Phase Modulation Homodyne Interferometer with a 10-pm Resolution Using a Tunable Laser Diode

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We propose a new displacement measurement method using a phase modulation homodyne interferometer and a tunable laser diode as a light source to determine an arbitrary length with a resolution in the order of 10 pm. In the proposed instrument, the displacement of a movable mirror in the interferometer can be converted to a frequency shift of the tunable laser diode. We discuss the principles of the proposed method, the instrumentation, and the experimental results, and compare the proposed method with two commercial displacement sensors. The commercial sensors used are a heterodyne interferometer, the interpolation error of which is also measured, and a capacitive sensor.

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NOMENCLATURE

L = optical path difference
N = mode number
f = optical frequency
λ = optical wavelength
c = speed of light
n = refractive index

1. Introduction

The science of nanotechnology has progressed rapidly, and a new method for measuring length or displacement with picometer resolution will be required in the near future.[1] At the present time, laser interferometers conform to an industrial standard and many commercial interferometers are used. It is, however, very difficult to determine an arbitrary length or displacement with nanometer accuracy using a commercial interferometer, as they suffer from the problem of nonlinearity in the fringe interpolation.[2–4]

A phase modulation homodyne interferometer (PMHI), a type of Michelson interferometer, was proposed by Basile et al. in 1991.[5] In the PMHI, the phase of an incident light is modulated using an electrooptic modulator (EOM), and the output signal of a photodiode at the end of the interferometer is demodulated with the modulation signal, using a lock-in amplifier (LIA). Using this configuration, it is possible to determine an optical path difference L of wavelength $\lambda \times$ integer N (mode number) with an accuracy of 10 pm or less using the null method. This is because the lock-in demodulation can transfer a dark fringe (in this case, $L = \lambda \times N$) into a null cross-point of the lock-in output signal. It is impossible, however, to determine an arbitrary length or displacement using this method. We propose a new displacement measurement method using the PMHI and a tunable laser diode (TLD) as a light source to determine an arbitrary length with a resolution in the order of 10 pm. In the proposed instrument, the displacement of a movable mirror (target) in the interferometer can be converted to a frequency shift of the TLD. The features of the proposed system are as follows:

(1) Displacement of the target mirror can be measured by an optical frequency change of the TLD.
(2) No periodic errors are caused by the phase interpolation.
(3) The principles of the system ensure the traceability of the meter.

Research on measuring a displacement with a sub-nanometer resolution or better using Fabry–Perot optical cavities and tunable light sources has been performed by several research groups.[6–9] Haitjema et al.[6] measured the change in the length of the optical resonator by applying the principles of the Pound–Drever–Hall method,[10] which is the technique of stabilizing the laser frequency using the resonance condition of an optical resonator. Howard et al.[7] measured the atomic height of a silicon surface by combining the above method and a scanning tunneling microscope (STM). Lawall[8] has tried to improve resolution and expand the measurement into the 10-mm range using the method of Bay et al.,[11] and Bitou et al.[9] have expanded the measurement range of this method using an optical frequency comb[12] in order to measure optical frequency. However, since these methods require fine and high-cost Fabry–Perot cavity mirrors and precise alignment, it is difficult to extend them to commercial application. Conversely, conventional Michelson interferometers do not require special optics. Some research groups,[13,14] including ours,[14] have attempted to measure displacements with a resolution in the order of 10 pm using conventional Michelson interferometers and tunable light sources. Silver et al.[13] proposed a method of measuring the optical path difference as an optical frequency change using a polarization encoding and the null method.

In this paper, we discuss the principles of the proposed method, the instrumentation, and the experimental results. In the experiments, we compare the proposed method with two commercial displacement sensors, a capacitive sensor[15] and a heterodyne interferometer,[16] as well as measure the interpolation error of the commercial interferometer.

