Bull. Korean Math. Soc. **44** (2007), No. 1, pp. 185–194

ON STABILITY OF THE FUNCTIONAL EQUATIONS HAVING RELATION WITH A MULTIPLICATIVE DERIVATION

EUN HWI LEE, ICK-SOON CHANG, AND YONG-SOO JUNG

ABSTRACT. In this paper we study the Hyers-Ulam-Rassias stability of the functional equations related to a multiplicative derivation.

1. Introduction

In 1940, the stability problem of functional equations has originally been stated by S. M. Ulam [26]. As an answer to the problem of Ulam, D. H. Hyers has proved the stability of the linear functional equation [8] in 1941, which states that if $\delta > 0$ and $f : X \to Y$ is mapping with X, Y Banach spaces, such that

$$\|f(x+y) - f(x) - f(y)\| \le \delta \tag{1.1}$$

for all $x, y \in X$, then there exists a unique additive mapping $T : X \to Y$ such that

$$\|f(x) - T(x)\| \le \delta$$

for all $x, y \in X$.

In such a case, the additive functional equation $f(x+y) = f(x) + f(y)$ is said to have the Hyers-Ulam stability property on (X, Y). This terminology is applied to all kinds of functional equations which have been studied by many authors (for instance, [9]-[11], [17]-[23]).

In 1978, Th. M. Rassias [17] succeeded in generalizing the Hyers' result by weakening the condition for the bound of the left side of the inequality (1.1). Due to the fact, the additive functional equation $f(x+y) = f(x) + f(y)$ is said to have the Hyers-Ulam-Rassias stability property on (X, Y). Since then, a number of results concerning the stability of different functional equations can be found in [3, 4, 5, 7, 9, 11, 14, 17].

Received September 20, 2006.
2000 *Mathematics Subject Classification.* 39B52, 39B72.
Key words and phrases. Hyers-Ulam-Rassias stability, multiplicative (Jordan) derivation.

We now consider functional equations which define multiplicative derivations and multiplicative Jordan derivations in algebras:

$$d(xy) = xd(y) + yd(x), \tag{1.2}$$

$$g(x^2) = 2xg(x). \tag{1.3}$$

It is immediate to observed that the real-valued function $f(x) = x \ln x$ is a solution of the functional equations (1.2) and (1.3).

During the *34-th International Symposium on Functional Equations*, Gy. Maksa [1] posed the Hyers-Ulam stability problem for the functional equation (1.2) on the interval (0,1]. The first result concerning the superstability of this equation for functions between operator algebras was obtained by P. Šemrl [24]. On the other hand, Zs. Páles [16] remarked that the functional equation (1.2) for real-valued functions on $[1, \infty)$ is stable in the sense of Hyers and Ulam. In 1997, C. Borelli [2] demonstrated the stability of the equation (1.2). In particular, J. Tabor gave an answer to the question of Maksa in [25].

Here we introduce the next functional equation due to the functional equation (1.3):

$$h(rx^2 + 2x) = 2rxh(x) + 2h(x), \tag{1.4}$$

where r is a nonzero real number, and consider the following functional equation motivated by the functional equation (1.2):

$$h(x + y + rxy) = h(x) + h(y) + rxh(y) + ryh(x), \tag{1.5}$$

where r is a nonzero real number.

The purpose of this paper is to solve the functional equation (1.4), (1.5) and investigate the Hyers-Ulam-Rassias stability of the functional equation (1.4), (1.5), respectively.

2. Stability of Eq. (1.4) and Eq. (1.5)

It is easy to see that the real-valued function $f(x) = (rx+1)\ln(rx+1)$, where r is a nonzero real number, is a solution of the functional equation (1.4) on the interval. Now we are ready to find out the general solution of the functional equation (1.4).

Theorem 2.1. *Let X be a real (complex) vector space and $r > 0$. A function $h : (-\frac{1}{r}, \infty) \to X$ satisfies the functional equation* (1.4) *for all $x \in (-\frac{1}{r}, \infty)$ if and only if there exists a solution $G : (0, \infty) \to X$ of the functional equation* (1.3) *such that*

$$h(x) = G(rx + 1)$$

for all $x \in (-\frac{1}{r}, \infty)$.

