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A GENERALIZATION OF INSERTION-OF-FACTORS-PROPERTY

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Abstract. We in this note introduce the concept of g-IFP rings which is a generalization of IFP rings. We show that from any IFP ring there can be constructed a right g-IFP ring but not IFP. We also study the basic properties of right g-IFP rings, constructing suitable examples to the situations raised naturally in the process.

1. Introduction

Throughout this paper all rings are associative with identity unless otherwise stated. Given a ring R we use $J(R)$, $N_*(R)$, and $N(R)$ to represent the Jacobson radical, the prime radical (i.e., lower nilradical), and the set of all nilpotent elements in R, respectively; and $r_R(-)$ ($l_R(-)$) is used for the right (left) annihilator over R, i.e., $r_R(S) = \{a \in R \mid sa = 0 \text{ for all } s \in S\}$ ($l_R(S) = \{b \in R \mid bs = 0 \text{ for all } s \in S\}$), where $S \subseteq R$ or S is a subset of a right (left) R-module. If $S = \{a\}$ then we write $r_R(a)$ ($l_R(a)$) in place of $r_R(\{a\})$ ($l_R(\{a\})$). $a \in R$ is said to be right (left) regular if $r_R(a) = 0$ ($l_R(a) = 0$). $a \in R$ is called a left (right) zero-divisor if $r_R(a) \neq 0$ ($l_R(a) \neq 0$). A zero-divisor means an element that is neither right nor left regular.

In a commutative ring the set of nilpotent elements forms an ideal that coincides with the prime radical with the help of [7, Proposition 3.2.1]. This property is also possessed by certain noncommutative rings, which are called 2-primal. Shin [11, Proposition 1.11] proved that given a ring R, $N_*(R) = N(R)$ if and only if every minimal prime ideal P of R is completely prime (i.e., R/P is a domain): Birkenmeier et al. [2] called such rings *2-primal*; while Hirano [5] used the term *N-ring* for the concept.

A well-known property between "commutative" and "2-primal" is the *insertion-of-factors-property* (or simply *IFP*) due to Bell [1]; A right (or left) ideal I of a ring R is said to have the *IFP* if $ab \in I$ implies $aRb \subseteq I$ for $a, b \in R$. A ring R is called *IFP* if the zero ideal of R has the IFP. Shin [11] used the term *SI* for the IFP; while Habeb [4] used the term *zero insertive* (or simply *zi*) for it, in the study of QF-3 rings. IFP rings are also known as *semicommutative* in

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Narbonne's paper [10]. Shin proved that IFP rings are 2-primal [11, Theorem 1.5]. A ring is called *reduced* if it has no nonzero nilpotent elements. It is trivial to check that reduced rings are IFP, whence the IFP condition is also between "reduced" and "2-primal". It is trivial that subrings of IFP rings are also IFP, so we use this fact freely in this note.

We in this note introduce another generalization of the IFP condition that is different from the 2-primal condition. We in this note call a ring R *right generalized IFP* (or simply, *right g-IFP*) provided that there is $0 \neq b' \in R$ with $aRb' = 0$ whenever $ab = 0$ for $a, b \in R$ with $b \neq 0$. The left g-IFP ring can be defined symmetrically. A ring is called *g-IFP* if it is both left and right g-IFP.

2. Basic structure and examples of right g-IFP rings

In this section we observe the ring-theoretic properties of g-IFP rings, and relationship between g-IFP rings and concerned concepts. Denote the set of all left (right) zero-divisors in a ring R by $zd_l(R)$ ($zd_r(R)$). We start with the following lemma.

Lemma 2.1. *For a ring R the following conditions are equivalent:*

(1) *R is right g-IFP;*

(2) *$r_R(a)$ contains a nonzero ideal of R for each $a \in zd_l(R)$;*

(3) *$r_R(aR) \neq 0$ for each $a \in zd_l(R)$.*

Proof. (1)⇒(2): $a \in zd_l(R)$ implies $r_R(a) \neq 0$, so $r_R(a)$ contains a nonzero left ideal of R, say Rb, if R is right g-IFP. Thus $0 = aRb = aRbR$ and $RbR \subseteq r_R(a)$.

(2)⇒(3): If $r_R(a)$ contains a nonzero ideal I of R then $0 = aI = aRI$ and $I \subseteq r_R(aR)$.

