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Fuzzy (r, s)-irresolute maps

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Abstract

Using the idea of degree of openness and degree of nonopenness, Coker and Demirci [5] defined intuitionistic fuzzy topological spaces in Sostak's sense as a generalization of smooth topological spaces and intuitionistic fuzzy topological spaces. M. N. Mukherjee and S. P. Sinha [10] introduced the concept of fuzzy irresolute maps on Chang's fuzzy topological spaces. In this paper, we introduce the concepts of fuzzy (r, s)-irresolute, fuzzy (r, s)-presemiopen, fuzzy almost (r, s)-open, and fuzzy weakly (r, s)-continuous maps on intuitionistic fuzzy topological spaces in Sostak's sense. Using the notions of fuzzy (r, s)-neighborhoods and fuzzy (r, s)-semineighborhoods of a given intuitionistic fuzzy points, characterizations of fuzzy (r, s)-irresolute maps are displayed. The relations among fuzzy (r, s)-irresolute maps, fuzzy (r, s)-continuous maps, fuzzy almost (r, s)-continuous maps, and fuzzy weakly (r, s)-cotinuous maps are discussed.

Key words : fuzzy (r, s)-continuous, fuzzy almost (r, s)-continuous, fuzzy (r, s)-irresolute, fuzzy (r, s)-presemiopen, fuzzy almost (r, s)-open, fuzzy weakly (r, s)-continuous

1. Introduction

The concept of fuzzy set was introduced by Zadeh [14]. Chang [2] defined fuzzy topological spaces. These spaces and its generalizations are later studied by several authors, one of which, developed by Sostak [13], used the idea of degree of openness. This type of generalization of fuzzy topological spaces was later rephrased by Chattopadhyay and his colleagues [3], and by Ramadan [11].

As a generalization of fuzzy sets, the concept of intuitionistic fuzzy sets was introduced by Atanassov [1]. Recently, Coker and his colleagues [4, 6] introduced intuitionistic fuzzy topological spaces using intuitionistic fuzzy sets. Using the idea of degree of openness and degree of nonopenness, Coker and Demirci [5] defined intuitionistic fuzzy topological spaces in Sostak's sense as a generalization of smooth topological spaces and intuitionistic fuzzy topological spaces. M. N. Mukherjee and S. P. Sinha [10] introduced the concept of fuzzy irresolute maps on Chang's fuzzy topological spaces. Jun and his colleagues [7] introduced various kinds of fuzzy mappings on intuitionistic fuzzy topological spaces.

In this paper, we introduce the concepts of fuzzy (r, s)-irresolute, fuzzy (r, s)-presemiopen, fuzzy almost (r, s)-open, and fuzzy weakly (r, s)-continuous maps on intuitionistic fuzzy topological spaces in Sostak's sense. Using the notions of fuzzy (r, s)-neighborhoods and fuzzy (r, s)-semineighborhoods of a given intuitionistic fuzzy points, characterizations of fuzzy (r, s)-irresolute maps are displayed. The relations among fuzzy (r, s)-irresolute maps, fuzzy (r, s)-continuous maps, fuzzy almost (r, s)-continuous maps, and fuzzy weakly (r, s)-cotinuous maps are discussed.

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2. Preliminaries

We will denote the unit interval $[0, 1]$ of the real line by I. A member μ of I^X is called a *fuzzy set* in X. For any $\mu \in I^X$, μ^c denotes the complement $1 - \mu$. By $\tilde{0}$ and $\tilde{1}$ we denote constant maps on X with value 0 and 1, respectively. All other notations are standard notations of fuzzy set theory.

Let X be a nonempty set. An *intuitionistic fuzzy set* A is an ordered pair

$$A = (\mu_A, \gamma_A)$$

where the functions $\mu_A : X \to I$ and $\gamma_A : X \to I$ denote the degree of membership and the degree of nonmembership, respectively and $\mu_A + \gamma_A \leq 1$. Obviously every fuzzy set μ in X is an intuitionistic fuzzy set of the form $(\mu, \tilde{1} - \mu)$.

Definition 2.1 ([1]) Let $A = (\mu_A, \gamma_A)$ and $B = (\mu_B, \gamma_B)$ be intuitionistic fuzzy sets in X. Then

(1) $A \subseteq B$ iff $\mu_A \leq \mu_B$ and $\gamma_A \geq \gamma_B$.

(2) $A = B$ iff $A \subseteq B$ and $B \subseteq A$.

(3) $A^c = (\gamma_A, \mu_A)$.

(4) $A \cap B = (\mu_A \wedge \mu_B, \gamma_A \vee \gamma_B)$.

(5) $A \cup B = (\mu_A \vee \mu_B, \gamma_A \wedge \gamma_B)$.

(6) $\underline{0} = (\tilde{0}, \tilde{1})$ and $\underline{1} = (\tilde{1}, \tilde{0})$.

Let f be a map from a set X to a set Y. Let $A = (\mu_A, \gamma_A)$ be an intuitionistic fuzzy set in X and $B = (\mu_B, \gamma_B)$ an intuitionistic fuzzy set in Y. Then

(1) The image of A under f, denoted by $f(A)$, is an intuitionistic fuzzy set in Y defined by

$$f(A) = (f(\mu_A), \tilde{1} - f(\tilde{1} - \gamma_A)).$$

(2) The inverse image of B under f, denoted by $f^{-1}(B)$, is an intuitionistic fuzzy set in X defined by

$$f^{-1}(B) = (f^{-1}(\mu_B), f^{-1}(\gamma_B)).$$

A *smooth fuzzy topology* on X is a map $T : I^X \to I$ which satisfies the following properties :

(1) $T(\tilde{0}) = T(\tilde{1}) = 1$.

(2) $T(\mu_1 \wedge \mu_2) \geq T(\mu_1) \wedge T(\mu_2)$.

(3) $T(\bigvee \mu_i) \geq \bigwedge T(\mu_i)$.

The pair (X, T) is called a *smooth fuzzy topological space.*

An *intuitionistic fuzzy topology* on X is a family T of intuitionistic fuzzy sets in X which satisfies the following properties :

(1) $\underline{0}, \underline{1} \in T$.

(2) If $A_1, A_2 \in T$, then $A_1 \cap A_2 \in T$.

(3) If $A_i \in T$ for each i, then $\bigcup A_i \in T$.

The pair (X, T) is called an *intuitionistic fuzzy topological space.*

Let $I(X)$ be a family of all intuitionistic fuzzy sets in X and let $I \otimes I$ be the set of the pair (r, s) such that $r, s \in I$ and $r + s \leq 1$.

Definition 2.2 ([5]) Let X be a nonempty set. An *intuitionistic fuzzy topology in Sostak's sense*(SoIFT for short) $\mathcal{T} = (\mathcal{T}_1, \mathcal{T}_2)$ on X is a map $\mathcal{T} : I(X) \to I \otimes I$ which satisfies the following properties :

(1) $\mathcal{T}_1(\underline{0}) = \mathcal{T}_1(\underline{1}) = 1$ and $\mathcal{T}_2(\underline{0}) = \mathcal{T}_2(\underline{1}) = 0$.

(2) $\mathcal{T}_1(A \cap B) \geq \mathcal{T}_1(A) \wedge \mathcal{T}_1(B)$ and $\mathcal{T}_2(A \cap B) \leq \mathcal{T}_2(A) \vee \mathcal{T}_2(B)$.