Proof. Assume that a function $h : (-\frac{1}{r}, \infty) \to X$ satisfies (1.4) for all $x \in (-\frac{1}{r}, \infty)$. Then we can define the mapping $G : (0, \infty) \to X$ by $G(x) = h(\frac{x-1}{r})$.

So we get

$$\begin{aligned} G(x^2) &= h\Big(\frac{x^2-1}{r}\Big) = h\Big(r\Big(\frac{x-1}{r}\Big)^2 + 2\Big(\frac{x-1}{r}\Big)\Big) \\ &= 2r\Big(\frac{x-1}{r}\Big)h\Big(\frac{x-1}{r}\Big) + 2h\Big(\frac{x-1}{r}\Big) \\ &= 2xG(x) \end{aligned}$$

for all $x \in (0,\infty)$. Therefore G is a solution of the functional equation (1.3), as desired, and $h(x) = G(rx+1)$ for all $x \in (-\frac{1}{r},\infty)$.

The converse is obvious. □

We here present the general solution of the functional equation (1.5).

Theorem 2.2. *Let X be a real (complex) vector space and $r > 0$. A function $h : (-\frac{1}{r},\infty) \to X$ satisfies the functional equation (1.5) for all $x \in (-\frac{1}{r},\infty)$ if and only if there exists a solution $D : (0,\infty) \to X$ of the functional equation (1.2) such that*

$$h(x) = D(rx+1)$$

for all $x \in (-\frac{1}{r},\infty)$.

Proof. The arguments used in Theorem 2.1 carry over almost verbatim. □

In particular, the previous two theorems hold for the case $r < 0$. Throughout this paper, $\mathbb{R}^+$ denotes the set of all nonnegative real numbers and X a real Banach space with the norm $|\ |$.

Theorem 2.3. [15, Theorem 2.1] *Let $f : [c,\infty) \to X$ be a given function for some $c \geq 1$ and let $\varphi : [c,\infty) \to \mathbb{R}^+$ be a function such that*

$$|f(x^2) - 2xf(x)| \leq \varphi(x) \tag{2.1}$$

for all $x \in [c,\infty)$. If the series $\sum_{i=1}^{\infty} 2^{-i}\varphi(x^{2^{i-1}})$ converges, then there exists a unique solution $g : [c,\infty) \to X$ of equation (1.3) such that

$$|f(x) - g(x)| \leq \sum_{i=1}^{\infty} 2^{-i}\varphi(x^{2^{i-1}}) \tag{2.2}$$

for all $x \in [c,\infty)$.

Theorem 2.4. *Let $f : [0,\infty) \to X$ be a given function and $r > 0$. Assume that $\varphi : [0,\infty) \to \mathbb{R}^+$ is a function such that*

$$|f(rx^2+2x) - 2rxf(x) - 2f(x)| \leq \varphi(x) \tag{2.3}$$

for all $x \in [0,\infty)$. If the series $\sum_{i=1}^{\infty} 2^{-i}\varphi(\frac{(rx+1)^{2^{i-1}}-1}{r})$ converges, then there exists a unique solution $h : [0,\infty) \to X$ of equation (1.4) such that

$$|f(x) - h(x)| \leq \sum_{i=1}^{\infty} 2^{-i}\varphi\Big(\frac{(rx+1)^{2^{i-1}}-1}{r}\Big) \tag{2.4}$$

for all $x \in [0,\infty)$.

Proof. Now put $x = \frac{t-1}{r}$ in (2.3) to obtain

$$\left|f\Big(\frac{t^2-1}{r}\Big) - 2tf\Big(\frac{t-1}{r}\Big)\right| \le \varphi\Big(\frac{t-1}{r}\Big).$$

Let us define functions $e, \psi : [1,\infty) \to X$ by

$$e(t) = f\Big(\frac{t-1}{r}\Big),\ \psi(t) = \varphi\Big(\frac{t-1}{r}\Big).$$

Then, by Theorem 2.3, there exists a unique solution $g : [1,\infty) \to X$ of equation (1.3) such that

$$\left|e(t) - g(t)\right| \le \sum_{i=1}^{\infty} 2^{-i}\psi(t^{2^{i-1}})$$

for all $t \in [1,\infty)$. Since $t = rx + 1$, we have

$$\left|f(x) - g(rx+1)\right| \le \sum_{i=1}^{\infty} 2^{-i}\varphi\Big(\frac{(rx+1)^{2^{i-1}} - 1}{r}\Big).$$