2. Principles and Instrumentation

Since interference signals of the Michelson interferometer always have an offset part, it is difficult to accurately determine a bright (or dark) fringe in the signals. However, the application of an EOM and lock-in detection to the homodyne interferometer allows the determination of the optical path difference L of wavelength $\lambda \times$ integer N with picometer resolution using the null method.[5]

Figure 1 shows the system of the phase modulation homodyne interferometer. This system includes a frequency-stabilized laser (FSL), an EOM (Pockels cell), a polarized beam splitter (PBS), quarter-wave plates (QWPs), corner cube prisms (CCs), a photodetector (PD), a lock-in amplifier (LIA), an oscillator, and a polarizer (P). The light beam from the FSL passing through the EOM is divided into the reference and the measurement beams by the PBS. The light beams reflected from the fixed and moving CCs are also mixed by the PBS. The mixed beam passing through the P then enters the PD. The light from the FSL is linearly polarized, with a polarization angle 45° from the vertical. Its Jones vector can be written as

$$\vec{E}_0 = \begin{pmatrix} E_x \\ E_y \end{pmatrix}. \tag{1}$$

The light passing through the EOM and the interferometer is written as

$$\vec{E}_1 = \begin{pmatrix} 1 & 0 \\ 0 & e^{i(\delta + a\cos(\omega t))} \end{pmatrix} \begin{pmatrix} E_x \\ E_Y \end{pmatrix} = \begin{pmatrix} E_x \\ E_y e^{i(\delta + a\cos(\omega t))} \end{pmatrix}, \tag{2}$$

where δ, a, and ω are the phase difference for the optical path difference of the interferometer, the modulation depth of the EOM, and the modulation frequency, respectively. The intensity of the light detected in the PD is written as

$$\begin{aligned} I_1 &= |E_1|^2 \\ &= \frac{1}{2}\{E_x^2 + E_y^2 + 2E_xE_y\cos(\delta + a\cos(\omega t))\} \\ &= \frac{1}{2} I_0 \{1 + \Gamma \cos(\delta + a\cos(\omega t))\}, \end{aligned} \tag{3}$$

where

$$I_0 = E_x^2 + E_y^2, \tag{4}$$

$$\Gamma = \frac{2E_xE_y}{E_x^2 + E_y^2}. \tag{5}$$

The output of the interferometer shown in equation (3) is synchronously detected by the LIA. Then, the output of the LIA is written as

$$\begin{aligned} I_{llock-in} &= \frac{1}{2\pi}\int\left[\frac{1}{2} I_0\{1 + \Gamma\cos(\delta + \cos(\omega t))\}\right]\cos(\omega t)\cdot dt(\omega t) \\ &= \frac{1}{2} I_0 \Gamma \sin(\delta) \times J_1(a), \end{aligned} \tag{6}$$

where $J_1(a)$ is a first-order Bessel function.

The newly proposed system is shown in Fig. 2. The system includes the PMHI, the TLD as a light source of the PMHI, the FSL, a servo unit to tune the TLD, and a beat-signal detection unit. The beat-signal detection unit has a high-speed photodetector (HSPD) and a frequency counter, and is used to observe the beat signals between the TLD and the FSL. At null cross-points of the PMHI, we obtain

$$L = \frac{c}{2fn} N, \tag{7}$$

where c, n, and N are the speed of light, the refractive index, and the mode number, respectively. When the moving CC is displaced, the optical frequency of the TLD is controlled to maintain the same null cross-point, and the equation becomes

$$\frac{\Delta L}{L} = -\frac{\Delta f}{f} + \frac{\Delta N}{N}, \tag{8}$$

where ΔL, Δf, and ΔN are changes of the optical path, the frequency, and the mode number, respectively. In this case, it is assumed that $\Delta n = 0$. In the system, since one of the null cross-points must be maintained by adjusting the frequency of the TLD, the change in the mode number (= ΔN) must be null. Then ΔL, can be determined from

$$\frac{\Delta L}{L} = -\frac{\Delta f}{f}. \tag{9}$$

Δf can be observed from the beat signal between the TLD and the FSL. Table 1 shows the specifications of each instrument shown in Fig. 2. In the system, the moving stage is linked with a displacement sensor. The measurement resolution of the system is limited by the line width of the TLD, which is related to the measurement of Δf, and is approximately 42 pm, when L, f, and Δf are 100 mm, 473 THz (λ = 633 nm), and Δf = 200 kHz, respectively.