(3) $\mathcal{T}_1(\bigcup A_i) \geq \bigwedge \mathcal{T}_1(A_i)$ and $\mathcal{T}_2(\bigcup A_i) \leq \bigvee \mathcal{T}_2(A_i)$.

The $(X, \mathcal{T}) = (X, \mathcal{T}_1, \mathcal{T}_2)$ is said to be an *intuitionistic fuzzy topological space in Sostak's sense*(SoIFTS for short). Also, we call $\mathcal{T}_1(A)$ a *gradation of openness* of A and $\mathcal{T}_2(A)$ a *gradation of nonopenness* of A.

Definition 2.3 ([8]) Let A be an intuitionistic fuzzy set in SoIFTS $(X, \mathcal{T}_1, \mathcal{T}_2)$ and $(r, s) \in I \otimes I$. Then A is said to be

(1) *fuzzy (r, s)-open* if $\mathcal{T}_1(A) \geq r$ and $\mathcal{T}_2(A) \leq s$,

(2) *fuzzy (r, s)-closed* if $\mathcal{T}_1(A^c) \geq r$ and $\mathcal{T}_2(A^c) \leq s$.

Definition 2.4 ([8]) Let $(X, \mathcal{T}_1, \mathcal{T}_2)$ be a SoIFTS. For each $(r, s) \in I \otimes I$ and for each $A \in I(X)$, the *fuzzy (r, s)-interior* is defined by

$$\begin{aligned} &\text{int}(A, r, s) \\ &= \bigcup\{B \in I(X) \mid B \subseteq A,\ \ B \text{ is fuzzy } (r, s)\text{-open}\} \end{aligned}$$

and the *fuzzy (r, s)-closure* is defined by

$$\begin{aligned} &\text{cl}(A, r, s) \\ &= \bigcap\{B \in I(X) \mid A \subseteq B,\ \ B \text{ is fuzzy } (r, s)\text{-closed}\}. \end{aligned}$$

Lemma 2.5 ([8]) For an intuitionistic fuzzy set A in a SoIFTS $(X, \mathcal{T}_1, \mathcal{T}_2)$ and $(r, s) \in I \otimes I$,

(1) $\text{int}(A, r, s)^c = \text{cl}(A^c, r, s)$.

(2) $\text{cl}(A, r, s)^c = \text{int}(A^c, r, s)$.

Let $(X, \mathcal{T}_1, \mathcal{T}_2)$ be an intuitionistic fuzzy topological space in Sostak's sense. Then it is easy to see that for each $(r, s) \in I \otimes I$, the family $\mathcal{T}_{(r,s)}$ defined by

$$\mathcal{T}_{(r,s)} = \{A \in I(X) \mid \mathcal{T}_1(A) \geq r \text{ and } \mathcal{T}_2(A) \leq s\}$$

is an intuitionistic fuzzy topology on X.

Let (X, T) be an intuitionistic fuzzy topological space and $(r, s) \in I \otimes I$. Then the map $T^{(r,s)} : I(X) \to I \otimes I$ defined by

$$T^{(r,s)}(A) = \begin{cases} (1, 0) & \text{if } A = \underline{0}, \underline{1}, \\ (r, s) & \text{if } A \in T - \{\underline{0}, \underline{1}\}, \\ (0, 1) & \text{otherwise} \end{cases}$$

becomes an intuitionistic fuzzy topology in Sostak's sense on X.

Let $\alpha, \beta \in [0,1]$ with $\alpha + \beta \leq 1$. An intuitionistic fuzzy point $x_{(\alpha,\beta)}$ in X is an intuitionistic fuzzy set in X defined by

$$x_{(\alpha,\beta)}(y) = \begin{cases} (\alpha,\beta) & \text{if } y = x, \\ (0,1) & \text{if } y \neq x. \end{cases}$$

In this case, x is called the *support* of $x_{(\alpha,\beta)}$, α the *value* of $x_{(\alpha,\beta)}$, and β the *nonvalue* of $x_{(\alpha,\beta)}$. An intuitionistic fuzzy point $x_{(\alpha,\beta)}$ is said to *belong* to an intuitionistic fuzzy set $A = (\mu_A, \gamma_A)$ in X, denoted by $x_{(\alpha,\beta)} \in A$, if $\mu_A(x) \geq \alpha$ and $\gamma_A(x) \leq \beta$. An intuitionistic fuzzy set A in X is the union of all intuitionistic fuzzy points which belong to A.

Definition 2.6 ([8]) Let A be an intuitionistic fuzzy set in a SoIFTS $(X, \mathcal{T}_1, \mathcal{T}_2)$ and $(r,s) \in I \otimes I$. Then A is said to be

(1) *fuzzy* (r,s)*-semiopen* if there is a fuzzy (r,s)-open set B in X such that $B \subseteq A \subseteq \text{cl}(B,r,s)$,

(2) *fuzzy* (r,s)*-semiclosed* if there is a fuzzy (r,s)-closed set B in X such that $\text{int}(B,r,s) \subseteq A \subseteq B$.

Theorem 2.7 ([8]) Let A be an intuitionistic fuzzy set in a SoIFTS $(X, \mathcal{T}_1, \mathcal{T}_2)$ and $(r,s) \in I \otimes I$. Then the following statements are equivalent :

(1) A is a fuzzy (r,s)-semiopen set.

(2) A^c is a fuzzy (r,s)-semiclosed set.

(3) $\text{cl}(\text{int}(A,r,s),r,s) \supseteq A$.

(4) $\text{int}(\text{cl}(A^c,r,s),r,s) \subseteq A^c$.

Theorem 2.8 ([8]) Let $(X, \mathcal{T}_1, \mathcal{T}_2)$ be a SoIFTS and $(r,s) \in I \otimes I$.

(1) If $\{A_i\}$ is a family of fuzzy (r,s)-semiopen sets in X, then $\bigcup A_i$ is fuzzy (r,s)-semiopen.

(2) If $\{A_i\}$ is a family of fuzzy (r,s)-semiclosed sets in X, then $\bigcap A_i$ is fuzzy (r,s)-semiclosed.

Definition 2.9 ([8]) Let $(X, \mathcal{T}_1, \mathcal{T}_2)$ be a SoIFTS. For each $(r,s) \in I \otimes I$ and for each $A \in I(X)$, the *fuzzy* (r,s)*-semiinterior* is defined by

$$\text{sint}(A,r,s) = \bigcup\{B \in I(X) \mid B \subseteq A,\ B \text{ is fuzzy } (r,s)\text{-semiopen}\}$$

and the *fuzzy* (r,s)*-semiclosure* is defined by

$$\text{scl}(A,r,s) = \bigcap\{B \in I(X) \mid A \subseteq B,\ B \text{ is fuzzy } (r,s)\text{-semiclosed}\}.$$

Definition 2.10 ([12]) Let A be an intuitionistic fuzzy set in a SoIFTS $(X, \mathcal{T}_1, \mathcal{T}_2)$ and $(r,s) \in I \otimes I$. Then A is said to be

(1) *fuzzy* (r,s)*-regular open* if $\text{int}(\text{cl}(A,r,s),r,s) = A$,

(2) *fuzzy* (r,s)*-regular closed* if $\text{cl}(\text{int}(A,r,s),r,s) = A$.