Hence we can define a function $h : [0,\infty) \to X$ by $h(x) = g(rx+1)$, and so

$$\begin{aligned} h(rx^2+2x) &= g((rx+1)^2) = 2(rx+1)g(rx+1) \\ &= 2rxg(rx+1) + 2g(rx+1) = 2rxh(x) + 2h(x). \end{aligned}$$

The proof of the theorem is complete. □

The following two corollaries are immediate consequences of Theorem 2.1.

Corollary 2.5. *Let $f : [0,\infty) \to \mathbb{R}$ be a given function and $r > 0$. Assume that $\Delta : [0,\infty)^2 \to \mathbb{R}^+$ is a function such that for any $x, y \in [0,\infty)$,*

$$(2.5) \qquad |f(x+y+rxy) - f(x) - f(y) - rxf(y) - ryf(x)| \le \Delta(x,y).$$

If the series

$$\sum_{i=1}^{\infty} 2^{-i}\Delta\Big(\frac{(rx+1)^{2^{i-1}} - 1}{r}, \frac{(rx+1)^{2^{i-1}} - 1}{r}\Big)$$

converges and

$$2^{-n}\Delta\Big(\frac{(rx+1)^{2^n} - 1}{r}, \frac{(ry+1)^{2^n} - 1}{r}\Big)$$

converges to zero for all $x \in [0,\infty)$ then there exists a unique solution $h : [0,\infty) \to \mathbb{R}$ of equation (1.5) such that

$$(2.6) \qquad \left|f(x) - h(x)\right| \le \sum_{i=1}^{\infty} 2^{-i}\Delta\Big(\frac{(rx+1)^{2^{i-1}} - 1}{r}, \frac{(rx+1)^{2^{i-1}} - 1}{r}\Big)$$

for all $x \in [0,\infty)$.

Proof. For $x = y$ in (2.5), we have

$$|f(rx^2 + 2x) - 2rxf(x) - 2f(x)| \le \Delta(x, x).$$

Putting $\varphi(x) = \Delta(x, x)$ and applying Theorem 2.4, one obtains

$$h(x) = g(rx + 1) = \lim_{n\to\infty} \frac{f(\frac{(rx+1)^{2^n}-1}{r})}{2^n(rx + 1)^{2^n-1}}$$

satisfying (2.6). We claim that h satisfies

$$h(x + y + rxy) = h(x) + h(y) + rxh(y) + ryh(x).$$

Note that

$$f\Big(\frac{(rx+1)^{2^n}(ry+1)^{2^n}-1}{r}\Big) = f\Big(\frac{(rx+1)^{2^n}-1}{r} + \frac{(ry+1)^{2^n}-1}{r} + r\cdot\frac{(rx+1)^{2^n}-1}{r}\cdot\frac{(ry+1)^{2^n}-1}{r}\Big). \tag{2.7}$$

In the inequality (2.5), replace x by $\frac{(rx+1)^{2^n}-1}{r}$, y by $\frac{(ry+1)^{2^n}-1}{r}$ and consider the equality (2.7) to find that

(2.8)

$$\begin{aligned} &\Big|f\Big(\frac{(rx+1)^{2^n}(ry+1)^{2^n}-1}{r}\Big) - (ry+1)^{2^n-1}f\Big(\frac{(rx+1)^{2^n}-1}{r}\Big) \\ &-(rx+1)^{2^n-1}f\Big(\frac{(ry+1)^{2^n}-1}{r}\Big) - rx(rx+1)^{2^n-1}f\Big(\frac{(ry+1)^{2^n}-1}{r}\Big) \\ &-ry(ry+1)^{2^n-1}f\Big(\frac{(rx+1)^{2^n}-1}{r}\Big)\Big| \\ \le\ &\Delta\Big(\frac{(rx+1)^{2^n}-1}{r}, \frac{(ry+1)^{2^n}-1}{r}\Big). \end{aligned}$$