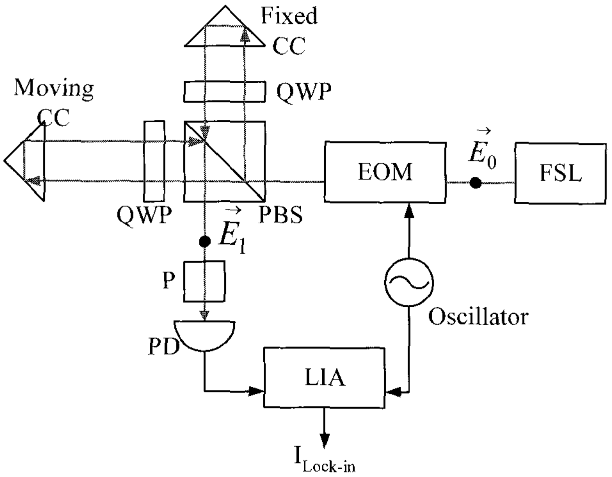


Fig. 1 Phase modulation homodyne interferometer

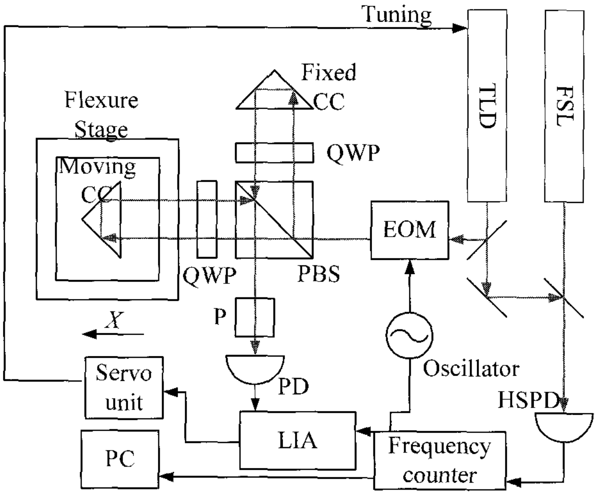


Fig. 2 Configuration of the proposed system

Table 1 Specifications of instruments in the proposed system

Instrument	Specification
TLD	New Focus, Inc. 6304
	Wavelength scan range: 632-640 nm
FSL	Spectra Physics, Inc. 117A
	Frequency: 473612537 MHz
EOM	New Focus, Inc. 4002M
	Modulation frequency range: DC - 100 MHz
Flexure stage	Resonance frequency: 2 kHz
	Pitch & yaw: <0.2 arcsec
	Working range: 1 μm
LIA	Modulation frequency: 600 kHz
	Cut off frequency: 2 kHz
	Mixer: R & K Co, Ltd. M1-X
	Low-pass filter: NF Electronic Instrument, Inc. E3201S
PD	Hamamatsu Photonics K.K. C5658
	Frequency range: 0.1–1 GHz
Frequency counter	Agilent Technologies, Inc. 53132A with 12.4 GHz option
	Frequency range : 0.02–12.4 GHz
HSPD	New Focus, Inc. 1434
	Frequency range : DC - 25 GHz

3. Experiment

3.1 Determination of the optical path difference *L*

L must be known prior to the experiment in order to determine ΔL for the system. *L* can be derived from the free spectral range (FSR), which is equal to the frequency difference between neighboring modes (null points). The FSR is written as

$$FSR = \frac{c}{2nL}. \tag{10}$$

By scanning the frequency of the TLD, the FSR can be measured in the LIA signal as shown in Fig. 3. In the figure, the frequency shift between neighboring null cross-points can be regarded as the FSR. From Fig. 3, the averaged FSR = 1.01528 GHz, and

$$L = \frac{2.9979\times10^{8}}{2\times1.00027\times1.01528\times10^{9}} = 147.60\,[mm], \tag{11}$$

where, after taking measurements of the temperature, humidity, pressure, and CO_2 content, the refractive index of air is compensated using Ciddor's equation.[17] Thus, the uncertainty U_L in the determination of *L* was approximately 84.8 μm and the predominant cause of the uncertainty was fluctuations in the FSR measurement (≒ 600 kHz).