Theorem 2.11 ([12]) Let A be an intuitionistic fuzzy set in a SoIFTS $(X, \mathcal{T}_1, \mathcal{T}_2)$ and $(r,s) \in I \otimes I$. Then the following statements are equivalent :

(1) A is fuzzy (r,s)-regular open.

(2) A^c is fuzzy (r,s)-regular closed.

Theorem 2.12 ([12]) (1) The fuzzy (r,s)-closure of a fuzzy (r,s)-open set is fuzzy (r,s)-regular closed for each $(r,s) \in I \otimes I$.
(2) The fuzzy (r,s)-interior of a fuzzy (r,s)-closed set is fuzzy (r,s)-regular open for each $(r,s) \in I \otimes I$.

Definition 2.13 ([9, 12]) Let $f : (X, \mathcal{T}_1, \mathcal{T}_2) \to (Y, \mathcal{U}_1, \mathcal{U}_2)$ be a map from a SoIFTS X to a SoIFTS Y and $(r,s) \in I \otimes I$. Then f is called

(1) a *fuzzy* (r,s)*-continuous* map if $f^{-1}(B)$ is a fuzzy (r,s)-open set in X for each fuzzy (r,s)-open set B in Y,

(2) a *fuzzy* (r,s)*-open* map if $f(A)$ is a fuzzy (r,s)-open set in Y for each fuzzy (r,s)-open set A in X,

(3) a *fuzzy* (r,s)*-semicontinuous* map if $f^{-1}(B)$ is a fuzzy (r,s)-semiopen set in X for each fuzzy (r,s)-open set B in Y,

(4) a *fuzzy* (r,s)*-semiopen* map if $f(A)$ is a fuzzy (r,s)-semiopen set in Y for each fuzzy (r,s)-open set A in X,

(5) a *fuzzy almost* (r,s)*-continuous* map if $f^{-1}(B)$ is a fuzzy (r,s)-open set in X for each fuzzy (r,s)-regular open set B in Y.

Theorem 2.14 ([12]) Let $f : (X, \mathcal{T}_1, \mathcal{T}_2) \to (Y, \mathcal{U}_1, \mathcal{U}_2)$ be a map from a SoIFTS X to a SoIFTS Y and $(r,s) \in I \otimes I$. Then the following statements are equivalent :

(1) f is a fuzzy almost (r,s)-continuous map.

(2) $f^{-1}(B) \subseteq \text{int}(f^{-1}(\text{int}(\text{cl}(B,r,s),r,s)),r,s)$ for each fuzzy (r,s)-open set B in Y.

(3) $\text{cl}(f^{-1}(\text{cl}(\text{int}(B,r,s),r,s)),r,s) \subseteq f^{-1}(B)$ for each fuzzy (r,s)-closed set B in Y.

Theorem 2.15 ([8]) Let $f : (X,\mathcal{T}_1,\mathcal{T}_2) \to (Y,\mathcal{U}_1,\mathcal{U}_2)$ be a map and $(r,s) \in I \otimes I$. Then the following statements are equivalent :

(1) f is a fuzzy (r,s)-semicontinuous map.

(2) $f^{-1}(B)$ is a fuzzy (r,s)-semiclosed set in X for each fuzzy (r,s)-closed set B in Y.

(3) $\text{int}(\text{cl}(f^{-1}(B),r,s),r,s) \subseteq f^{-1}(\text{cl}(B,r,s))$ for each intuitionistic fuzzy set B in Y.

(4) $f(\text{int}(\text{cl}(A,r,s),r,s)) \subseteq \text{cl}(f(A),r,s)$ for each intuitionistic fuzzy set A in X.

3. Fuzzy (r,s)-irresolute maps

Now, we define the notions of fuzzy (r,s)-irresolute, fuzzy (r,s)-presemiopen, fuzzy almost (r,s)-open, and fuzzy weakly (r,s)-continuous maps on intuitionistic fuzzy topological spaces in Sostak's sense, and then we investigate some of their properties.

Definition 3.1 Let $f : (X,\mathcal{T}_1,\mathcal{T}_2) \to (Y,\mathcal{U}_1,\mathcal{U}_2)$ be a map from a SoIFTS X to a SoIFTS Y and $(r,s) \in I \otimes I$. Then f is called

(1) a *fuzzy (r,s)-irresolute* map if $f^{-1}(B)$ is a fuzzy (r,s)-semiopen set in X for each fuzzy (r,s)-semiopen set B in Y,

(2) a *fuzzy (r,s)-presemiopen* map if $f(A)$ is a fuzzy (r,s)-semiopen set in Y for each fuzzy (r,s)-semiopen set A in X,

(3) a *fuzzy almost (r,s)-open* map if $f(A)$ is a fuzzy (r,s)-open set in Y for each fuzzy (r,s)-regular open set A in X,

(4) a *fuzzy weakly (r,s)-continuous* map if for every fuzzy (r,s)-open set B in Y, $f^{-1}(B) \subseteq \text{int}(f^{-1}(\text{cl}(B,r,s)),r,s)$.

Definition 3.2 Let $x_{(\alpha,\beta)}$ be an intuitionistic fuzzy point in a SoIFTS $(X,\mathcal{T}_1,\mathcal{T}_2)$ and $(r,s) \in I \otimes I$. Then an intuitionistic fuzzy set A in X is called

(1) a *fuzzy (r,s)-neighborhood* of $x_{(\alpha,\beta)}$ if there is a fuzzy (r,s)-open set B in X such that $x_{(\alpha,\beta)} \in B \subseteq A$,

(2) a *fuzzy (r,s)-semineighborhood* of $x_{(\alpha,\beta)}$ if there is a fuzzy (r,s)-semiopen set B in X such that $x_{(\alpha,\beta)} \in B \subseteq A$.

Theorem 3.3 Let $f : (X,\mathcal{T}_1,\mathcal{T}_2) \to (Y,\mathcal{U}_1,\mathcal{U}_2)$ be a map from a SoIFTS X to a SoIFTS Y and $(r,s) \in I \otimes I$. Then the following statements are equivalent :

(1) f is fuzzy (r,s)-irresolute.

(2) $f^{-1}(B)$ is a fuzzy (r,s)-semiclosed set in X for each fuzzy (r,s)-semiclosed set B in Y.

(3) For every intuitionistic fuzzy point $x_{(\alpha,\beta)}$ in X and every fuzzy (r,s)-semiopen set B in Y such that $f(x_{(\alpha,\beta)}) \in B$, there is a fuzzy (r,s)-semiopen set A in X such that $x_{(\alpha,\beta)} \in A$ and $f(A) \subseteq B$.

(4) For every intuitionistic fuzzy point $x_{(\alpha,\beta)}$ in X and every fuzzy (r,s)-semineighborhood B of $f(x_{(\alpha,\beta)})$ in Y, $f^{-1}(B)$ is a fuzzy (r,s)-semineighborhood of $x_{(\alpha,\beta)}$ in X.

(5) For every intuitionistic fuzzy point $x_{(\alpha,\beta)}$ in X and every fuzzy (r,s)-semineighborhood B of $f(x_{(\alpha,\beta)})$ in Y, there is a fuzzy (r,s)-semineighborhood A of $x_{(\alpha,\beta)}$ in X such that $f(A) \subseteq B$.

(6) $f(\text{scl}(A,r,s)) \subseteq \text{scl}(f(A),r,s)$ for each intuitionistic fuzzy set A in X.

