i}$ is given in (34) and M is obtained by

$$M = \begin{bmatrix} -0.3987\gamma_1 & -0.3987\gamma_2 \\ 0.1679\gamma_1 & 0.1679\gamma_2 \end{bmatrix} \tag{46}$$

Applying (9), the perturbation bounds are found

$$|\gamma_1| < 1.0971 \text{ and } |\gamma_2| < 1.5270 \tag{47}$$

Using (47) into (46), the perturbed system matrix is obtained

$$M = \begin{bmatrix} -0.4374 & -0.6088 \\ 0.1842 & 0.2564 \end{bmatrix} \tag{48}$$

Using (48) into full-order system, we obtain the following eigenvalues

$$\lambda_1 = 0.8321$$
$$\lambda_2 = 0.2987$$
$$\lambda_3 = 0.0104$$
$$\lambda_4 = 0.0000$$

These eigenvalues show that the state feedback system under perturbations is stable.

ii) Robustness of Output Feedback Control

Applying output feedback control to stabilize the system (42), the stable system matrix is obtained as

$$A_{\delta ci} = \begin{bmatrix} -0.0119 & -0.0413 & -0.0069 & -0.0157 \\ 0.0393 & -0.0005 & -0.2250 & 0.4141 \\ -0.0356 & 0.0012 & -0.6340 & 0.2652 \\ 0.1128 & -0.0020 & -0.1317 & -1.0572 \end{bmatrix} \tag{49}$$

The range of γ_i should be assigned to hold stability of the system as follows

$$|\gamma_1| < 0.0682 \quad |\gamma_2| < 0.0695$$
$$|\gamma_3| < 0.0745 \quad |\gamma_4| < 0.0746$$

Applying these perturbations to the matrix (49), we have

$$A_{\delta ci} = \begin{bmatrix} 0.0563 & -0.0413 & -0.0069 & -0.0157 \\ 0.0393 & 0.0690 & -0.2250 & 0.4141 \\ -0.0356 & 0.0012 & -0.5594 & 0.2652 \\ 0.1128 & -0.0020 & -0.1317 & -0.9826 \end{bmatrix}$$

and its eigenvalues are given by

$$\lambda_1 = -0.8715$$
$$\lambda_2 = -0.6693$$
$$\lambda_3 = 0.0621 + 0.0596i$$
$$\lambda_4 = 0.0621 - 0.0596i$$

It is shown that the investigated range of perturbations maintains the stability of the system under perturbations. These perturbation bounds are not in explicit range, but can keep the system stability as long as the perturbations are inside this range.

Ⅳ. CONCLUSION

In this paper, investigation of stability robustness of unified systems is mainly discussed. The goal of this paper is to keep stability of the system with the stabilizing controller when perturbations are added to the closed-loop system. The perturbation bounds are obtained based on the solutions of the unified Lyapunov matrix equation (6). Numerical results are presented by an example. From the example, it is shown that the system stability is still maintained even if perturbations are applied within the obtained range.

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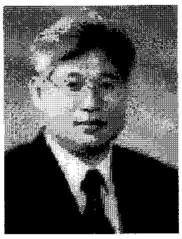

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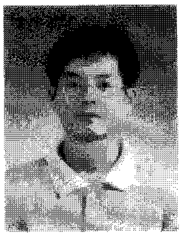

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