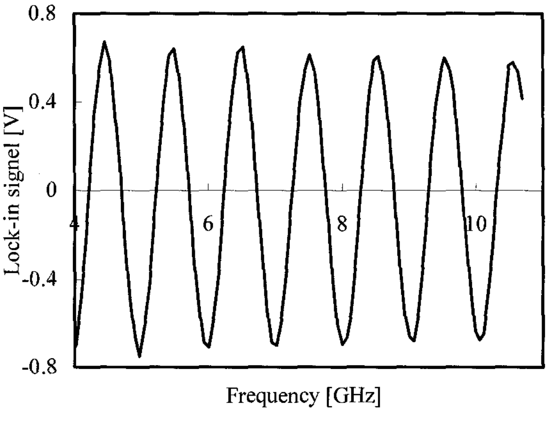


Fig. 3 Measurement of the free spectral range

3.2 Comparison measurement

The proposed system was compared with two commercial displacement sensors, a capacitive sensor[15] and a heterodyne interferometer.[16] In the comparisons, the refractive index of air was again compensated using Ciddor's equation. Figure 4 shows the experimental configuration used for comparison with a capacitive sensor. Figures 5(a) and (b) show a comparison of the experimental results.

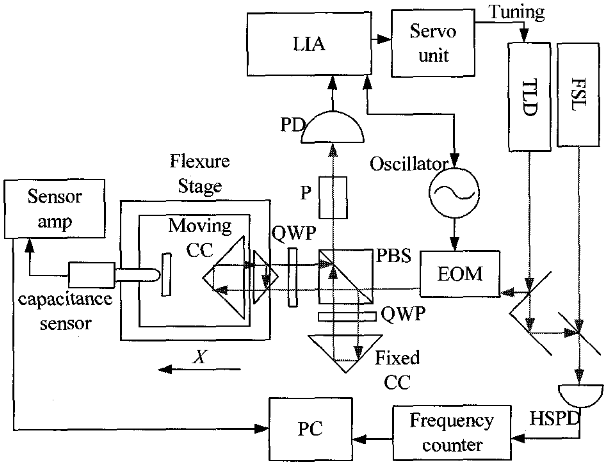
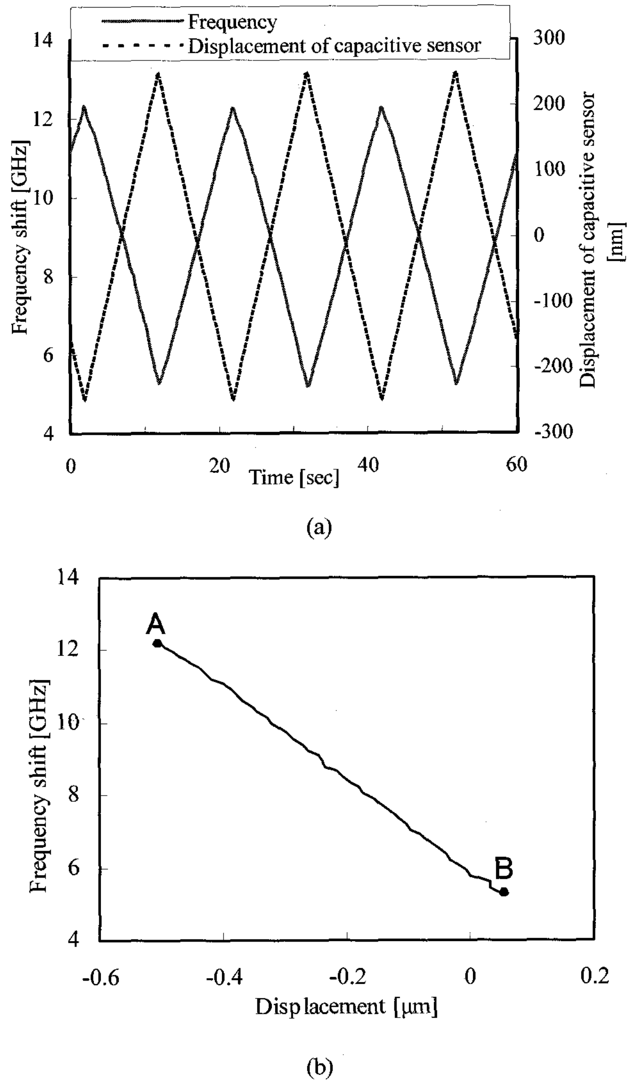