(7) $\text{scl}(f^{-1}(B),r,s) \subseteq f^{-1}(\text{scl}(B,r,s))$ for each intuitionistic fuzzy set B in Y.

Proof. (1) $\Leftrightarrow$ (2) It is obvious.
(3) $\Rightarrow$ (1) Let B be a fuzzy (r,s)-semiopen set in Y and $x_{(\alpha,\beta)}$ an intuitionistic fuzzy point in X such that $x_{(\alpha,\beta)} \in f^{-1}(B)$. Then $f(x_{(\alpha,\beta)}) \in B$. Thus there is a fuzzy (r,s)-semiopen set A in X such that $x_{(\alpha,\beta)} \in A$ and $f(A) \subseteq B$. Then $A \subseteq f^{-1}(B)$. Thus

$$\begin{aligned} x_{(\alpha,\beta)} \in A &\subseteq \text{cl}(\text{int}(A,r,s),r,s) \\ &\subseteq \text{cl}(\text{int}(f^{-1}(B),r,s),r,s). \end{aligned}$$

Hence

$$\begin{aligned} f^{-1}(B) &= \bigcup\{x_{(\alpha,\beta)} \mid x_{(\alpha,\beta)} \in f^{-1}(B)\} \\ &\subseteq \text{cl}(\text{int}(f^{-1}(B),r,s),r,s). \end{aligned}$$

Therefore f is a fuzzy (r,s)-irresolute map.
(1) $\Rightarrow$ (4) Let $x_{(\alpha,\beta)}$ be an intuitionistic fuzzy point in X and B a fuzzy (r,s)-semineighborhood of $f(x_{(\alpha,\beta)})$ in Y. Then there is a fuzzy (r,s)-semiopen set C in Y such that $f(x_{(\alpha,\beta)}) \in C \subseteq B$ and hence $x_{(\alpha,\beta)} \in f^{-1}(C) \subseteq$

$f^{-1}(B)$. Since f is fuzzy (r,s)-irresolute, $f^{-1}(C)$ is a fuzzy (r,s)-semiopen set in X. Thus $f^{-1}(B)$ is a fuzzy (r,s)-semineighborhood of $x_{(\alpha,\beta)}$.
(4) $\Rightarrow$ (5) Let $x_{(\alpha,\beta)}$ be an intuitionistic fuzzy point in X and B a fuzzy (r,s)-semineighborhood of $f(x_{(\alpha,\beta)})$ in Y. By (4), $f^{-1}(B)$ is a fuzzy (r,s)-semineighborhood of $x_{(\alpha,\beta)}$ in X. Let $f^{-1}(B)=A$. Then $f(A)=f(f^{-1}(B))\subseteq B$.
(5) $\Rightarrow$ (3) Let $x_{(\alpha,\beta)}$ be an intuitionistic fuzzy point in X and B a fuzzy (r,s)-semiopen set in Y such that $f(x_{(\alpha,\beta)})\in B$. Then since B is a fuzzy (r,s)-semineighborhood of $f(x_{(\alpha,\beta)})$, by (5), there is a fuzzy (r,s)-semineighborhood A of $x_{(\alpha,\beta)}$ in X such that $f(A)\subseteq B$. Then there is a fuzzy (r,s)-semiopen set C in X such that $x_{(\alpha,\beta)}\in C\subseteq A$ and hence $f(C)\subseteq f(A)\subseteq B$.
(2) $\Rightarrow$ (6) Let A be an intuitionistic fuzzy set in X. Since $\mathrm{scl}(f(A),r,s)$ is fuzzy (r,s)-semiclosed in Y, by (2), $f^{-1}(\mathrm{scl}(f(A),r,s))$ is a fuzzy (r,s)-semiclosed set in X. Since $f(A)\subseteq \mathrm{scl}(f(A),r,s)$, we have $A\subseteq f^{-1}(f(A))\subseteq f^{-1}(\mathrm{scl}(f(A),r,s))$. Hence

$$\begin{aligned}\mathrm{scl}(A,r,s) &\subseteq \mathrm{scl}(f^{-1}(\mathrm{scl}(f(A),r,s)),r,s)\\ &= f^{-1}(\mathrm{scl}(f(A),r,s)).\end{aligned}$$

Therefore

$$\begin{aligned}f(\mathrm{scl}(A,r,s)) &\subseteq f(f^{-1}(\mathrm{scl}(f(A),r,s)))\\ &\subseteq \mathrm{scl}(f(A),r,s).\end{aligned}$$

(6) $\Rightarrow$ (2) Let B be a fuzzy (r,s)-semiclosed set in Y. Then $f^{-1}(B)$ is an intuitionistic fuzzy set in X. By (6),

$$\begin{aligned}f(\mathrm{scl}(f^{-1}(B),r,s)) &\subseteq \mathrm{scl}(f(f^{-1}(B)),r,s)\\ &\subseteq \mathrm{scl}(B,r,s)=B.\end{aligned}$$

Thus $\mathrm{scl}(f^{-1}(B),r,s)\subseteq f^{-1}(f(\mathrm{scl}(f^{-1}(B),r,s)))\subseteq f^{-1}(B)$. Hence $f^{-1}(B)=\mathrm{scl}(f^{-1}(B),r,s)$. Therefore $f^{-1}(B)$ is a fuzzy (r,s)-semiclosed set in X.
(6) $\Rightarrow$ (7) Let B be an intuitionistic fuzzy set in Y. Then $f^{-1}(B)$ is an intuitionistic fuzzy set in X. By (6),

$$\begin{aligned}f(\mathrm{scl}(f^{-1}(B),r,s)) &\subseteq \mathrm{scl}(f(f^{-1}(B)),r,s)\\ &\subseteq \mathrm{scl}(B,r,s).\end{aligned}$$

Hence

$$\begin{aligned}\mathrm{scl}(f^{-1}(B),r,s) &\subseteq f^{-1}(f(\mathrm{scl}(f^{-1}(B),r,s)))\\ &\subseteq f^{-1}(\mathrm{scl}(B,r,s)).\end{aligned}$$

(7) $\Rightarrow$ (6) Let A be an intuitionistic fuzzy set in X. Then $f(A)$ is an intuitionistic fuzzy set in Y. By (7),

$$\begin{aligned}\mathrm{scl}(A,r,s) &\subseteq \mathrm{scl}(f^{-1}(f(A)),r,s)\\ &\subseteq f^{-1}(\mathrm{scl}(f(A),r,s)).\end{aligned}$$

Hence

$$\begin{aligned}f(\mathrm{scl}(A,r,s)) &\subseteq f(f^{-1}(\mathrm{scl}(f(A),r,s)))\\ &\subseteq \mathrm{scl}(f(A),r,s).\end{aligned}$$

Lemma 3.4 Let $f:(X,\mathcal{T}_1,\mathcal{T}_2)\to(Y,\mathcal{U}_1,\mathcal{U}_2)$ be a map from a SoIFTS X to a SoIFTS Y and let A and B be intuitionistic fuzzy sets in X and Y, respectively. Then $f^{-1}(B)\subseteq A$ if and only if $(f(A^c))^c\supseteq B$.