(3)⇒(1): Let $r_R(a) \neq 0$ for $a \in R$. Then we get $r_R(aR) \neq 0$ by the condition, so R is right g-IFP. □

IFP rings are clearly g-IFP but the converse need not hold by the following. The example below also shows that the g-IFP condition is not left-right symmetric. Given a ring R we use $R[x]$ ($R[[x]]$) to denote the polynomial (power series) ring with an indeterminate x over R.

Example 2.2. Let D be a division ring and let $T = D[x]/\langle x^2 \rangle$, where $\langle x^2 \rangle$ is the ideal of $D[x]$ generated by x^2. Write $\delta = x + \langle x^2 \rangle$. Then $T = D \oplus D\delta$ with $\delta^2 = 0$ and each element of the form $a + b\delta$ is invertible when a is nonzero. Now consider the ring $R = \begin{pmatrix} T/D\delta & T/D\delta \\ 0 & T \end{pmatrix}$. Notice that all nonzero proper ideals of R are

$$I_1 = \begin{pmatrix} 0 & T/D\delta \\ 0 & D\delta \end{pmatrix}, I_2 = \begin{pmatrix} 0 & T/D\delta \\ 0 & T \end{pmatrix}, I_3 = \begin{pmatrix} 0 & T/D\delta \\ 0 & 0 \end{pmatrix},$$

$$I_4 = \begin{pmatrix} 0 & 0 \\ 0 & D\delta \end{pmatrix}, I_5 = \begin{pmatrix} T/D\delta & T/D\delta \\ 0 & 0 \end{pmatrix} \text{ and } I_6 = \begin{pmatrix} T/D\delta & T/D\delta \\ 0 & D\delta \end{pmatrix}.$$

It is easily checked that each I_i is the set of zero divisors for all $1 \le i \le 6$. Note that every left zero-divisor of R is contained in I_k for some $k \in \{1, 2, \dots, 6\}$, and that $r_R(I_j) \neq 0$ for all j. Thus R is right g-IFP by Lemma 2.1.

Next we show that R is not left g-IFP. Note $\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} = 0$ and $R\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & T/D\delta \\ 0 & T \end{pmatrix}$. Let $\begin{pmatrix} a & b \\ 0 & c + d\delta \end{pmatrix} \in l_R(\begin{pmatrix} 0 & T/D\delta \\ 0 & T \end{pmatrix})$. From

$$\begin{pmatrix} a & b \\ 0 & c + d\delta \end{pmatrix} \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} = 0,$$

we get $b = 0$ and $c + d\delta = 0$; hence $\begin{pmatrix} a & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} = 0$ forces $a = 0$. Thus $l_R(\begin{pmatrix} 0 & T/D\delta \\ 0 & T \end{pmatrix}) = 0$ and R is not left g-IFP by Lemma 2.1.

A ring is called *abelian* if every idempotent is central. It is trivial to check that IFP rings are abelian, but right or left g-IFP rings need not be abelian by Example 2.2.

A ring R is called *directly finite* if $xy = 1$ implies $yx = 1$ for $x, y \in R$. It is trivial to check that abelian rings are directly finite, and 2-primal rings are also directly finite by [2, Proposition 2.10]. A ring R is called *von Neumann regular* if for each $a \in R$ there exists $x \in R$ such that $a = axa$. Abelian von Neumann regular rings are reduced (hence g-IFP) by [3, Theorem 3.2].

Lemma 2.3. (1) *Right or left g-IFP rings are directly finite.*

(2) *Direct sums (possibly without identity) and direct products of right g-IFP rings are also right g-IFP.*

Proof. (1) Let R be a right g-IFP ring. Assume that $xy = 1$ but $yx \neq 1$ for some $x, y \in R$. Then yx is a non-identity idempotent and $yx(1 - yx) = 0$ with $1 - yx \neq 0$. Since R is right g-IFP, we have $yxRb = 0$ for some nonzero $b \in R$; but $xRb = xyxRb = 0$ implies $0 \neq b = xyb \in xRb = 0$, a contradiction. Thus R is directly finite. The proof of left case is similar.