Fig. 4 Configuration for comparing measurements between the capacitance sensor and the proposed system



(a)

(b)

Fig. 5 Comparison results between the capacitive sensor and the proposed system. (a) Time variation of the both signals. (b) Frequency shift and displacement of the capacitive sensor

Figure 5(a) presents the relationship between the measured beat-signal frequency (equal to the frequency shift of the TLD) and the displacement measured by the capacitive sensor. Figure 5(a) shows that the displacement change can be converted to a frequency shift of the TLD. Figure 5(b) shows the relationship between the frequency shift and the displacement measured by the capacitive sensor. The measured frequency shift between points A and B in Fig. 5(b) was converted to 0.5406 μm using equations (9) and (11). However, the measured displacement by the capacitive sensor was 0.5524 μm, giving an approximate difference of 2%. This discrepancy might have been caused by the imperfection in the alignment of the capacitive sensor.

The proposed system was then compared with a commercial heterodyne interferometer[16] to investigate the periodic interpolation error. The experimental configuration is shown in Fig. 6. In the configuration, the moving CCs of the PMHI and the heterodyne interferometer are set on the same X-axis stage. When the X-axis stage is scanned, both interferometers can synchronously measure the displacement of the stage. The lateral diameters of both beams of the interferometers were approximately 1.5 mm, which was enough to eliminate the Abbe offset. The antiparallelism between the two beams was adjusted to less than 0.03°,and thus, the cosine error was estimated to be in the order of less than 10^{-7}. In the comparison measurement, the scanning frequency and the range for the X-axis stage were 0.05 Hz and 800 nm, respectively. To reduce the effect from the Δn fluctuation, the experimental system was set in a heat-insulated and air-sealed cell. Fluctuations of temperature and air pressure were estimated to be less than 5 mK and 3 Pa, respectively, during the 110 s that the experiment ran. Figure 7(a) shows the time variations of the displacement using the heterodyne interferometer and the frequency change of the proposed system. Figure 7(b) shows the relationship between the displacement from the heterodyne interferometer and the frequency change of the TLD. Using equations (9) and (11), a periodic error (equal to the output of the heterodyne interferometer – the output of the proposed method) was obtained as shown in Fig. 8. In this figure, a half-wavelength periodicity and a periodic amplitude of approximately ±2.5 nm are clearly recognized. Since ΔL can be written in terms of f, Δf, n, Δn, N, and ΔN, the measurement uncertainty of $\Delta L(U_{\Delta L})$ should be related to values of f, Δf, Δn, and ΔN. However, the system can maintain the same null point ($\Delta N = 0$). Therefore, the causes of the measurement uncertainty ($U_{\Delta L}$: the combined standard uncertainty) are a combination of the measurement uncertainty of f (U_f: 0.093 nm), Δf ($U_{\Delta f}$: 0.10 nm), Δn ($U_{\Delta n}$: 0.20 nm), and L (U_L: 0.42 nm). The combined standard uncertainty ($U_{\Delta L}$) and the expanded measurement uncertainty ($2U_{\Delta L}$: $k = 2$) are approximately 0.48 nm and 0.96 nm, respectively. The predominant causes of the uncertainty were in the determination of L and the fluctuation in Δn. We believe that these uncertainties can be reduced using the direct determination method for L with double tunable lasers as light sources for one PMHI.