Proof.

$$\begin{aligned}f^{-1}(B)\subseteq A &\Leftrightarrow f^{-1}(B^c)=f^{-1}(B)^c\supseteq A^c\\ &\Leftrightarrow f(A^c)\subseteq f(f^{-1}(B^c))\subseteq B^c\\ &\Leftrightarrow (f(A^c))^c\supseteq B.\end{aligned}$$

Theorem 3.5 Let $f:(X,\mathcal{T}_1,\mathcal{T}_2)\to(Y,\mathcal{U}_1,\mathcal{U}_2)$ be a map from a SoIFTS X to a SoIFTS Y and $(r,s)\in I\otimes I$. Then f is fuzzy almost (r,s)-open if and only if for each intuitionistic fuzzy set B in Y and each fuzzy (r,s)-regular closed set A in X such that $f^{-1}(B)\subseteq A$, there is a fuzzy (r,s)-closed set C in Y such that $B\subseteq C$ and $f^{-1}(C)\subseteq A$.

Proof. Let f be a fuzzy almost (r,s)-open map, B an intuitionistic fuzzy set in Y, and A a fuzzy (r,s)-regular closed set in X such that $f^{-1}(B)\subseteq A$. Let $C=(f(A^c))^c$. Then C is a fuzzy (r,s)-closed set in Y and by Lemma 3.4, $B\subseteq C$. Also, we have

$$\begin{aligned}f^{-1}(C)=f^{-1}((f(A^c))^c) &= (f^{-1}(f(A^c)))^c\\ &\subseteq (A^c)^c=A.\end{aligned}$$

Conversely, let A be a fuzzy (r,s)-regular open set in X. Let $B=f(A)^c$ and $D=A^c$. Then we have

$$f^{-1}(B)=f^{-1}(f(A)^c)=(f^{-1}(f(A)))^c\subseteq A^c=D.$$

By hypothesis, there is a fuzzy (r,s)-closed set C in Y such that $f(A)^c=B\subseteq C$ and $f^{-1}(C)\subseteq D=A^c$. Then $A\subseteq f^{-1}(C)^c=f^{-1}(C^c)$. Hence $f(A)=C^c$. Therefore $f(A)$ is a fuzzy (r,s)-open set in Y and consequently f is a fuzzy almost (r,s)-open map.

Theorem 3.6 Let $f:(X,\mathcal{T}_1,\mathcal{T}_2)\to(Y,\mathcal{U}_1,\mathcal{U}_2)$ be a map from a SoIFTS X to a SoIFTS Y and $(r,s)\in I\otimes I$. Then f is a fuzzy almost (r,s)-open map if and only if $f(\mathrm{int}(A,r,s))\subseteq \mathrm{int}(f(A),r,s)$ for each fuzzy (r,s)-semiclosed set A in X.

Proof. Let f be a fuzzy almost (r,s)-open map and A a fuzzy (r,s)-semiclosed set in X. Then $\mathrm{int}(A,r,s)\subseteq \mathrm{int}(\mathrm{cl}(A,r,s),r,s)\subseteq A$. Note that $\mathrm{cl}(A,r,s)$ is a fuzzy (r,s)-closed set in X. By Theorem 2.12 (2), $\mathrm{int}(\mathrm{cl}(A,r,s),r,s)$ is a fuzzy (r,s)-regular open set in X. Since f is a fuzzy almost (r,s)-open map,

$f(\text{int}(\text{cl}(A,r,s),r,s))$ is a fuzzy (r,s)-open set in Y. Thus we have

$$\begin{aligned} f(\text{int}(A,r,s)) &\subseteq f(\text{int}(\text{cl}(A,r,s),r,s)) \\ &= \text{int}(f(\text{int}(\text{cl}(A,r,s),r,s)),r,s) \\ &\subseteq \text{int}(f(A),r,s). \end{aligned}$$

Conversely, let A be a fuzzy (r,s)-regular open set in X. Then A is fuzzy (r,s)-open and hence $\text{int}(A,r,s) = A$. Since $\text{int}(\text{cl}(A,r,s),r,s) = A$, A is a fuzzy (r,s)-semiclosed set. So

$$f(A) = f(\text{int}(A,r,s)) \subseteq \text{int}(f(A),r,s) \subseteq f(A).$$

Thus $f(A) = \text{int}(f(A),r,s)$ and so $f(A)$ is fuzzy (r,s)-open in Y. Hence f is a fuzzy almost (r,s)-open map.

Theorem 3.7 Let $f : (X,\mathcal{T}_1,\mathcal{T}_2) \to (Y,\mathcal{U}_1,\mathcal{U}_2)$ be a map from a SoIFTS X to a SoIFTS Y and $(r,s) \in I \otimes I$. Then f is a fuzzy almost (r,s)-continuous map if and only if for every intuitionistic fuzzy point $x_{(\alpha,\beta)}$ in X and every fuzzy (r,s)-neighborhood B of $f(x_{(\alpha,\beta)})$, there is a fuzzy (r,s)-neighborhood A of $x_{(\alpha,\beta)}$ such that $f(A) \subseteq \text{int}(\text{cl}(B,r,s),r,s)$.

Proof. Let $x_{(\alpha,\beta)}$ be an intuitionistic fuzzy point in X and B a fuzzy (r,s)-neighborhood of $f(x_{(\alpha,\beta)})$. Then there is a fuzzy (r,s)-open set C in Y such that $f(x_{(\alpha,\beta)}) \in C \subseteq B$. So $x_{(\alpha,\beta)} \in f^{-1}(C) \subseteq f^{-1}(B)$. Since f is a fuzzy almost (r,s)-continuous map, by Theorem 2.14,

$$\begin{aligned} f^{-1}(C) &\subseteq \text{int}(f^{-1}(\text{int}(\text{cl}(C,r,s),r,s)),r,s) \\ &\subseteq \text{int}(f^{-1}(\text{int}(\text{cl}(B,r,s),r,s)),r,s). \end{aligned}$$

Put $A = f^{-1}(\text{int}(\text{cl}(B,r,s),r,s))$. Then $x_{(\alpha,\beta)} \in f^{-1}(C) \subseteq \text{int}(A,r,s) \subseteq A$. By Theorem 2.12 (2), $\text{int}(\text{cl}(B,r,s),r,s)$ is fuzzy (r,s)-regular open. Since f is fuzzy almost (r,s)-continuous, $A = f^{-1}(\text{int}(\text{cl}(B,r,s),r,s))$ is a fuzzy (r,s)-open set. Thus A is a fuzzy (r,s)-neighborhood of $x_{(\alpha,\beta)}$ and

$$\begin{aligned} f(A) &= f(f^{-1}(\text{int}(\text{cl}(B,r,s),r,s))) \\ &\subseteq \text{int}(\text{cl}(B,r,s),r,s). \end{aligned}$$

Conversely, let B be a fuzzy (r,s)-regular open set in Y and $x_{(\alpha,\beta)} \in f^{-1}(B)$. Then B is fuzzy (r,s)-open and a fuzzy (r,s)-neighborhood of $f(x_{(\alpha,\beta)})$. By hypothesis, there is a fuzzy (r,s)-neighborhood of $A_{x_{(\alpha,\beta)}}$ of $x_{(\alpha,\beta)}$ such that $f(A_{x_{(\alpha,\beta)}}) \subseteq \text{int}(\text{cl}(B,r,s),r,s) = B$. Since $A_{x_{(\alpha,\beta)}}$ is a fuzzy (r,s)-neighborhood of $x_{(\alpha,\beta)}$, there is a fuzzy (r,s)-open set $C_{x_{(\alpha,\beta)}}$ in X such that

$$\begin{aligned} x_{(\alpha,\beta)} \in C_{x_{(\alpha,\beta)}} \subseteq A_{x_{(\alpha,\beta)}} &\subseteq f^{-1}(f(A_{x_{(\alpha,\beta)}})) \\ &\subseteq f^{-1}(B). \end{aligned}$$

So we have

$$\begin{aligned} f^{-1}(B) &= \bigcup\{x_{(\alpha,\beta)} \mid x_{(\alpha,\beta)} \in f^{-1}(B)\} \\ &\subseteq \bigcup\{C_{x_{(\alpha,\beta)}} \mid x_{(\alpha,\beta)} \in f^{-1}(B)\} \\ &\subseteq f^{-1}(B). \end{aligned}$$

Thus $f^{-1}(B) = \bigcup\{C_{x_{(\alpha,\beta)}} \mid x_{(\alpha,\beta)} \in f^{-1}(B)\}$ and so $f^{-1}(B)$ is a fuzzy (r,s)-open set in X. Hence f is a fuzzy almost (r,s)-continuous map.