(2) Suppose that R_i $(i \in I)$ are right g-IFP rings, and let $R = \prod_{i \in I} R_i$ be the direct product of R_i's. Set $ab = 0$ for $a = (a_i), b = (b_i) \in R$. Then $a_i b_i = 0$ for all $i \in I$. Since each R_i is right g-IFP, we get $a_i R_i b_i' = 0$ for some $0 \neq b_i' \in R_i$. Let $b' = (b_i') \in R$, then we have $aRb' = (a_i R_i b_i') = 0$ for some $0 \neq b' \in R$, showing that R is right g-IFP. The case of direct sums is similar. □

Remark. As a byproduct of Lemma 2.3(1) we get that von Neumann regular rings need not be one-sided g-IFP. Let F be a field and R be the column finite infinite matrix ring over F. Note that R is von Neumann regular. Let $a \in R$ be the matrix with $(i, i+1)$-entry 1 and zero elsewhere, and $b \in R$ be the matrix with $(i+1, i)$-entry 1 and zero elsewhere, where $i = 1, 2, \dots$. Then $ab = 1$ but

$ba \neq 1$; hence R is not directly finite. Thus R is neither right nor left g-IFP by Lemma 2.3(1).

Note that IFP rings are both g-IFP and 2-primal, however one of the classes of g-IFP rings and 2-primal rings need not contain the other as can be seen by the following.

Example 2.4. (1) There is a g-IFP ring that is not 2-primal. Let K be a field and $D_n = K\{x_n\}$ with relation $x_n^{n+2} = 0$, where n is any nonnegative integer and $K\{x_n\}$ is the free algebra generated by x_n over K. Note $D_n \cong K[x]/\langle x^{n+2}\rangle$ where $\langle x^{n+2}\rangle$ is the ideal of $K[x]$ generated by x^{n+2}. We use the ring in [6, Example 1.6]. Define $R_n = \begin{pmatrix} D_n & D_n x_n \\ D_n x_n & D_n \end{pmatrix}$. Notice that $J(R_n) = \begin{pmatrix} D_n x_n & D_n x_n \\ D_n x_n & D_n x_n \end{pmatrix}$ and $\frac{R_n}{J(R_n)} \cong \begin{pmatrix} K & 0 \\ 0 & K \end{pmatrix}$; hence $\begin{pmatrix} f_1 & f_2 \\ f_3 & f_4 \end{pmatrix} \in R_n$ is invertible when the constants of f_1 and f_4 are both nonzero.

Now we will show that R_n is g-IFP. Let $0 \neq \begin{pmatrix} f_1 & f_2 \\ f_3 & f_4 \end{pmatrix} \in R_n$ with $f_i \in D_n x_n$ for all i, and say that the smallest degree of nonzero f_i's is k for some k with $1 \leq k < n+2$. Then $\begin{pmatrix} f_1 & f_2 \\ f_3 & f_4 \end{pmatrix} R_n \begin{pmatrix} x_n^{n+2-k} & 0 \\ 0 & x_n^{n+2-k} \end{pmatrix} = 0$ with $\begin{pmatrix} x_n^{n+2-k} & 0 \\ 0 & x_n^{n+2-k} \end{pmatrix} \neq 0$. Let $0 \neq \begin{pmatrix} f_1 & f_2 \\ f_3 & f_4 \end{pmatrix} \in R_n$ with $f_1 \notin D_n x_n$ and $f_i \in D_n x_n$ for $i \in \{2,3,4\}$. Then $\begin{pmatrix} f_1 & f_2 \\ f_3 & f_4 \end{pmatrix} R_n \begin{pmatrix} 0 & 0 \\ 0 & x_n^{n+1} \end{pmatrix} = 0$ because each matrix in $\begin{pmatrix} f_1 & f_2 \\ f_3 & f_4 \end{pmatrix} R_n$ is of the form $\begin{pmatrix} f & h \\ g & k \end{pmatrix}$ with $h, k \in D_n x_n$. Next let $0 \neq \begin{pmatrix} f_1 & f_2 \\ f_3 & f_4 \end{pmatrix} \in R_n$ with $f_4 \notin D_n x_n$ and $f_i \in D_n x_n$ for $i \in \{1,2,3\}$. Then we have $\begin{pmatrix} f_1 & f_2 \\ f_3 & f_4 \end{pmatrix} R_n \begin{pmatrix} x_n^{n+1} & 0 \\ 0 & 0 \end{pmatrix} = 0$ because each matrix in $\begin{pmatrix} f_1 & f_2 \\ f_3 & f_4 \end{pmatrix} R_n$ is of the form $\begin{pmatrix} f & h \\ g & k \end{pmatrix}$ with $f, g \in D_n x_n$.