Now if we divide the inequality (2.8) by $2^n(rx+1)^{2^n-1}(ry+1)^{2^n-1}$, then, since

$$\frac{1}{2^n(rx+1)^{2^n-1}(ry+1)^{2^n-1}} \le 1,$$

we get

$$\begin{aligned} &\Big|\frac{1}{2^n(rx+1)^{2^n-1}(ry+1)^{2^n-1}} f\Big(\frac{(rx+1)^{2^n}(ry+1)^{2^n}-1}{r}\Big) \\ &-\frac{1}{2^n(rx+1)^{2^n-1}} f\Big(\frac{(rx+1)^{2^n}-1}{r}\Big) - \frac{1}{2^n(ry+1)^{2^n-1}} \end{aligned}$$

$$\cdot f\Big(\frac{(ry+1)^{2^n}-1}{r}\Big) - \frac{rx}{2^n(ry+1)^{2^n-1}}f\Big(\frac{(ry+1)^{2^n}-1}{r}\Big)$$
$$-\frac{ry}{2^n(rx+1)^{2^n-1}}f\Big(\frac{(rx+1)^{2^n}-1}{r}\Big)\Big|$$
$$\leq 2^{-n}\Delta\Big(\frac{(rx+1)^{2^n}-1}{r}, \frac{(ry+1)^{2^n}-1}{r}\Big).$$

Taking the limit in the last inequality as $n \to \infty$, we have

$$h(x+y+rxy) - h(x) - h(y) - rxh(y) - ryh(x) = 0.$$

The proof of the corollary is complete. □

Corollary 2.6. *Let $f : [0,\infty) \to X$ be a given function such that for some $r > 0$, $\theta \geq 0$ and p, $q \leq 0$,*

$$|f(x+y+rxy) - f(x) - f(y) - rxf(y) - ryf(x)| \leq \theta(x^p + y^q) \tag{2.9}$$

for all $x, y \in [0,\infty)$. Then there exists a unique solution $h : [0,\infty) \to X$ of equation (1.5) *such that*

$$|f(x) - h(x)| \leq \sum_{i=1}^{\infty} 2^{-i}\theta\Big[\Big(\frac{(rx+1)^{2^{i-1}}-1}{r}\Big)^p + \Big(\frac{(rx+1)^{2^{i-1}}-1}{r}\Big)^q\Big] \tag{2.10}$$

for all $x \in [0,\infty)$.

Proof. Setting $\Delta(x,y) = \theta(x^p + y^q)$ in the previous Corollary 2.5, we can obtain the desired result. □

Theorem 2.7. [15, Theorem 2.5] *Let $f : (0,1] \to X$ be a given function and let $\varphi : (0,1] \to \mathbb{R}^+$ be a function satisfying*

$$|f(x^2) - 2xf(x)| \leq \varphi(x)$$

for all $x \in (0,1]$. If the series $\sum_{i=0}^{\infty} 2^i\varphi(x^{2^{-i-1}})$ converges, then there exists a unique solution $h : (0,1] \to X$ of the equation (1.3) *such that*

$$|f(x) - h(x)| \leq \sum_{i=0}^{\infty} 2^i\varphi(x^{2^{-i-1}}) \tag{2.11}$$

for all $x \in (0,1]$.

Theorem 2.8. *Let $f : (-1/r, 0] \to X$ be a given function and let $\varphi : (-1/r, 0] \to \mathbb{R}^+$ be a function satisfying for some $r > 0$,*

$$|f(rx^2 + 2x) - 2rxf(x) - 2f(x)| \leq \varphi(x) \tag{2.12}$$

for all $x \in (-1/r, 0]$. If the series $\sum_{i=1}^{\infty} 2^i\varphi\big(\frac{(rx+1)^{2^{-i-1}}-1}{r}\big)$ converges, then there exists a unique solution $h : (-1/r, 0] \to X$ of equation (1.4) *such that*

$$|f(x) - h(x)| \leq \sum_{i=0}^{\infty} 2^i\varphi\Big(\frac{(rx+1)^{2^{-i-1}}-1}{r}\Big) \tag{2.13}$$

for all $x \in (-1/r, 0]$.