Theorem 3.8 Let $(X,\mathcal{T})$ and $(Y,\mathcal{U})$ be SoIFTSs and $(r,s) \in I \otimes I$. If $f : (X,\mathcal{T}) \to (Y,\mathcal{U})$ is a fuzzy (r,s)-irresolute map, then f is a fuzzy (r,s)-semicontinuous map.

Proof. Let B be a fuzzy (r,s)-open set in Y. Then B is a fuzzy (r,s)-semiopen set in Y. Since f is a fuzzy (r,s)-irresolute map, $f^{-1}(B)$ is a fuzzy (r,s)-semiopen set in X. Hence f is a fuzzy (r,s)-semicontinuous map.

The following example shows that the converse of Theorem 3.8 need not be true.

Example 3.9 Let $X = \{x,y,z\}$ and let A_1, A_2 and B be intuitionistic fuzzy sets in X defined as

$$A_1(x) = (0,0.9),\ A_1(y) = (0.3,0.6),\ A_1(z) = (0.3,0.6);$$

$$A_2(x) = (0.9,0),\ A_2(y) = (0.3,0.6),\ A_2(z) = (0.3,0.6);$$

and

$$B(x) = (0.9,0),\ B(y) = (0.7,0.3),\ B(z) = (0.7,0.3).$$

Define $\mathcal{T} : I(X) \to I \otimes I$ and $\mathcal{U} : I(X) \to I \otimes I$ by

$$\mathcal{T}(A) = (\mathcal{T}_1(A),\mathcal{T}_2(A)) = \begin{cases} (1,0) & \text{if } A = \underline{0},\underline{1}, \\ (\frac{1}{2},\frac{1}{3}) & \text{if } A = A_1, \\ (0,1) & \text{otherwise;} \end{cases}$$

and

$$\mathcal{U}(A) = (\mathcal{U}_1(A),\mathcal{U}_2(A)) = \begin{cases} (1,0) & \text{if } A = \underline{0},\underline{1}, \\ (\frac{1}{2},\frac{1}{3}) & \text{if } A = A_2, \\ (0,1) & \text{otherwise.} \end{cases}$$

Then clearly $\mathcal{T}$ and $\mathcal{U}$ are SoIFTs on X. Consider a map $f : (X,\mathcal{T}) \to (X,\mathcal{U})$ defined by $f(x) = x$, $f(y) = y$ and $f(z) = z$. It is easy to see that f is a fuzzy $(\frac{1}{2},\frac{1}{3})$-semicontinuous map and B is a fuzzy $(\frac{1}{2},\frac{1}{3})$-semiopen set in $(X,\mathcal{U})$. But $f^{-1}(B) = B$ is not a fuzzy $(\frac{1}{2},\frac{1}{3})$-semiopen set in $(X,\mathcal{T})$. Hence f is not a fuzzy $(\frac{1}{2},\frac{1}{3})$-irresolute map.

Theorem 3.10 Let $f : (X,\mathcal{T}) \to (Y,\mathcal{U})$ be a map from a SoIFTS X to a SoIFTS Y and $(r,s) \in I \otimes I$. If f is fuzzy (r,s)-semicontinuous and fuzzy almost (r,s)-open, then f is a fuzzy (r,s)-irresolute map.

Proof. Let B be a fuzzy (r,s)-semiclosed set in Y. Then $\text{int}(\text{cl}(B,r,s),r,s) \subseteq B$. Since f is fuzzy (r,s)-semicontinuous, by Theorem 2.15,

$$\text{int}(\text{cl}(f^{-1}(B),r,s),r,s) \subseteq f^{-1}(\text{cl}(B,r,s)).$$

Thus we have

$$\begin{aligned}\text{int}(\text{cl}(f^{-1}(B),r,s),r,s) &\\ &= \text{int}(\text{int}(\text{cl}(f^{-1}(B),r,s),r,s),r,s)\\ &\subseteq \text{int}(f^{-1}(\text{cl}(B,r,s)),r,s).\end{aligned}$$

Since f is fuzzy (r,s)-semicontinuous and $\text{cl}(B,r,s)$ is fuzzy (r,s)-closed, $f^{-1}(\text{cl}(B,r,s))$ is fuzzy (r,s)-semiclosed in X. Since f is a fuzzy almost (r,s)-open map,

$$\begin{aligned}f(\text{int}(f^{-1}(\text{cl}(B,r,s)),r,s)) &\\ &\subseteq \text{int}(f(f^{-1}(\text{cl}(B,r,s))),r,s)\\ &\subseteq \text{int}(\text{cl}(B,r,s),r,s) \subseteq B.\end{aligned}$$

Hence we have

$$\begin{aligned}\text{int}(\text{cl}(f^{-1}(B),r,s),r,s) &\\ &\subseteq f^{-1}(f(\text{int}(\text{cl}(f^{-1}(B),r,s),r,s)))\\ &\subseteq f^{-1}(f(\text{int}(f^{-1}(\text{cl}(B,r,s)),r,s)))\\ &\subseteq f^{-1}(B).\end{aligned}$$

Thus $f^{-1}(B)$ is a fuzzy (r,s)-semiclosed set in X. Therefore f is a fuzzy (r,s)-irresolute map.

Remark 3.11 Clearly a fuzzy (r,s)-continuous map is a fuzzy almost (r,s)-continuous map for each $(r,s) \in I \otimes I$. That the converse need not be true is shown by the following example. Also, the example shows that a fuzzy almost (r,s)-continuous map need not be a fuzzy (r,s)-irresolute map for each $(r,s) \in I \otimes I$.