Thus R_n is right g-IFP. We can also show that R_n is left g-IFP by a similar method.

Next let $R = \prod_{n=0}^{\infty} R_n$. Then R is g-IFP by Lemma 2.3(2) since every R_n is g-IFP. Consider two sequences $(a_n), (b_n) \in R$ such that $a_n = \begin{pmatrix} 0 & x_n \\ 0 & 0 \end{pmatrix}$ and $b_n = \begin{pmatrix} 0 & 0 \\ x_n & 0 \end{pmatrix}$ for all n. Then $(a_n), (b_n) \in N(R)$ since $(a_n)^2 = 0 = (b_n)^2$; but $(a_n) \notin N_*(R)$ or $(b_n) \notin N_*(R)$ since each component of $(a_n) + (b_n)$ is $\begin{pmatrix} 0 & x_n \\ x_n & 0 \end{pmatrix}$ and $(a_n) + (b_n)$ is not nilpotent. Thus R is not 2-primal.

(2) There is a 2-primal ring that is neither right nor left g-IFP. Consider the 2 by 2 upper triangular ring $R = \begin{pmatrix} \mathbb{Z}_2 & \mathbb{Z}_2 \\ 0 & \mathbb{Z}_2 \end{pmatrix}$ over the field $\mathbb{Z}_2$ of integers modulo 2. Consider two proper ideals $I = R\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}R = \begin{pmatrix} \mathbb{Z}_2 & \mathbb{Z}_2 \\ 0 & 0 \end{pmatrix}$ and $J = R\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}R = \begin{pmatrix} 0 & \mathbb{Z}_2 \\ 0 & \mathbb{Z}_2 \end{pmatrix}$ of R, where $\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \in zd_l(R)$ and $\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \in zd_r(R)$. Then every element of I and J is a zero-divisor. However there cannot exist nonzero elements $x, y \in R$ such that $Ix = 0$ and $yJ = 0$. Thus R is neither right nor left g-IFP by Lemma 2.1. But R is 2-primal by [2, Proposition 2.5].

Remark. (1) Subrings of right g-IFP rings need not be right g-IFP. Consider the right g-IFP ring $R = \begin{pmatrix} T/D\delta & T/D\delta \\ 0 & T \end{pmatrix}$ with $T = D \oplus D\delta$ in Example 2.2. Set $D = \mathbb{Z}_2$, the field of integers modulo 2. Then $\begin{pmatrix} \mathbb{Z}_2 & \mathbb{Z}_2 \\ 0 & \mathbb{Z}_2 \end{pmatrix}$ is a subring of R. But $\begin{pmatrix} \mathbb{Z}_2 & \mathbb{Z}_2 \\ 0 & \mathbb{Z}_2 \end{pmatrix}$ is not right g-IFP by Example 2.4(2).

(2) Direct products of 2-primal rings need not be 2-primal by Marks [9] and [8, Example 1.7]. We here can obtain this result as a byproduct of Example 2.4(1). In fact $J(R_n) = P(R_n)$ since $J(R_n)$ is nilpotent, so R_n is 2-primal. But $\prod_{i=0}^{\infty} R_n$ is not 2-primal by Example 2.4(1).

Denote the n by n matrix ring over a ring R by $Mat_n(R)$ for a positive integer n. Let R be a simple ring and $S = Mat_2(R)$. Take $a = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \in S$. Then a is nilpotent, but $SaS = S$ and $r_S(SaS) = 0$, $l_S(SaS) = 0$. Thus S is neither right nor left g-IFP by Lemma 2.1. By a similar manner, $Mat_n(R)$ cannot be neither right nor left g-IFP for all $n \geq 2$.