Proof. As the proof of Theorem 2.4, if we set $t = rx+1$ in (2.12), then we have

$$\Big|f\Big(\frac{t^2-1}{r}\Big) - 2tf\Big(\frac{t-1}{r}\Big)\Big)\Big| \le \varphi\Big(\frac{t-1}{r}\Big).$$

Define $e, \psi : (0,1] \to X$ by

$$e(t) = f\Big(\frac{t-1}{r}\Big),\ \psi(t) = \varphi\Big(\frac{t-1}{r}\Big).$$

Then, by Theorem 2.7, there exists a unique solution $d : (0,1] \to X$ of the equation (1.3) such that

$$|e(t) - d(t)| \le \sum_{i=0}^{\infty} 2^i \varphi\Big(\frac{t^{2^{-i-1}} - 1}{r}\Big),$$

where

$$d(t) = \lim_{n\to\infty} 2^n t^{1-2^{-n}} f\Big(\frac{t^{2^{-n}} - 1}{r}\Big).$$

Since $e(t) = f(\frac{t-1}{r})$ and $t = rx+1$,

$$|f(x) - d(rx+1)| \le \sum_{i=0}^{\infty} 2^i \varphi\Big(\frac{(rx+1)^{2^{-i-1}} - 1}{r}\Big).$$

Now we can define $h : (-1/r, 0] \to X$ by $h(x) = d(rx+1)$. Then

$$\begin{aligned} h(rx^2+2x) &= d((rx+1)^2) = 2(rx+1)d(rx+1) \\ &= 2rxd(rx+1) + 2d(rx+1) = 2rxh(x) + 2h(x), \end{aligned}$$

which completes the proof. □

Corollary 2.9. *Let* $f : (-1/r, 0] \to \mathbb{R}$ *be a given function and let* $\Delta : (-1/r, 0]^2 \to \mathbb{R}^+$ *be a function satisfying for some* $r > 0$,

(2.14) $$|f(x+y+rxy) - f(x) - f(y) - rxf(y) - ryf(x)| \le \Delta(x,y)$$

for all $x, y \in (-1/r, 0]$. *If the series*

$$\sum_{i=1}^{\infty} 2^i \Delta\Big(\frac{(rx+1)^{2^{-i-1}} - 1}{r}, \frac{(ry+1)^{2^{-i-1}} - 1}{r}\Big)$$

converges and

$$2^n \Delta\Big(\frac{(rx+1)^{2^{-i-1}} - 1}{r}, \frac{(ry+1)^{2^{-i-1}} - 1}{r}\Big)$$

converges to zero, then there exists a unique solution $h : (-1/r, 0] \to \mathbb{R}$ *of equation* (1.5) *such that*

(2.15) $$\big|f(x) - h(x)\big| \le \sum_{i=1}^{\infty} 2^i \Delta\Big(\frac{(rx+1)^{2^{-i-1}} - 1}{r}, \frac{(ry+1)^{2^{-i-1}} - 1}{r}\Big)$$

for all $x \in (-1/r, 0]$.

Proof. For $y = x$ in (2.14), we have

$$|f(rx^2 + 2x) - 2rxf(x) - 2f(x)| \leq \Delta(x, x).$$

Putting $\varphi(x) = \Delta(x, x)$ and applying Theorem 2.8, one obtains

$$h(x) = \lim_{n\to\infty} 2^n (rx+1)^{1-2^{-n}} f\Big(\frac{(rx+1)^{2^{-n}} - 1}{r}\Big),$$

which satisfies (2.15). We claim that h satisfies

$$h(x + y + rxy) = h(x) + h(y) + rxh(y) + ryh(x).$$

Observed that

$$f\Big(\frac{(rx+1)^{2^{-n}}(ry+1)^{2^{-n}} - 1}{r}\Big) = f\Big(\frac{(rx+1)^{2^{-n}} - 1}{r} + \frac{(ry+1)^{2^{-n}} - 1}{r} + r \cdot \frac{(rx+1)^{2^{-n}} - 1}{r} \cdot \frac{(ry+1)^{2^{-n}} - 1}{r}\Big).$$