Example 3.12 Let $X = \{x,y,z\}$ and let A and B be intuitionistic fuzzy sets in X defined as

$$A(x) = (0,0.5),\ \ A(y) = (0.3,0.5),\ \ A(z) = (0.3,0.5);$$

and

$$B(x) = (0,0.7),\ \ B(y) = (0.2,0.7),\ \ B(z) = (0.2,0.7).$$

Define $\mathcal{T} : I(X) \to I \otimes I$ and $\mathcal{U} : I(X) \to I \otimes I$ by

$$\mathcal{T}(C) = (\mathcal{T}_1(C),\mathcal{T}_2(C)) = \begin{cases} (1,0) & \text{if } C = \underline{0},\underline{1},\\ (\frac{1}{2},\frac{1}{3}) & \text{if } C = A,\\ (0,1) & \text{otherwise};\end{cases}$$

and

$$\mathcal{U}(C) = (\mathcal{U}_1(C),\mathcal{U}_2(C)) = \begin{cases} (1,0) & \text{if } C = \underline{0},\underline{1},\\ (\frac{1}{2},\frac{1}{3}) & \text{if } C = A,B,\\ (0,1) & \text{otherwise.}\end{cases}$$

Then clearly $\mathcal{T}$ and $\mathcal{U}$ are SoIFTs on X. Consider a map $f : (X,\mathcal{T}) \to (X,\mathcal{U})$ defined by $f(x) = x$, $f(y) = y$ and $f(z) = z$. Note that $\text{int}(\text{cl}(A,\frac{1}{2},\frac{1}{3}),\frac{1}{2},\frac{1}{3}) = \text{int}(A^c,\frac{1}{2},\frac{1}{3}) = A$ and $\text{int}(\text{cl}(B,\frac{1}{2},\frac{1}{3}),\frac{1}{2},\frac{1}{3}) = \text{int}(A^c,\frac{1}{2},\frac{1}{3}) = A \neq B$ in $(X,\mathcal{U})$. So A is a fuzzy $(\frac{1}{2},\frac{1}{3})$-regular open set but B is not a fuzzy $(\frac{1}{2},\frac{1}{3})$-regular open set in $(X,\mathcal{U})$. Since $f^{-1}(A) = A$ is a fuzzy $(\frac{1}{2},\frac{1}{3})$-open set in $(X,\mathcal{T})$, f is a fuzzy almost $(\frac{1}{2},\frac{1}{3})$-continuous map. But since $f^{-1}(B) = B$ is not a fuzzy $(\frac{1}{2},\frac{1}{3})$-open set in $(X,\mathcal{T})$, f is not a fuzzy $(\frac{1}{2},\frac{1}{3})$-continuous map. Since $f^{-1}(B) = B$ is not fuzzy $(\frac{1}{2},\frac{1}{3})$-semiopen in $(X,\mathcal{T})$, f is not a fuzzy $(\frac{1}{2},\frac{1}{3})$-irresolute map.

The following example shows that a fuzzy (r,s)-continuous map need not be a fuzzy (r,s)-irresolute map for each $(r,s) \in I \otimes I$.

Example 3.13 Let $X = \{x,y,z\}$ and let A_1, A_2 and B be intuitionistic fuzzy sets in X defined as

$$A_1(x) = (0,0.7),\ \ A_1(y) = (0.3,0.5),\ \ A_1(z) = (0.3,0.5);$$

$$A_2(x) = (0,0.7),\ \ A_2(y) = (0.3,0.5),\ \ A_2(z) = (0.8,0.2);$$

and

$$B(x) = (0.1,0.7),\ \ B(y) = (0.8,0.1),\ \ B(z) = (0.8,0.1).$$

Define $\mathcal{T} : I(X) \to I \otimes I$ and $\mathcal{U} : I(X) \to I \otimes I$ by

$$\mathcal{T}(A) = (\mathcal{T}_1(A),\mathcal{T}_2(A)) = \begin{cases} (1,0) & \text{if } A = \underline{0},\underline{1},\\ (\frac{1}{2},\frac{1}{3}) & \text{if } A = A_1,\\ (0,1) & \text{otherwise};\end{cases}$$

and

$$\mathcal{U}(A) = (\mathcal{U}_1(A),\mathcal{U}_2(A)) = \begin{cases} (1,0) & \text{if } A = \underline{0},\underline{1},\\ (\frac{1}{2},\frac{1}{3}) & \text{if } A = A_2,\\ (0,1) & \text{otherwise.}\end{cases}$$

Then clearly $\mathcal{T}$ and $\mathcal{U}$ are SoIFTs on X. Consider a map $f : (X,\mathcal{T}) \to (X,\mathcal{U})$ defined by $f(x) = x$ and $f(y) = f(z) = y$. Then it is easy to see that f is a fuzzy $(\frac{1}{2},\frac{1}{3})$-continuous map and B is a fuzzy $(\frac{1}{2},\frac{1}{3})$-semiopen set in $(X,\mathcal{U})$. But since $f^{-1}(B) = B$ is not a fuzzy $(\frac{1}{2},\frac{1}{3})$-semiopen set in $(X,\mathcal{T})$, f is not a fuzzy $(\frac{1}{2},\frac{1}{3})$-irresolute map.

Remark 3.14 Clearly a fuzzy (r,s)-continuous map is a fuzzy weakly (r,s)-continuous map for each $(r,s) \in I \otimes I$. That the converse need not be true is shown by the following example. Also, the following example shows that a fuzzy weakly (r,s)-continuous map is neither a fuzzy

(r,s)-irresolute map nor a fuzzy almost (r,s)-continuous map for each $(r,s) \in I \otimes I$.

Example 3.15 Let $X = \{x,y,z\}$ and let A_1, A_2 and B be intuitionistic fuzzy sets in X defined as

$$A_1(x) = (0.4, 0.3),\ \ A_1(y) = (0.4, 0.4),\ \ A_1(z) = (0.1, 0.5);$$

$$A_2(x) = (0, 0.5),\ \ A_2(y) = (0.3, 0.5),\ \ A_2(z) = (0.1, 0.6);$$

and

$$B(x) = (0.3, 0),\ \ B(y) = (0.4, 0.3),\ \ B(z) = (0.5, 0.2).$$

Define $\mathcal{T} : I(X) \to I \otimes I$ and $\mathcal{U} : I(X) \to I \otimes I$ by

$$\mathcal{T}(A) = (\mathcal{T}_1(A), \mathcal{T}_2(A)) = \begin{cases} (1,0) & \text{if } A = \underline{0}, \underline{1}, \\ (\frac{1}{2}, \frac{1}{3}) & \text{if } A = A_1, \\ (0,1) & \text{otherwise;} \end{cases}$$

and

$$\mathcal{U}(A) = (\mathcal{U}_1(A), \mathcal{U}_2(A)) = \begin{cases} (1,0) & \text{if } A = \underline{0}, \underline{1}, \\ (\frac{1}{2}, \frac{1}{3}) & \text{if } A = A_2, \\ (0,1) & \text{otherwise.} \end{cases}$$

Then clearly $\mathcal{T}$ and $\mathcal{U}$ are SoIFTs on X. Consider a map $f : (X, \mathcal{T}) \to (X, \mathcal{U})$ defined by $f(x) = x$, $f(y) = y$ and $f(z) = z$. Note that

$$f^{-1}(\underline{0}) = \underline{0} \subseteq \text{int}(f^{-1}(\text{cl}(\underline{0}, \frac{1}{2}, \frac{1}{3})), \frac{1}{2}, \frac{1}{3}) = \underline{0},$$

$$f^{-1}(\underline{1}) = \underline{1} \subseteq \text{int}(f^{-1}(\text{cl}(\underline{1}, \frac{1}{2}, \frac{1}{3})), \frac{1}{2}, \frac{1}{3}) = \underline{1},$$

and

$$f^{-1}(A_2) = A_2 \subseteq \text{int}(f^{-1}(\text{cl}(A_2, \frac{1}{2}, \frac{1}{3})), \frac{1}{2}, \frac{1}{3}) = A_1.$$