In the following we find a kind of subring of n by n matrix ring that can be right or left g-IFP. Given a ring R we consider a ring extension

$$R_n = \left\{ \begin{pmatrix} a & a_{12} & a_{13} & \cdots & a_{1n} \\ 0 & a & a_{23} & \cdots & a_{2n} \\ 0 & 0 & a & \cdots & a_{3n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & a \end{pmatrix} \mid a, a_{ij} \in R \right\},$$ where $n(\geq 2)$ is a positive integer. About R_n we have the following useful results:

(i) R_n is IFP for $n \leq 3$ by [6, Proposition 1.2] when R is a reduced ring;

(ii) R_n need not be IFP for $n \geq 2$ by [6, Example 1.3] when R is an IFP ring;

(iii) R_n cannot be IFP for $n \geq 4$ by [6, Example 1.3] over any ring R.

With the help of these results and the following proposition, we can say that from any IFP ring there can be constructed a right g-IFP ring but not IFP.

Proposition 2.5. *A ring R is right g-IFP if and only if R_n over R is right g-IFP for any n.*

Proof. Suppose that R is right g-IFP and let

$$\begin{pmatrix} a & a_{12} & \cdots & a_{1n} \\ 0 & a & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & a \end{pmatrix}\begin{pmatrix} b & b_{12} & \cdots & b_{1n} \\ 0 & b & \cdots & b_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & b \end{pmatrix} = 0 \text{ with } \begin{pmatrix} b & b_{12} & \cdots & b_{1n} \\ 0 & b & \cdots & b_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & b \end{pmatrix} \neq 0$$

in R_n. If $a = 0$ then we have

$$\begin{pmatrix} 0 & a_{12} & \cdots & a_{1n} \\ 0 & 0 & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 0 \end{pmatrix} R_n \begin{pmatrix} 0 & 0 & \cdots & 1 \\ 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 0 \end{pmatrix} = 0.$$

Assume $a \neq 0$. By the condition $aRb' = 0$ for some nonzero $b' \in R$ since $ab = 0$. So

$$\begin{pmatrix} a & a_{12} & \cdots & a_{1n} \\ 0 & a & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & a \end{pmatrix} R_n \begin{pmatrix} 0 & 0 & \cdots & b' \\ 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 & \cdots & aRb' \\ 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 0 \end{pmatrix} = 0.$$

Thus R_n is right g-IFP.

Conversely assume that R_n is right g-IFP and let $ab = 0$ for $a, b \in R$. Then $\begin{pmatrix} a & 0 & \cdots & 0 \\ 0 & a & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & a \end{pmatrix}\begin{pmatrix} b & 0 & \cdots & 0 \\ 0 & b & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & b \end{pmatrix} = 0$ and so we have $AR_nB = 0$ for some nonzero $B \in R_n$ by the condition, where $A = \begin{pmatrix} a & 0 & \cdots & 0 \\ 0 & a & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & a \end{pmatrix}$. Say $B = \begin{pmatrix} b & b_{12} & \cdots & b_{1n} \\ 0 & b & \cdots & b_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & b \end{pmatrix}$. If $b \neq 0$ then $aRb = 0$. Next set $b = 0$ and say that j and k are smallest with respect to the property $b_{jk} \neq 0$. Then since AR_nB contains

$$\begin{pmatrix} 0 & \cdots & 0 & arb_{jk} & \cdots & arb_{jn} \\ 0 & \cdots & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & \cdots & 0 & 0 & \cdots & 0 \end{pmatrix}$$

for all $r \in R$, we also get $aRb_{jk} = 0$ from $AR_nB = 0$. Thus R is right g-IFP. □

We also have that R_n is 2-primal for any n if and only if R is a 2-primal ring, with the help of [2, Propositions 2.2 and 2.5].

Proposition 2.6. *Let R be a local ring with nilpotent $J(R)$. Then R is g-IFP.*

Proof. Let k be smallest with respect to $J(R)^k = 0$. Put $ab = 0$ for $a, b \in R$ with $b \neq 0$. Then $a \in J(R)$ and we get $aJ(R)^{k-1} = 0$, concluding that R is right g-IFP. Similarly R is left g-IFP. □

Proposition 2.7. *Let R be a semiprime right (resp. left) g-IFP ring. Then every left (resp. right) zero-divisor is a zero-divisor.*

Proof. Let $r_R(a) \neq 0$ for $a \in R$. Since R is right g-IFP, there is a nonzero ideal I of R such that $aI = 0$ by Lemma 2.1. Then $(IaR)^2 = IaRIaR = IaIaR = 0$, but R is semiprime and so $Ia = 0$, showing $l_R(a) \neq 0$. The proof of the other case is similar. □

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