Now replacing x and y by $\frac{(rx+1)^{2^{-n}}-1}{r}$ and $\frac{(ry+1)^{2^{-n}}-1}{r}$ in (2.14), then

$$\Big|f\Big(\frac{(rx+1)^{2^{-n}}(ry+1)^{2^{-n}} - 1}{r}\Big) - (rx+1)^{2^{-n}-1} \cdot f\Big(\frac{(ry+1)^{2^{-n}} - 1}{r}\Big) - (ry+1)^{2^{-n}-1} f\Big(\frac{(rx+1)^{2^{-n}} - 1}{r}\Big)$$
$$-rx(rx+1)^{2^{-n}-1} f\Big(\frac{(ry+1)^{2^{-n}} - 1}{r}\Big) - ry(ry+1)^{2^{-n}-1} \cdot f\Big(\frac{(rx+1)^{2^{-n}} - 1}{r}\Big)\Big| \leq \Delta\Big(\frac{(rx+1)^{2^{-n}} - 1}{r}, \frac{(ry+1)^{2^{-n}} - 1}{r}\Big).$$

Multiplying in the last inequality by $2^n(rx+1)^{1-2^{-n}}(ry+1)^{1-2^{-n}} (\leq 1)$, we have

$$\Big|2^n(rx+1)^{1-2^{-n}}(ry+1)^{1-2^{-n}} f\Big(\frac{(rx+1)^{2^{-n}}(ry+1)^{2^{-n}} - 1}{r}\Big)$$
$$-2^n(ry+1)^{1-2^{-n}} f\Big(\frac{(ry+1)^{2^{-n}} - 1}{r}\Big) - 2^n(rx+1)^{1-2^{-n}} \cdot f\Big(\frac{(rx+1)^{2^{-n}} - 1}{r}\Big) - 2^n rx(ry+1)^{1-2^{-n}} f\Big(\frac{(ry+1)^{2^{-n}} - 1}{r}\Big)$$
$$-2^n ry(rx+1)^{1-2^{-n}} f\Big(\frac{(rx+1)^{2^{-n}} - 1}{r}\Big)\Big|$$
$$\leq 2^n \Delta\Big(\frac{(rx+1)^{2^{-n}} - 1}{r}, \frac{(ry+1)^{2^{-n}} - 1}{r}\Big).$$

Taking the limit in the last inequality as $n \to \infty$, one obtains

$$h(x + y + rxy) - h(x) - h(y) - rxh(y) - ryh(x) = 0.$$

This completes the proof of the theorem. □

Example 1. For some θ, $p \le 0$, let

$$f(x) = (rx+1)\ln(rx+1) + \theta(rx+1)^{p-1}, \; x \le 0, \; r > 0.$$

Note that

$$|f(rx^2+2x) - 2rxf(x) - 2f(x)| = \theta[2(rx+1)^p - (rx+1)^{2(p-1)}].$$

In Theorem 2.4 setting $\varphi(x) = \theta[2(rx+1)^p - (rx+1)^{2(p-1)}]$, we obtain the desired mapping $h(x) = (rx+1)\ln(rx+1)$ satisfying (1.4).

Example 2. Consider

$$f(x) = (rx+1)\ln(rx+1) + (\ln(rx+1))^2, \; -\frac{1}{r} < x \le 0, \; r > 0.$$

Then

$$|f(rx^2+2x) - 2rxf(x) - 2f(x)| = 2(\ln(rx+1))^2 - 2rx(\ln(rx+1))^2.$$

Taking $\varphi(x) = 2(\ln(rx+1))^2 - 2rx(\ln(rx+1))^2$ in Theorem 2.8, we have the desired mapping $h(x) = (rx+1)\ln(rx+1)$ satisfying (1.4).

Acknowledgement. The authors would like to thank referees for their valuable comments. The second author dedicates this paper to his late father.

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Eun Hwi Lee
Department of Mathematics
Jeonju University
Jeonju 302-729, Korea
E-mail address: `ehl@jj.ac.kr`

Ick-Soon Chang
Department of Mathematics
Mokwon University
Taejon 302-729, Korea
E-mail address: `ischang@mokwon.ac.kr`

Yong-Soo Jung
Department of Mathematics
Chungnam National University
Taejon 305-764, Korea
E-mail address: `ysjung@math.cnu.ac.kr`