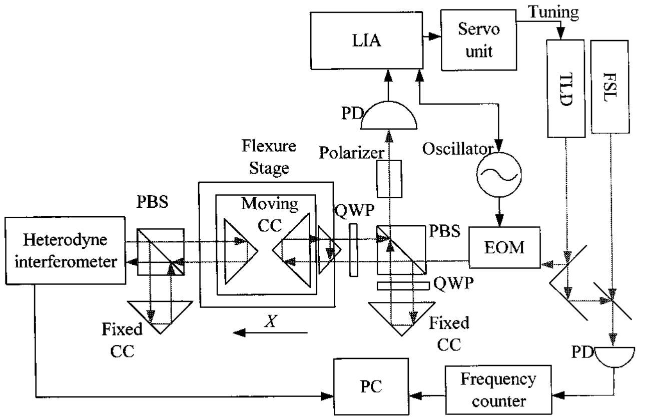
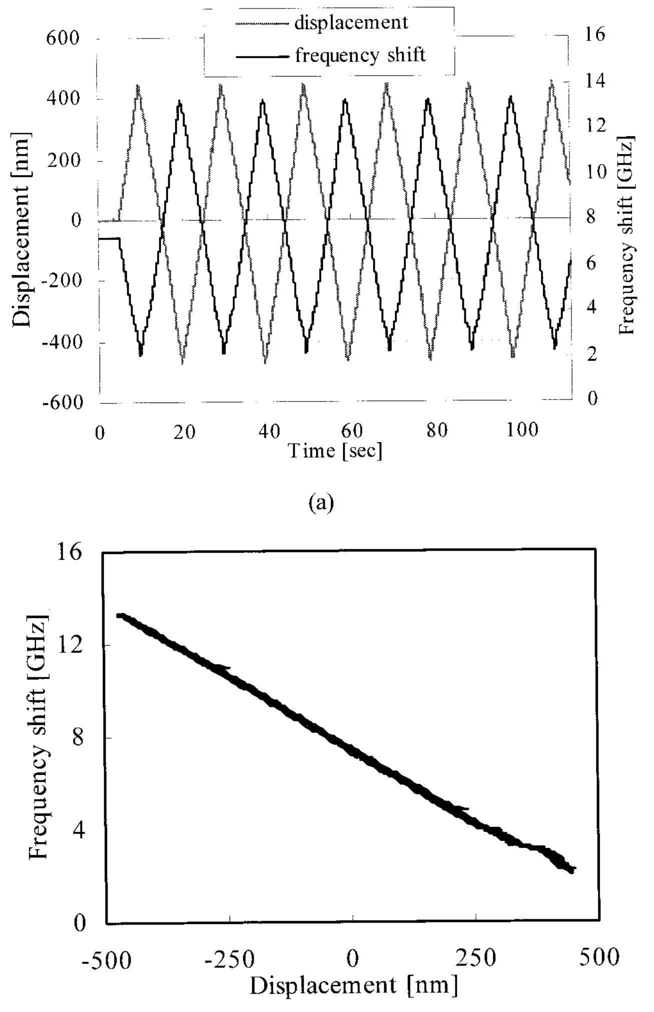


Fig. 6 Configuration of comparison measurement between the commercial heterodyne interferometer and the proposed system



(a)

(b)

Fig. 7 Comparison results between the commercial heterodyne interferometer and the proposed system. (a), (b) Relationship between the signal of commercial heterodyne interferometer and frequency shift

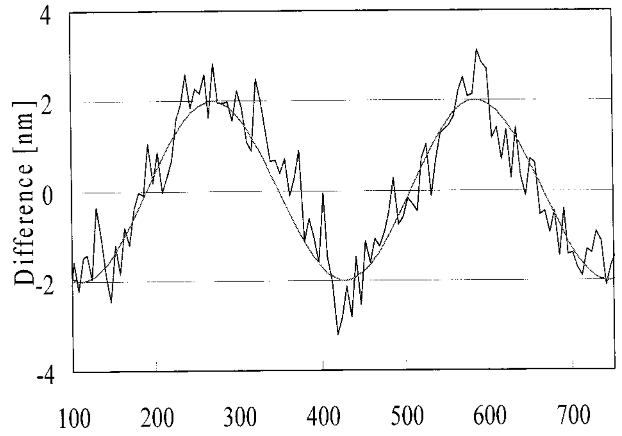


Fig. 8 Periodic error of the commercial heterodyne interferometer

4. Conclusion

We proposed a method for measuring a displacement with high resolution by adjusting the optical frequency of a TLD, and constructed the system by combining a PMHI, a TLD, frequency-tuning electronics, and a beat detection unit. To measure a displacement, the optical path difference was determined by measuring the FSR from the lock-in output when the optical frequency of the TLD was scanned. Comparative measurements between the proposed system and two commercial displacement sensors, a capacitive sensor and a heterodyne interferometer, were carried out to evaluate the performance of the proposed system. The experimental results demonstrated both a clear periodic error in the commercial interferometer, and that the proposed method has a high potential to measure a displacement with a resolution on the order of 10 pm.