Hence f is a fuzzy weakly $(\frac{1}{2}, \frac{1}{3})$-continuous map. On the other hand, since $f^{-1}(A_2) = A_2$ is not fuzzy $(\frac{1}{2}, \frac{1}{3})$-open in $(X, \mathcal{T})$, f is not a fuzzy $(\frac{1}{2}, \frac{1}{3})$-continuous map. It is easy to see that B is a fuzzy $(\frac{1}{2}, \frac{1}{3})$-semiopen set in $(X, \mathcal{U})$. But since $f^{-1}(B) = B$ is not a fuzzy $(\frac{1}{2}, \frac{1}{3})$-semiopen set in $(X, \mathcal{T})$, f is not fuzzy $(\frac{1}{2}, \frac{1}{3})$-irresolute. Since $\text{int}(\text{cl}(A_2, \frac{1}{2}, \frac{1}{3}), \frac{1}{2}, \frac{1}{3}) = A_2$, A_2 is a fuzzy $(\frac{1}{2}, \frac{1}{3})$-regular open set in $(X, \mathcal{U})$. But since $f^{-1}(A_2) = A_2$ is not fuzzy $(\frac{1}{2}, \frac{1}{3})$-open in $(X, \mathcal{T})$, f is not a fuzzy almost $(\frac{1}{2}, \frac{1}{3})$-continuous map.

The following example shows that a fuzzy (r,s)-irresolute map is neither a fuzzy (r,s)-continuous map nor a fuzzy weakly (r,s)-continuous map for each $(r,s) \in I \otimes I$. Also, the example shows that a fuzzy (r,s)-irresolute map need not be a fuzzy almost (r,s)-continuous map for each $(r,s) \in I \otimes I$.

Example 3.16 Let $X = \{x,y,z\}$ and let A_1 and A_2 be intuitionistic fuzzy sets in X defined as

$$A_1(x) = (0, 1),\ \ A_1(y) = (0.2, 0.7),\ \ A_1(z) = (0.1, 0.7);$$

and

$$A_2(x) = (0, 1),\ \ A_2(y) = (0.3, 0.7),\ \ A_2(z) = (0.1, 0.7).$$

Define $\mathcal{T} : I(X) \to I \otimes I$ and $\mathcal{U} : I(X) \to I \otimes I$ by

$$\mathcal{T}(A) = (\mathcal{T}_1(A), \mathcal{T}_2(A)) = \begin{cases} (1,0) & \text{if } A = \underline{0}, \underline{1}, \\ (\frac{1}{2}, \frac{1}{3}) & \text{if } A = A_1, \\ (0,1) & \text{otherwise;} \end{cases}$$

and

$$\mathcal{U}(A) = (\mathcal{U}_1(A), \mathcal{U}_2(A)) = \begin{cases} (1,0) & \text{if } A = \underline{0}, \underline{1}, \\ (\frac{1}{2}, \frac{1}{3}) & \text{if } A = A_2, \\ (0,1) & \text{otherwise.} \end{cases}$$

Then clearly $\mathcal{T}$ and $\mathcal{U}$ are SoIFTs on X. Consider a map $f : (X, \mathcal{T}) \to (X, \mathcal{U})$ defined by $f(x) = x$, $f(y) = y$ and $f(z) = z$. Then it is easy to see that f is a fuzzy $(\frac{1}{2}, \frac{1}{3})$-irresolute map. Since $f^{-1}(A_2) = A_2$ is not fuzzy $(\frac{1}{2}, \frac{1}{3})$-open in $(X, \mathcal{T})$, f is not a fuzzy $(\frac{1}{2}, \frac{1}{3})$-continuous map. Since $f^{-1}(A_2) = A_2 \not\subseteq \text{int}(f^{-1}(\text{cl}(A_2, \frac{1}{2}, \frac{1}{3})), \frac{1}{2}, \frac{1}{3}) = A_1$, f is not a fuzzy weakly $(\frac{1}{2}, \frac{1}{3})$-continuous map. Since $\text{int}(\text{cl}(A_2, \frac{1}{2}, \frac{1}{3}), \frac{1}{2}, \frac{1}{3}) = A_2$, A_2 is a fuzzy $(\frac{1}{2}, \frac{1}{3})$-regular open set in $(X, \mathcal{U})$. But $f^{-1}(A_2) = A_2$ is not a fuzzy $(\frac{1}{2}, \frac{1}{3})$-open set in $(X, \mathcal{T})$. Hence f is not a fuzzy almost $(\frac{1}{2}, \frac{1}{3})$-continuous map.

In view of Example 3.12, Example 3.13, Example 3.15, and Example 3.16, we have the following result.

Theorem 3.17 (1) Fuzzy (r,s)-irresolute maps and fuzzy (r,s)-continuous maps are independent notions.
(2) Fuzzy (r,s)-irresolute maps and fuzzy almost (r,s)-continuous maps are independent notions.
(3) Fuzzy (r,s)-irresolute maps and fuzzy weakly (r,s)-continuous maps are independent notions.

Remark 3.18 It is clear that every fuzzy (r,s)-presemiopen map is a fuzzy (r,s)-semiopen map for each $(r,s) \in I \otimes I$. However, the converse may be false as shown by the following example.

Example 3.19 Let $X = \{x,y,z\}$ and let A_1, A_2 and B be intuitionistic fuzzy sets in X defined as

$$A_1(x) = (1, 0),\ \ A_1(y) = (0.3, 0.5),\ \ A_1(z) = (0.1, 0.5);$$

$$A_2(x) = (0, 1),\ \ A_2(y) = (0.3, 0.5),\ \ A_2(z) = (0.1, 0.5);$$

and

$$B(x) = (1, 0),\ \ B(y) = (0.4, 0.2),\ \ B(z) = (0.1, 0.1).$$

Define $\mathcal{T} : I(X) \to I \otimes I$ and $\mathcal{U} : I(X) \to I \otimes I$ by

$$\mathcal{T}(A) = (\mathcal{T}_1(A), \mathcal{T}_2(A)) = \begin{cases} (1,0) & \text{if } A = \underline{0}, \underline{1}, \\ (\frac{1}{2}, \frac{1}{3}) & \text{if } A = A_1, \\ (0,1) & \text{otherwise;} \end{cases}$$

and

$$\mathcal{U}(A) = (\mathcal{U}_1(A), \mathcal{U}_2(A)) = \begin{cases} (1,0) & \text{if } A = \underline{0}, \underline{1}, \\ (\frac{1}{2}, \frac{1}{3}) & \text{if } A = A_2, \\ (0,1) & \text{otherwise.} \end{cases}$$

Then clearly $\mathcal{T}$ and $\mathcal{U}$ are SoIFTs on X. Consider a map $f : (X,\mathcal{T}) \to (X,\mathcal{U})$ defined by $f(x) = x$, $f(y) = y$ and $f(z) = z$. Then it is easy to see that f is a fuzzy $(\frac{1}{2}, \frac{1}{3})$-semiopen map. Obviously, B is a fuzzy $(\frac{1}{2}, \frac{1}{3})$-semiopen set in $(X,\mathcal{T})$. But since $f(B) = B$ is not a fuzzy $(\frac{1}{2}, \frac{1}{3})$-semiopen set in $(X,\mathcal{U})$, f is not fuzzy $(\frac{1}{2}, \frac{1}{3})$-presemiopen map.

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