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REFERENCES

1. International Technology Roadmap for Semiconductors: http://public.itrs.net/
2. Bobroff, N., “Recent Advances in Displacement Measuring Interferometry,” Measurement Science and Technology, Vol. 4, pp. 907–926, 1993.
3. Wu, C., Lawall, J. and Deslattes, R. D., “Heterodyne Interferometer with Subatomic Periodic Nonlinearity,” Applied Optics, Vol. 38, No. 12, pp. 4089–4094, 1999.
4. Badami, V. G. and Patterson, S. R., “A Frequency Domain Method for the Measurement of Nonlinearity in Heterodyne Interferometer,” Precision Engineering, Vol. 24, pp. 41–49, 2000.
5. Basile, G., Bergamin, A., Cavagnero, G. and Mana, G., “Phase Modulation in High-resolution Optical Interferometry,” Metrologia, Vol. 28, pp. 455–461, 1991.
6. Haitjema, H., Schellekens, P. H. J. and Wetzels, S. F. C. L., “Calibration of Displacement Sensors up to 300 μm with Nanometer Accuracy and Direct Traceability to a Primary Standard of Length,” Metrologia, Vol. 37, pp. 25–33, 2000.
7. Howard, L., Stone, J. and Fu, J., “Real-time Displacement Measurements with a Fabry–Perot Cavity and a Diode Laser,” Precision Engineering, Vol. 25, No. 4, pp. 321–335, 2001.
8. Lawall, J. R., “Fabry–Perot Metrology for Displacements up to 50 mm,” Journal of the Optical Society of America A, Vol. 22, No. 12, pp. 2786–2798, 2005.
9. Bitou, Y., Schibli, T. R. and Minoshima, K., “Accurate Wide-range Displacement Measurement Using Tunable Diode Laser and Optical Frequency Comb Generator,” Optics Express, Vol. 14, No. 2, pp. 644–654, 2006.
10. Drever, R. W. P., Hall, J. L., Kowalski, F. V., Hough, J., Ford, G. M., Munley, A. J. and Ward, H., “Laser Phase and Frequency Stabilization Using an Optical Resonator,” Applied Physics B: Photophysics Laser Chemistry, Vol. 31, No. 2, pp. 97–105, 1983.
11. Bay, Z., Luther, G. G. and White, J. A., “Measurement of an Optical Frequency and the Speed of Light,” Physical Review Letters, Vol. 29, No. 3, pp. 189–192, 1972.
12. Udem, Th., Reichert, J., Holzwarth, R. and Hünsch, T. W., “Accurate Measurement of Large Optical Frequency Differences with a Mode-locked Laser,” Optics Letters, Vol. 14, No. 13, pp. 881–883, 1999.
13. Silver, R. M., Zou, H., Gonda, S., Damazo, B., Jun, J., Jensen, C. and Howard, L., “Atomic-resolution Measurements with a New Tunable Diode Laser-based Interferometer,” Optical Engineering, Vol. 43, No. 1, pp. 79–86, 2004.
14. Matsuura, F., Ishige, M., Kawasugi, M., Hoshino, Y. and Aketagawa, M., “Length Measurement Using a Tunable Laser Diode,” Japanese Society for Precision Engineering, Spring Meeting, pp. N07, 2006.
15. ADE corporation: http://www.ade.com/#menu
16. Agilent Technologies: http://www.home.agilent.com/agilent/home.jspx?lc=eng&NEWCCLC=USeng&cc=US
17. Ciddor, P. E., “Refractive Index of Air: New Equations for the Visible and Near Infrared,” Applied Optics, Vol. 35, No. 9, pp. 1566–1573, 1996.